FUZZY RECURSIVE SYMBOL-BY-SYMBOL DETECTOR FOR SINGLE USER CDMA RECEIVERS

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Abstract

The Bayesian or maximum a posteriori (MAP) symbol-by-symbol detector allows minimum BER single user detection in CDMA systems with less memory requirements than the multiuser Maximum Likelihood Sequence Estimator. This work proposes a further complexity reduction by developing a fuzzy recursive implementation of the Bayesian detector. Simulation studies demonstrate the almost optimal performance of the developed fuzzy detector. The resulting system offers a performance v.s. complexity trade-off very appealing for detection in a 3rd generation mobile handset.

1.- Introduction

For single user detection in CDMA scenarios, the sheer complexity of the Maximum Likelihood Sequence Estimator (MLSE) prohibites its use. An alternative is the Bayesian symbol-by-symbol detector, which offers optimum single user detection (i.e. optimum in a minimum BER sense) without requiring the demodulation of the other users [1]. The present work focuses on synchronous CDMA reception like the one in downlink from the base station to the mobile handset in 3rd generation mobile systems. Specifically, the aim is to obtain a low complexity single user detector presenting a performance near to the optimal one. For this purpose, fuzzy logic is applied, whose max/min inference suits the decision process reducing the computational complexity.
Firstly, Section 2 presents the signal model. Next, Section 3 formulates the Bayesian detector. Section 4 designs the recursive fuzzy CDMA detector. This section shows that following the algorithm of Hayes et al. [2], the fuzzy detector can be implemented in a similar manner to the familiar Viterbi algorithm trellis for MLSE. Section 5 shows some simulation results and, finally conclusions come.

2.- Signal Model description.

In a block transmission CDMA system each received burst is divides in sets of Nc chips. The signal in each set can be formulated as **r**:

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_c^{(k)} \mathbf{s}_c^{(k)} + \mathbf{w} \qquad (1)$$

Where K is the number of intracell users, $\mathbf{s}_c^{(k)} \in R^{Ncx1}$ is the temporal chip sequence transmitted by user k of length N_c; $\mathbf{H}_c^{k} \in R^{(Nc+L-1)xNc}$ is the channel matrix for user k of length L, $\mathbf{w} \in R^{(Nc+L-1)x1}$ and $\mathbf{r} \in R^{(Nc+L-1)x1}$ are the Gaussian noise and the total received signals respectively. The signal model of Eq. (1) can also be written as follows:

$$\mathbf{r} = \mathbf{H}_c \mathbf{s}_c + \mathbf{w} \qquad (2)$$

where $\mathbf{H}_c = \left[\mathbf{H}_c^{(1)} \cdots \mathbf{H}_c^{(K)} \right]$, and

$$\mathbf{s}_c = \left[\mathbf{s}_c^{(1)} \cdots \mathbf{s}_c^{(K)} \right]^T$$

Assuming that a short spreading code is being used, the transmitted chip sequence by user k can be expressed as $\mathbf{s}_c^{(k)} = \mathbf{C}^{(k)} \mathbf{s}^{(k)}$, where $\mathbf{s}^{(k)} \in R^{Nx1}$ corresponds to the bipolar (±1) transmitted symbol sequence. $\mathbf{C}^{(k)} = \mathbf{I}_N \otimes \mathbf{c}^{(k)}$, being $\mathbf{c}^{(k)} \in R^{Gx1}$ the spreading code of user k with a processing gain of G. Note that $N_c = NG$, where N_c and N represents the

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number of chips and symbols transmitted by each user respectively. Finally, the last notations can be compacted using stacked vectors as $\mathbf{s}_c = \mathbf{C}\mathbf{s}$, where $\mathbf{C} = \mathrm{diag}\{[\mathbf{C}^{(1)} \cdots \mathbf{C}^{(K)}]\}$. Then, the received signal model can be expressed at bit time as:

$$\mathbf{r} = \mathbf{H}_c\mathbf{s}_c + \mathbf{w} = \mathbf{H}_s\mathbf{s} + \mathbf{w} \qquad (3)$$

where $\mathbf{H}_s = \mathbf{H}_c\mathbf{C}$ being $\mathbf{H}_s$ the channel matrix at bit time.

