SUBSPACE ADAPTIVE DETECTION FOR ASYNCHRONOUS MULTIUSER CDMA WITH FREQUENCY DOMAIN CHANNEL RESPONSE AND DELAY ESTIMATION

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ABSTRACT

Channel estimation, particularly delay estimation in asynchronous CDMA channel, is essential for most receivers. The correlated time domain data sequence will become asymptotically uncorrelated in frequency domain. By taking Discrete Fourier Transform (DFT) of the received signals, we can take advantage of this asymptotical property to simplify the estimation algorithm. The proposed scheme can jointly estimate the path delay and the complex channel response with lower computational complexity. Furthermore, having such estimates, it is possible to use adaptive subspace MOE detector, previously implemented in synchronous channels, for asynchronous multiuser CDMA. Using simulation, it was shown that this scheme can provide comparable performance to that obtained with Wiener filter.

1. INTRODUCTION

Multiuser detection technologies have been studied and used heavily to overcome the near-far problem in CDMA systems. All of these detection schemes require the knowledge of user's parameters, such as propagation delay, spreading code, complex channel impulse response, etc. In particular, estimating propagation delay and channel impulse response has become one of the important area in multiuser detection technologies. In up-link DS-CDMA channel for example, signal arrival time is randomly distributed due to the varying distance between user and base station even if the transmitting time is synchronized. In [1,2], subspace based MUSIC algorithm has been used to estimate the path delay, assuming the number of users and the noise covariance matrix are known. In [3], both delay and channel phase shift are estimated but require special code design, which increases implementation complexity. In this work, an estimation scheme based on signal's frequency-domain property is proposed. It uses maximum likelihood (ML) for channel parameters estimation. By exploiting the asymptotically uncorrelation property of the received signal in frequency-domain shown in [4], the proposed estimator can jointly estimate the propagation delay of desired user and its complex channel response (a combination of channel amplitude and phase shift generally caused by Doppler shift) with relatively reduced complexity.

One important issue in multiuser detection is to reduce receiver's complexity while keeping the performance near optimal. This initiates the interest to adaptive minimum output energy (MOE) scheme as in [6], where only desired user's information is required. In slow variant fading channel, where signal subspace remains almost unchanged over a large time period, rank reduction can result in decreasing computational complexity of the detector. As was shown in [7], the subspace based MOE outperforms in synchronous case blind LMS with faster convergence rate and higher output signal-to-interference-and-noise ratio (SINR). By applying to asynchronous frequency non-selective fading channel, it is shown in this work using simulation that the subspace based MOE detector can provide performance as comparable to that with Wiener filter with reduced complexity.

2. SYSTEM MODEL

The received continuous asynchronous signal in time-variant Rayleigh fading channel can be expressed as follows,

$$r(t) = \sum_{k=1}^{K} \sum_{i} \sqrt{a_k}\gamma_k(t)e^{j\phi_k(t)} b_k(i) s_k(t - iT - \tau_k) + n(t) \tag{1}$$

where K is the number of the users, a_k is symbol's energy of user k. $b_k(i)$ is kth user's ith data symbol which can be BPSK or M-ary quadrature modulated signals. $s_k(t)$ is signature waveform, which has the form of $\sum_{j=1}^{N} c_k(j)g(t - jT_c)$, where $\{c_k(j)\}$ is kth user's spreading codes sequence, T_c is chip time and $g(t)$ is the chip impulse shape. $\gamma_k(t)$ and $\phi_k(t)$ are time-variant channel fading and channel phase shift respectively. The later assumes uniformly distributed within $[0, 2\pi)$. τ_k is the delay of kth user. Without loss of generality, we assume all delays are limited within one symbol interval, or $0 \leq \tau_k < T$. $n(t)$ is zero mean AWGN with variance σ^2.

