MULTIPLE SIGNAL DETECTION UNDER A SPECIFIED FALSE ALARM CONSTRAINT

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ABSTRACT

This paper presents a sequential detection scheme for sinusoidal signals observed in spatially colored noise which controls the probability of overestimating the number of signals. The scheme is based on the Sequentially Rejective Bonferroni Test together with the nonlinear weighted least squares approach for estimation of the signal parameters under the hypotheses and alternatives respectively. The power of the method is compared via computer simulations to the commonly used minimum description length method, MDL. The proposed scheme performs well in comparison to the MDL method and gives at the same time control of the false alarm probability.

1. INTRODUCTION

The present paper considers detection of the number of sinusoidal sources in array signal processing. An important application is, e.g., radar array processing, using a coherent pulsed Doppler radar, where it is essential to control the probability of false alarms. Sequential detection procedures are often suggested in the literature, where in each step a threshold is selected to give the test a prescribed level of significance, see, e.g., [1, 2, 3]. However, exact control of the global level, i.e., the probability of rejecting the "true hypothesis", of the detection scheme is a complicated problem. The reason for this is that the probability of rejecting the hypothesis is dependent on the probability of rejecting the preceding (false) hypotheses. Recognizing this problem, we will use the Sequentially Rejective Bonferroni Test, [4], to bound the global level of significance. For examples of other applications where this procedure has been used , see, e.g, [5, 6]. As test statistics in the individual stages of the procedure, we use the Generalized Likelihood Ratio, GLRT. Order statistics is used for deriving the asymptotic distribution of the statistic under the hypothesis in each stage. Instead of using the maximum likelihood method for parameter estimation, a two step nonlinear weighted least square approach is employed which gives consistent and efficient estimates.

2. SIGNAL MODEL

Consider an array of m sensors which receives the sinusoidal waveforms generated by p sources. The m-vector $\mathbf{y}(t)$ of noise corrupted sensor outputs is modeled by

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta}_0)\mathbf{B}_0\mathbf{s}(\boldsymbol{\omega}_0, t) + \mathbf{e}(t)\,, \tag{1}$$

where

$$\mathbf{A}(\boldsymbol{\theta}_0) = \begin{bmatrix} \mathbf{a}(\theta_{01}) & \ldots & \mathbf{a}(\theta_{0p}) \end{bmatrix}, \tag{2}$$

$\mathbf{B}$ is a diagonal matrix with the complex amplitudes $\mathbf{b}_0 = \begin{bmatrix} b_{01}, & \ldots & , b_{0p} \end{bmatrix}^T$ on the diagonal and

$$\mathbf{s}(\boldsymbol{\omega}_0, t) = \begin{bmatrix} e^{j\omega_{01}t} & \ldots & e^{j\omega_{0p}t} \end{bmatrix}^T. \tag{3}$$

The vector $\mathbf{a}(\theta_{0i})$ is the array steering vector of the i:th source. The amplitudes, b_{0i}, the frequencies, ω_{0i} and the directions of arrival (DOA:s), θ_{0i}, are considered to be unknown. The zero subscripts denote the true parameter values. The noise is temporally white, zero-mean, complex Gaussian random variables with second order moment (the superscript "*" denotes complex conjugate and transpose)

$$\mathrm{E}[\mathbf{e}(t)\mathbf{e}^*(t)] \quad = \quad \mathbf{Q}\,. \tag{4}$$

The spatial covariance matrix $\mathbf{Q}$ is Hermitian and positive definite, but otherwise arbitrary. Now, based on N snapshots of the data, $\mathbf{y}(t)$, the task is to decide on the number of sources and estimate the unknown parameters. Concerning the detection procedure, a scheme that limits the probability of overestimating the number of sources is desired. In the following, we will call this probability for the "false alarm probability". We collect the unknown signal parameters, corresponding to the hypothesis that there are i signals, in the vector

$$\begin{aligned}\boldsymbol{\eta}_i \quad = \quad & \big[\ \mathrm{Re}(b_1) \quad \ldots \quad \mathrm{Re}(b_i) \quad \mathrm{Im}(b_1) \quad \ldots \\ & \mathrm{Im}(b_i) \quad \theta_1 \quad \ldots \quad \theta_i \quad \omega_1 \quad \ldots \quad \omega_i\ \big]^T\end{aligned} \tag{5}$$

3. HYPOTHESES

Assuming that the maximum number of possible signals, p_{max} is given, the following mutually exclusive set of hypotheses, H_i, and alternatives, K_i, describes the detection problem:

$$\begin{array}{llll} H_1: & \text{no signal} & K_1: & \text{one signal} \\ H_2: & \text{one signal} & K_2: & \text{two signals} \\ \vdots & & \vdots & \\ H_{p_{max}}: & p_{max}-1 \text{ signals} & K_{p_{max}}: & p_{max} \text{ signals} \end{array}$$

By mutually exclusive, we mean that only one of the hypotheses can be true at a time. Assuming that some arbitrary detection scheme has rejected all but q of the hypotheses, we shall use the rule that the hypothesis corresponding to the minimum number of signals is true.