3.- Bayesian detector for CDMA

Given a set of observations collected in the vector $\mathbf{r}_k = [r_{k-G+1} \cdots r_k]^T$, the Bayesian or maximum a posteriori (MAP) symbol-by-symbol decision rule for the symbol transmitted by the desired user 1 at instant k ($s^1(k)$) is to choose the symbol a_p which is most probable given these observations, i.e.:

$$p = \arg\, max_i\; P_{x/\mathbf{r}}(s^1_{k-d} = a_i / \mathbf{r}_k) \qquad (4)$$

where *d* is the delay parameter which is chosen to exceed the signal bitwise memory m=L/G+2. Without loss of generality and in order to make formulation easier *d* = 0. By applying the Bayes rule and assuming binary transmission, the Bayesian decision function is defined as :

$$g_k = f_{\mathbf{r}/x}(\mathbf{r}_k / s^1_k = 1) - f_{\mathbf{r}/x}(\mathbf{r}_k / s^1_k = -1) \qquad (5)$$

where *f(.)* stands for probability density function. For the signal generation process defined by eq.(3), the conditional densities that constitute the decision function are straightforward to evaluate. The set of noise free output states of $\mathbf{c} = \mathbf{H}_s\mathbf{s}$ is partitioned into two sets conditioned on the transmitted symbol of interest: $S^+ = \{\mathbf{c} / s^1{}_k = 1\}$ and $S^- = \{\mathbf{c} / s^1{}_k = -1\}$. Thus, the Bayesian decision function becomes

$$g_k = \sum_{i=1}^{n_s} w_i \exp\left(-\frac{\left\|\mathbf{r}_k - \mathbf{c}_i\right\|^2}{2\sigma_n^2}\right) \qquad (6)$$

where n_s is the number of channel states and w_i are the weights associated with each of the centers: $w_i = 1\; if\; \mathbf{c}_i \in S^+$ and $w_i = -1\; if\; \mathbf{c}_i \in S^-$. To work out the total number of states (n_s) we have to know the length of the channel memory (L). If the memory is less than the processing gain (G), we will always have only one bit of ISI. Consequently $n_s = 2^{(2K)}$. However if the memory is bigger than the processing gain, we will have $n_s = 2^{((L/G+2)K)}$, where L/G is the number of times that the channel memory is bigger than processing gain.

Moreover we have to note that the channel states are vectors because the received data are processed at chip time and we dispense with the matched filter.

4.-Fuzzy implementation of a CDMA receiver.

The non-recursive Bayesian detector has a finite memory. These means that to minimize the expected number of symbol errors it is not used all the available information, and, therefore, we get a sub-optimum value of BER. If we want to improve the detection quality (BER sense) we have to increase the detector memory. However, this solution produces an exponential growing of the memory requirements and consequently, it is computationally unaffordable. One way to overcome this drawback is to seek a recursive algorithm for detecting the symbol s^1_k to avoid the storage of the whole data sequence. In [3][4] the authors have developed recursive algorithms in single-user scenarios with intersymbol interference. In the present work, we have extended their formulation to multi-user environments, and therefore to co-channel interference. The optimal detector in (5) can also be formulated as :

$$g_k = \sum_{s^1_k} \cdots \sum_{s^K_k} \cdots \sum_{s^K_{k-m+1}} f_r\left(r_k \mid s^1_k, \cdots, s^1_{k-m+1}, \cdots, s^K_k, \cdots, s^K_{k-m+1}\right) \qquad (7)$$

The recursive algorithm is then designed based on the fact that recursive expressions for $f_r\left(r_k \mid s^1_k, \cdots, s^1_{k-(m-1)}, \cdots, s^K_k, \cdots, s^K_{k-(m-1)}\right)$ can be derived. Applying the total probability theorem and, using standard probability techniques leads us to the following recursion:

$$f_r\left(\mathbf{r}_k \mid s^1_k, \cdots s^1_{k-(m-1)}, \cdots, s^K_k, \cdots s^K_{k-(m-1)}\right) = \frac{1}{2} f_r\left(\mathbf{r}_k \mid s^1_k, \cdots s^1_{k-m} = -1, \cdots, s^K_k, \cdots s^K_{k-m}\right) + \frac{1}{2} f_r\left(\mathbf{r}_k \mid s^1_k, \cdots s^1_{k-m} = 1, \cdots, s^K_k, \cdots s^K_{k-m}\right) \qquad (8)$$