For simplicity, we assume user 1 is the desired user. The chip rate matched-filter samples the received signal over

This work is partially supported by NSF grant # CCR-9903381 and NJCWT

one entire symbol interval $[0, T)$. Assuming the channel is quasi-static then the channel fading $\gamma_k(t)$ and phase shift $\phi_k(t)$ can be approximated as a constant within channel training period provided this period is short enough. Relative to some time reference, let user 1's delay $\tau_1 = (m_1 + \delta_1)T_c$, where $m_1 = \lfloor \frac{\tau_1}{T_c} \rfloor$ denotes the largest integer which is no more than $\frac{\tau_1}{T_c}$ and $0 \leq \delta_1 < 1$. In estimation part, we use lower case and upper case characters to denote time-domain, and frequency domain variables respectively. Define right cycshift operation of vector $\underline{x} = [x(0), x(1), ..., x(N-1)]^T$ as $C_r^m \underline{x} = [x(N-m), x(N-m+1), ..., x(N-1), x(0), ..., x(N-m-1)]^T$. The desired user transmits identical data symbol during the channel training period. For simplicity, all one's data are assumed. Then the output vector $\underline{y}(n)$ is given as follow,

$$\begin{aligned} \underline{y} &= \sqrt{a_1}\gamma_1 e^{j\phi_1}[\delta_1 C_r^{m_1+1}\underline{s}_1 + (1-\delta_1)C_r^{m_1}\underline{s}_1] + \underline{u} \\ &= h_1[\delta_1 C_r^{m_1+1}\underline{s}_1 + (1-\delta_1)C_r^{m_1}\underline{s}_1] + \underline{u} \end{aligned} \quad (2)$$

where, $h_1 = \sqrt{a_1}\gamma_1 e^{j\phi_1}$ is complex channel response corresponding to desired user, and signature vector $\underline{s}_1 = [s_1(0), ..., s_1(N-1)]^T = [s_1(0), s_1(T_c), ..., s_1((N-1)T_c)]^T$. $\underline{u}$ is comprised of contribution from noise and multiple access interference (MAI) from other active users.

Let $\underline{w}_i = [1, e^{j\omega i}, ..., e^{j\omega i(N-1)}]^T$, $\omega = -\frac{2\pi}{N}$ and $i = 0, ..., N-1$. We define the DFT transform matrix as $W = [\underline{w}_0, \underline{w}_1, ..., \underline{w}_{N-1}]$. With imposing circular time shift properties, it is easy to show that N points DFT of equation (2) can be expressed as

$$\begin{aligned} \underline{Y} &= h_1[\delta_1 diag(\underline{w}_{m_1+1}) + (1-\delta_1)diag(\underline{w}_{m_1})]W\underline{s}_1 + \underline{U} \\ &= h_1\underline{S}_1(\tau_1) + \underline{U} \end{aligned} \quad (3)$$

where, $diag(\underline{x})$ is a diagonal matrix with diagonal elements equal to vector $\underline{x}$. Following [4,5], the additive noise $\underline{U}$ in the frequency domain has frequency components which are asymptotically uncorrelated with distribution approximately Gaussian $\sim N(\underline{0}, C)$ and variance $\sigma_U^2(l)$, l=0,...,N-1, equal to $\sigma_U^2(l) = U(l)U^*(l) = \| \underline{u}^T\underline{w}_l \|^2$.

3. SIMPLIFIED MAXIMUM LIKELIHOOD CHANNEL ESTIMATE

To apply ML algorithm for estimating the unknown channel parameters, we note that the pdf of $\underline{Y}$ is given by

$$p(\underline{Y}) = \frac{1}{\pi^N \mid C \mid} exp\{-(\underline{Y} - h_1\underline{S}_1)^H C^{-1}(\underline{Y} - h_1\underline{S}_1)\} \quad (4)$$

Then the estimate of the unknown channel response h_1 is found by equating the derivative of the log-likelihood function to zero with respect to h_1, which resulting

$$\hat{h}_1 = \frac{\underline{S}_1^H(\tau_1)C^{-1}}{\underline{S}_1^H(\tau_1)C^{-1}\underline{S}_1(\tau_1)}\underline{Y} \quad (5)$$

To estimate the path delay τ_1 of desired user, we propose a cost function F,

$$\begin{aligned} F &= \| \underline{Y} - \hat{h}_1\underline{S}_1(\tau_1) \|^2 \\ &= \| (I - \frac{\underline{S}_1(\tau_1)\underline{S}_1^H(\tau_1)C^{-1}}{\underline{S}_1^H(\tau_1)C^{-1}\underline{S}_1(\tau_1)})\underline{Y} \|^2 \end{aligned} \quad (6)$$