4. THE SEQUENTIALLY REJECTIVE BONFERRONI APPROACH

In [4], an approach, called the "Sequentially Rejective Bonferroni Test", is presented which guarantees a multiple level of significance. The following definition of multiple level of significance is also given therein.

Definition 4.1 *A multiple test procedure with critical regions C_1, C_2, ..., $C_{p_{max}}$ for testing hypotheses H_1, H_2, ..., $H_{p_{max}}$ is said to have a "multiple level of significance" α (for free combinations) if for any non-empty index set $I \subseteq \{1, 2, 3, \dots, p_{max}\}$ the supremum of the probability $P\left(\bigcup_{i \in I} C_i\right)$ when H_i are true for all $i \in I$ is smaller than or equal to α.*

This procedure sets a limit on the probability of rejecting a set of true hypothesis. It is clear that this scheme also limits the probability of overestimating the number of signals, the false alarm probability.

Here follows a brief description of the Sequentially Rejective Bonferroni Test. In each stage i, a test statistic is used which has a tendency of assuming larger values under the alternative, K_i. We denote the "critical level" for the outcome of the test statistic by P_i. Thus, P_i is the probability that a random sample of the test statistic exceeds the observed value when H_i is true. The following stepwise procedure is now followed:

1. Calculate the P_i-values.
2. Sort the P_i-values in ascending order and denote the sequence of ordered values by $P^{(1)} \leq P^{(2)} \leq \dots \leq P^{(p_{max})}$. The hypotheses corresponding to the sorted P-value sequence are denoted by $H^{(1)}, H^{(2)}, \dots, H^{(p_{max})}$.
3. Set $i = 1$.
4. If $P^{(i)} \leq \alpha/(p_{max} - i + 1)$, go to step 5, otherwise accept $H^{(i)}, H^{(i+1)}, \dots, H^{(p_{max})}$ and stop.
5. Reject $H^{(i)}$. If $i < p_{max}$, set $i = i + 1$, and go to step 4, otherwise stop.

As is discussed in [4], this test procedure has a multiple level of significance for any type of restricted combinations of the hypotheses. Herein, the restriction is that only one hypothesis may be true at the same time. Since we have postulated that among the hypotheses that may be passed by the procedure, all but the one corresponding to the minimum number of signals shall be rejected, we can calculate the P_i-values consecutively, accept H_i and stop when $P_i > \alpha/(p_{max} - i + 1)$. To clarify this, note that if the P-values of H_k, $k = i+1, \dots, p_{max}$, also should exceed the threshold $\alpha/(p_{max} - i + 1)$ in the original scheme, these hypotheses will be rejected since H_i represents the minimum number of signals.

5. DESCRIPTION OF THE PROPOSED SCHEME

As a test statistic in each stage of the scheme, we will use the Generalized Likelihood Ratio Test, GLRT, see, e.g., [7]. The concentrated test statistic, with respect to the unknown spatial covariance matrix, in stage i becomes

$$\lambda_i = 2N \ln |\mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i})| - 2N \ln |\mathbf{C}(\hat{\boldsymbol{\eta}}_{K_i})| , \tag{6}$$

where $\mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i})$ and $\mathbf{C}(\hat{\boldsymbol{\eta}}_{K_i})$ are the sample covariance matrices under the hypothesis and the alternative, and $\hat{\boldsymbol{\eta}}_{H_i} = \hat{\boldsymbol{\eta}}_{i-1}$ and $\hat{\boldsymbol{\eta}}_{K_i} = \hat{\boldsymbol{\eta}}_i$ are the Maximum Likelihood Estimates, MLE, of the signal parameters. The sample covariance matrix under H_i is computed as

$$\mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i}) = \frac{1}{N} \sum_{t=0}^{N-1} \Big(\mathbf{y}(t) - \mathbf{A}(\hat{\boldsymbol{\theta}}_{H_i}) \hat{\mathbf{B}}_{H_i} \mathbf{s}(\hat{\boldsymbol{\omega}}_{H_i}, t)\Big) \Big(\ \dots \ \Big)^* , \tag{7}$$

and equivalently under K_i. Instead of the maximum likelihood, ML, method, we will use a two step variant of the nonlinear weighted least squares, WLS, method which gives consistent and *efficient* estimates, [8]. In the first step, consistent parameter estimates are obtained which are used for obtaining a consistent estimate of the spatial covariance matrix. The inverse of the estimated covariance matrix is then used as weighting in a second step which produces efficient parameter estimates. Note that this approach gives the same performance as using the ML estimates since the two step WLS estimates are consistent and efficient. Under the alternative, K_i, we will fix the parameters of the $i-1$ first signals to the estimates obtained under H_i and only search for the parameters of the i:th signal. This is done in order to reduce the computational burden. This approach means that we try to find a best fit of an additional signal to the "residual" obtained under H_i.