If we assume that all symbols are known but s_k^1,then the uncertainty of the last sample r_k only comes from the white noise components, and, therefore, it is statistically independent of previous samples r_{k-1}. The recursive expression given in eq.(8) can be written as :

$$\begin{aligned} &f_r\left(r_k \mid s_k^1,\cdots s_{k-m+1}^1,\cdots,s_k^K,\cdots s_{k-m+1}^K\right)= \\ &\frac{1}{2}f_n\left(r_k - H_s\left[s_k^1,\cdots,s_{k-m}^1=-1,\cdots,s_k^K,\cdots,s_{k-m}^K\right]^T\right) \\ &f_r\left(\mathbf{r}_{k-1}\mid s_{k-1}^1,\cdots,s_{k-m}^1=-1,\cdots,s_k^K,\cdots s_{k-m}^K\right) \\ &+\frac{1}{2}f_n\left(r_k - H_s\left[s_k^1,\cdots,s_{k-m}^1=1,\cdots,s_k^K,\cdots,s_{k-m}^K\right]^T\right) \\ &f_r\left(\mathbf{r}_{k-1}\mid s_{k-1}^1,\cdots,s_{k-m}^1=1,\cdots,s_k^K,\cdots s_{k-m}^K\right) \end{aligned} \tag{9}$$

The last equation shows the relationship between the recursive conditional probabilities of the received vector as a function of the previous time sample. Finally, to get the symbol decision we can relate eq. (9) with eq. (7) .For Gaussian noise we obtain:

$$g_k=\sum_{i=1}^{n_s} f_{r_{k-1}}^i \underbrace{\exp\left(-\frac{\left\|\mathbf{r}_k-\mathbf{c}_i\right\|^2}{2\sigma_n^2}\right)}_{f_n(r_k)} \tag{10}$$

Where the symbol decision, has been associated with: the channel states, c_i, the recursive density probability function of last sample, $f_{r_{k-1}}^i$, and the noise probability density f_n.

This last expression is very close to expression (6), but now the weights w_i are the values of the recursive probabilities $f_{r_{k-1}}^i$.Furthermore, in line with the normalized radial basis function derived by Cha et al. [5], we can form a normalized Bayesian detector which forms an estimate of the transmitted symbols themselves rather than a decision function.

$$g_k=\frac{\sum_{i=1}^{n_s} w_i\exp\left(-\frac{\left\|\mathbf{r}_k-\mathbf{c}_i\right\|^2}{2\sigma_n^2}\right)}{\sum_{i=1}^{n_s}\exp\left(-\frac{\left\|\mathbf{r}_k-\mathbf{c}_i\right\|^2}{2\sigma_n^2}\right)} \tag{11}$$

This algorithm allows getting a detector with infinity memory without storing all the data sequence. However, it involves a large amount of computational burden, especially if the number of users and/or their amplitude levels

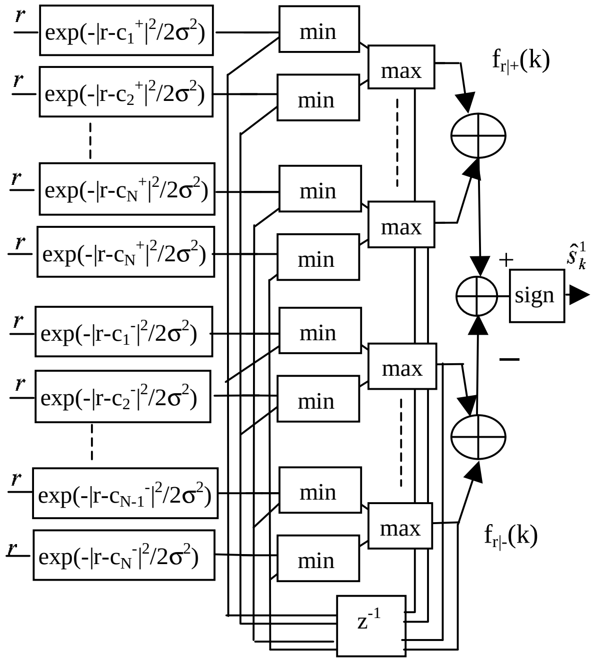

Figure 1. Recursive Fuzzy detector with maximum and minimum inferences.

are large. To reduce this complexity, this paper proposes the fuzzy recursive detector.

As noted by the authors in [6], expression (11) shows the direct relationship between Bayesian and Fuzzy terms. Eq.(11) can be seen as a fuzzy system that uses a singleton fuzzification, product inference, center of gravity (COG) defuzzifier, and Gaussian membership function, which are normalized to range [0 1]. This fact allows to approximate product and sum inferences by minimum and maximum inferences respectively (see Figure 1), which are more easy to calculate and consequently reduce the final computational burden.

Below, the present work contributes to that of [6] by designing a fuzzy detector that operates recursively in order to be able to operate in multiuser CDMA systems that transmit in burst mode.