Minimizing F, we get

$$\hat{\tau}_1 = arg\min_{\tau_1} \| (I - \frac{\underline{S}_1(\tau_1)\underline{S}_1^H(\tau_1)C^{-1}}{\underline{S}_1^H(\tau_1)C^{-1}\underline{S}_1(\tau_1)})\underline{Y} \|^2 \quad (7)$$

However the noise covariance matrix C is unknown. A simple approach to estimate the mean of $\underline{Y}$, or $\underline{\hat{m}}_Y$, and variance matrix $\hat{C}$ by using M-symbol length sample vectors as

$$\underline{\hat{m}}_Y = \frac{1}{M}\sum_{p=1}^{M}\underline{Y}_p \approx h_1\underline{S}_1(\tau_1) \quad (8)$$

$$\hat{C} = \frac{1}{M}\sum_{p=1}^{M}(\underline{Y}_p - \underline{\hat{m}}_Y)(\underline{Y}_p - \underline{\hat{m}}_Y)^H \quad (9)$$

By replacing C in equation 7 with equation 9, we can find $\hat{\tau}_1$ by solving the minimizing problem.

In general, the covariance matrix of noise $\underline{u}$ is not a diagonal matrix, which means that due to MAI presence the noise samples are correlated. However, as mentioned above, the frequency-domain noise samples $\underline{U}$ is asymptotically uncorrelated Gaussian distributed, which implies that C is (asymptotically) diagonal with $C = diag(\sigma_U^2(0), \sigma_U^2(1), ..., \sigma_U^2(N-1))$. With this property, the computation complexity of estimating of C with equation 9 is reduced by order of N. Hence the estimation in equation 7 can be rewritten as

$$\hat{\tau}_1 = arg\min_{\tau_1} \| (I - \frac{\underline{S}_1(\tau_1)\underline{S}_1^H(\tau_1)C^{-1}}{\sum_{l=0}^{N-1}\sigma_U^{-2}(l)\|S_1(l)\|^2})\underline{Y} \|^2 \quad (10)$$

where, $S_1(l), l = 0, ..., N-1$, are elements of vector $\underline{S}_1(\tau_1)$. Using a single sample of $\underline{Y}$ in equation 7 and equation 10 will result in poor estimation performance. Instead, we use the mean of $\underline{Y}$ over M samples to replace $\underline{Y}$. To solve (10) with higher accuracy and lower computation complexity, two stages processed is suggested. The first stage is to roughly allocate the global minimum area with larger search step size, while at the second, a fine search with small search step size is conducted within the global minimium area to find more accurate path delay.

After finding path $\hat{\tau}_1$, the channel impulse response $\hat{h}_1$ can be obtained from (5) by replacing $\underline{S}_1(\tau_1)$ with $\underline{S}_1(\hat{\tau}_1)$.

$$\hat{h}_1 = \frac{\underline{S}_1^H(\hat{\tau}_1)\hat{C}^{-1}}{\underline{S}_1^H(\hat{\tau}_1)\hat{C}^{-1}\underline{S}_1(\hat{\tau}_1)}\underline{Y} \quad (11)$$

4. SUBSPACE BASED BLIND DETECTION

Using eigen-decomposition(EVD) to received signal's covariance matrix,

$$R = E[\underline{y}\underline{y}^H] = E_s\Lambda_s E_s^H + E_n\Lambda_n E_n^H = R_s + \sigma_n^2 I_n \quad (12)$$

where the signal subspace is spaned by column vector of $E_s \in R^{N\times p}$ and the noise subspace spaned by column of $E_n \in R^{N\times(N-p)}$.

In [7], a subspace based MOE algorithm was derived, where it has shown the MOE detector lies in the signal subspace only. In other words, the detector's coefficient $\underline{v}$ can be represented as

$$\underline{v} = E_s\underline{v}_s \quad (13)$$

with $\underline{v}_s$ has a dimension of $K \times 1$. It was also shown this algorithm provides better performance than blind LMS in synchronous case.

In asynchronous situation, however, the direct extension of the subspace MOE algorithm will result in poor performance. Consider the asynchronous model in (1), the signal subspace E_s is no long with a rank of K, the number of users, as it is in synchronous case. We assume the rank of p. p is variant with the different delay distributions. Therefore, rank tracking becomes an important issue in asynchronous signal detection.