In Table 1, there is a comparative study of the computational burden of MLSE, Recursive Bayesian and Fuzzy detectors. Note that the recursive fuzzy detector requires less memory and algebraic operations than MLSE or Bayes. In addition, in table 1 have been considered also a possible reduction of the total number of centroids to get a more affordable recursive detector. That reduction has been carried out according the maximums value of the membership function degree that we want to use to estimate the information signal. In [6] it is shown that not all centroids are necessary to get reasonable BER results in Bayes detection.

	MLSE	Bayes	Fuzzy
+	$M((G-1)2^{LK}+1)$	$(G-1)\alpha 2^{mK}+1$	$(G-1)\ \alpha 2^{mK}$
-	$M(G2^{LK}+1)$	$G\alpha 2^{mK}+1$	$G\alpha 2^{mK}$
*	$M(G2^{LK}+1)$	$(G+2)\alpha 2^{mK}$	$\alpha 2^{mK}$
Exp.	------------------	$\alpha 2^{mK}$	$\alpha 2^{mK}$
Max	$M2^{LK}$	--------------	2^{mK-1}
Min	$M2^{LK}$	--------------	2^{mK}
Mem	2^{MK}	2^{mK}	2^{mK-1}

Table 1. Computational burden of studied systems. α=coefficient of centroids reduction. If the value of α is equal to 1, no centroid reduction is applied. If its value is 0.25 or 0.5, the reduction is the 75% and 50% respectively. m=(L/G+2) is the channel memory bitwise length. M is the interval between merges, that is, the number of bits detected at the same time in Viterbi algorithm: $3m \leq M \leq 5m$.

5.- Simulations

Figure 2 shows that the recursive fuzzy detector provides almost the same performance as the optimal Bayes or MLSE detectors. Downlink transmission in the TDD-Mode of UTRA has been considered with 3 equal power users and the standarized indoor channel $[0.9525\ 0.3047]^T$. The channel coefficients are assumed known. Nevertheless, in UMTS the mobile knows the spread spectrum code of all users and supervised clustering algorithms can be applied. Figure 3 shows the results when centroids reduction is carried out for the fuzzy detector. Note that all the centroids are not necessary to get a reasonable BER. Specifically, a reduction lower than 50% is possible without performance degradation.

6. Conclusions

This work presents a fuzzy recursive implementation of the Bayesian detector. Simulations compare this detector with the optimal Bayesian and MLSE ones and demonstrate the good performance and low complexity of the developed detector for multiuser UMTS channels in downlink. Further studies are to be carried out for unknown channels. Additionally, theoretical analysis based on robust statistics are in progress to study the performance of the max/min detector for non-Gaussian environments.

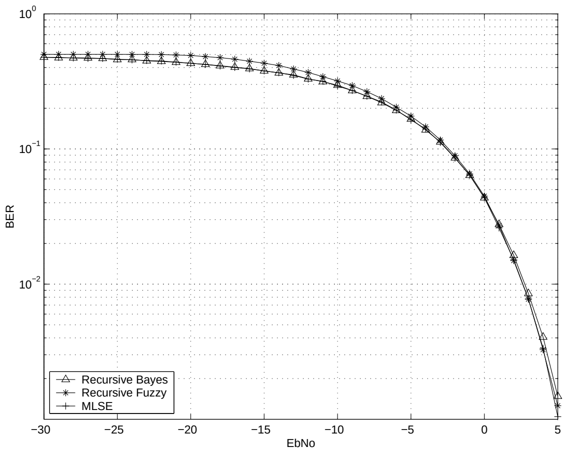

Figure 2. Recursive Fuzzy and Bayesian detectors vs MLSE. For each point 10^5 samples are considered for each of the 10 MonteCarlo runs.

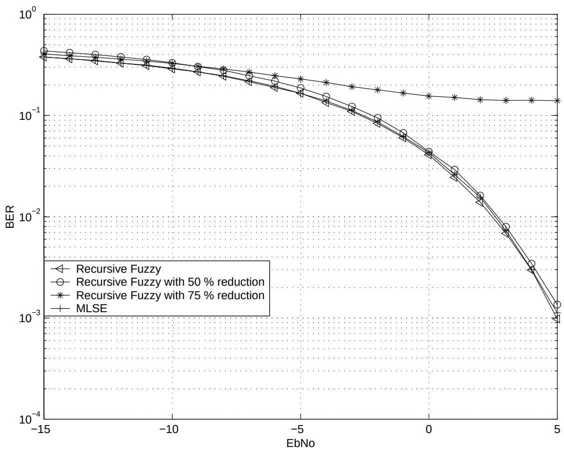

Figure 3.-.Recursive Fuzzy with centroids reduction.

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