Furthermore, since the rank p can be larger than number of users K, in some extreme case it may be close to the whole signal space N even with relatively small number of users. In such situation, subspace based algorithm does not benefit sufficiently from computational simplexity and performance improvement due to removing of noise subspace.

Another problem is that using received signal vector $\underline{y}$ with one symbol length as in synchronous case is not adequate for removing multiple access interference (MAI) because of the destruction of orthogonality properties among different users. To overcome MAI, we propose the receiver's coefficient $\underline{v}$ with a length of $2L+1$ symbols, and the desired signal $\hat{\underline{s}}_1$ as $\hat{\underline{s}}_1 = \underline{1}_L \otimes \underline{s}_1$, where $\otimes$ is Kronecker product, and vector $\underline{1}_L$ has dimension of $(2L+1)\times 1$ with Lth element being 1 and others being zeros. Consider the eignvalues in signal subspace $\lambda_s(i) = \lambda(R_s) + \sigma_n^2, i = 1, ..., p$, which is always larger than the eignvalues corresponding to noise subspace which is almost constant. So, the rank p and signal subspace E_s can be found with $\frac{\lambda(i)}{\lambda_{noise}} > c$, where constant $c > 1$.

Based on [7], the interference subspace is found by

$$\hat{\underline{y}}(n) = \underline{y}(n) - <\underline{y}(n), \hat{\underline{s}}_1> \hat{\underline{s}}_1 \quad (14)$$

$$\hat{R} = \frac{1}{M}\sum_{n=1}^{M} \hat{\underline{y}}(n)\hat{\underline{y}}^H(n) = \hat{E}_s\hat{\Lambda}_s\hat{E}_s^H + \hat{E}_n\hat{\Lambda}_n\hat{E}_n^H \quad (15)$$

where covariance matrix $\hat{R}$ is rank of $N-1$ matrix. The interference subspace is spaned by $p-1$ column vectors of $\hat{E}_s$ corresponding to $p-1$ largest eigenvalues. Hence, the coefficient $\underline{v} = E_s\underline{v}_s = \hat{\underline{s}}_1 + \hat{E}_s\underline{\alpha}$, where $\underline{\alpha} \in R^{p-1}$ so that $<\underline{v}, \hat{\underline{s}}_1> = 1$ is always satisfied.

With this signal subspace, the original MOE constrained optimal problem is modified to the following unconstrained problem,

$$MOE = \min_{\underline{\alpha}} E[(\underline{v}^H\underline{y})^2] = (\underline{s}_1 + \hat{E}_s\underline{\alpha})^H R(\underline{s}_1 + \hat{E}_s\underline{\alpha}) \quad (16)$$

Taking derivative of equation 16 with respect to $\underline{\alpha}$, it is easy to find the optimal solution of $\underline{\alpha}^*$ is

$$\underline{\alpha}^* = -(\hat{E}_s^H R\hat{E}_s)^{-1}\hat{E}_s^H R\underline{s}_1 = -\hat{\Lambda}_s^{-1}\hat{E}_s^H R\underline{s}_1 \quad (17)$$

Substituting for $\underline{\alpha}$, the coefficient of subspace MOE detector $\underline{v}$ is

$$\underline{v} = (I - \hat{E}_s\hat{\Lambda}_s^{-1}\hat{E}_s^H R)\underline{s}_1 \quad (18)$$

Once received signal's statistics is available, the coefficient $\underline{v}$ is obtained according to equation 18 for signal detection.

Using gradient descent algorithm, it can be shown that the subspace based MOE detector can also be implemented recursively

$$\underline{v}(n+1) = \underline{v}(n) - \mu z(n)\hat{E}_s\hat{E}_s^H\underline{y}(n) \quad (19)$$

where the MOE detector output $z(n) = \underline{v}^H(n)\underline{y}(n)$.

5. NUMERICAL RESULTS AND CONCLUSION

Defining root mean square (RMS) estimation error σ_ϵ as

$$\sigma_\epsilon = \sqrt{E[\{\hat{\tau}_1(i) - \tau_1(i)\}^2 | \hat{\tau}_1(i) - \tau_1(i) < \epsilon_{max}]} \quad (20)$$

where, ϵ_{max} is the maximum allowed delay estimation error. Here we set $\epsilon_{max} = 0.2T_c$, otherwise the synchronization is assumed failed. A total of 500 MonteCarlo runs were performed for each simulation. Length $N = 15$ gold codes were used as the users' signature codes. All users suffer from time-variant channel which result in uniform distributed channel phase shift but assumed to be quasi-static within channel training period. The desired user's signal-to-noise ratio has been set to 8dB. With a normalized spreading codes, the actual SNR per bit is about 11.8dB lower for code length of 15. All other multiple access interference users transmit at the same energy.

50 bits and 200 bits (length of M) acquisition time are used for interference-to-desired signal ratio (ISR) of 0dB and 10dB respectively. Original and simplified scheme refer to the estimation algorithm based on (7) and (10) respectively. Figure 1 shows that the delay RMS error stayed almost unchanged with the increasing of the number of users.

However stronger ISR requires longer acquisition time. The comparison of performance with Cramer-Rao bound (where only delay is assumed unknown) is given in figure 2, where $ISR = 0dB$ and acquisition time of 50bits. If channel response h_1 and noise statistics are unknown, Cramer-Rao bound will become higher.

Figure 3 shows detection performance of the subspace MOE detector in asynchronous situation with 5 users. Results were averaged over different delay patterns. As shown in figure, the subspace based detector presents the comparable performance as that of Wiener filter with only knowledge of desired user's signature, and less computational complexity. With $L = 1$, the detector can provide almost flat near-far resistance property.

6. REFERENCES

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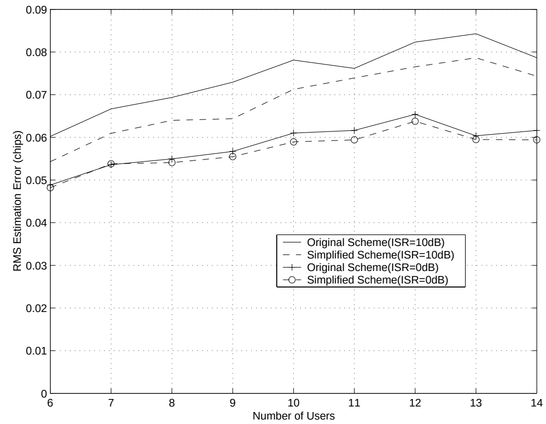

Fig. 1. RMS estimation error vs number of users.

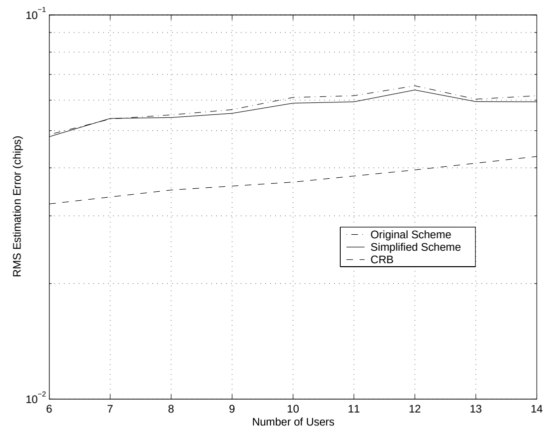

Fig. 2. Comparison with Cramer-Rao Bound for ISR=0dB.

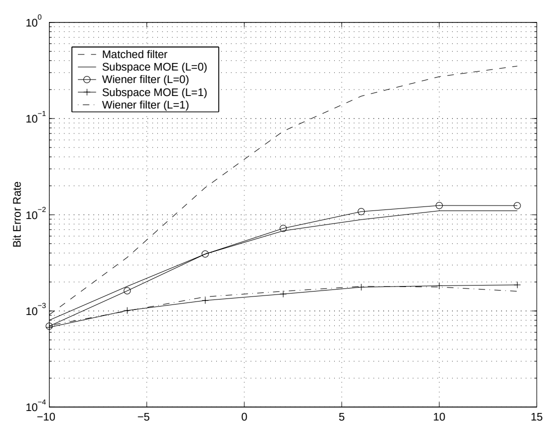

Fig. 3. BER vs. ISR