The proposed scheme is the following:

1. Set $i = 1$.
2. Define the hypothesis H_i: $i-1$ signals present, and the alternative K_i: i signals present.
3. Estimate the parameters under the hypothesis and the alternative respectively:
   Under H_i:
   If $i = 1$: H_1 corresponds to no signal. Estimate the sample covariance as $\mathbf{C}_{H_1} = 1/N \sum_{t=0}^{N-1} \mathbf{y}(t)\mathbf{y}^*(t)$.
   If $i \neq 1$:
   (a) Estimate $\hat{\boldsymbol{\eta}}_{H_i}$ using $\mathbf{C}^{-1}(\hat{\boldsymbol{\eta}}_{K_{i-1}})$ as weighting in the WLS method.
   (b) Update the estimate of the covariance matrix using the consistent estimates obtained in 3a. This gives a consistent estimate of the spatial covariance matrix under H_i, $\mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i})$.
   (c) Update the signal parameter estimates by using $\mathbf{C}^{-1}(\hat{\boldsymbol{\eta}}_{H_i})$ as weighting in the WLS method. This step produces efficient signal parameter estimates.

   Under K_i:
   (a) Fix the parameters of the $i-1$ first signals to the estimates obtained under H_i.
   (b) Using the WLS method with $\mathbf{C}^{-1}(\hat{\boldsymbol{\eta}}_{H_i})$ as weighting, compute mN estimates of the direction of arrival, θ, the angular frequency, ω, and the complex amplitude, b, of the i:th signal over an (ω_k, θ_l)-grid, $k = 1, \dots, N$, $l = 1, \dots, m$.
4. Compute the maximum of the corresponding mN test statistics:
   $$\hat{\lambda}_{i_{max}} = \max_{k,l} \Big(2N \ln |\mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i})| - 2N \ln |\mathbf{C}(\hat{\boldsymbol{\eta}}_{K_i,(k,l)})| \Big) .$$
   The maximizing k and l define the parameter estimates of the $i:th$ source under K_i, $\hat{b}_{K_i} = b_{(k,l)}$, $\hat{\theta}_{K_i} = \theta_{(k,l)}$ and $\hat{\omega}_{K_i} = \omega_{(k,l)}$, see also Equation (14).
5. Compute the P_i value, see Equation (22) in Section 6.
6. If $P_i \leq \alpha/(p_{max} - i + 1)$, go to Step 7, otherwise accept H_i and stop.
7. Reject H_i. If $i < p_{max}$, set $i = i + 1$ and go to Step 2, otherwise stop.

6. DISTRIBUTION OF THE TEST STATISTIC

To calculate the P_i-value, we need to know the distribution of the test statistic under H_i. This is not an easy task, since under H_i, the amplitude of the i:th signal is zero, which means that the corresponding ω and θ are not observable. A consequence of this is that the standard asymptotic theory of the GLRT is not applicable. However, conditioned on an arbitrary ω and θ, asymptotic GLRT theory states that the distribution is a χ^2-distribution with degrees of freedom equal to the difference in the number of parameters under the hypothesis and the alternative. In this case, this difference is 2, corresponding to the real and imaginary parts of b. To obtain the distribution when maximizing the test statistic over an (ω, θ)-grid, we will use the concept of order statistics. First, however, we will verify the "conditioned" χ_2^2-distribution when the detection scheme and corresponding WLS estimates in Section 5 are used. Consider

$$\begin{aligned}\mathbf{C}(\hat{\boldsymbol{\eta}}_{K_i}) &= \frac{1}{N}\sum_{t=0}^{N-1}\Big(\mathbf{y}(t)-\mathbf{A}(\hat{\boldsymbol{\theta}}_{K_i})\hat{\mathbf{B}}_{K_i}\mathbf{s}(\hat{\boldsymbol{\omega}}_{K_i},t)\Big)\\ &\quad\Big(\mathbf{y}(t)-\mathbf{A}(\hat{\boldsymbol{\theta}}_{K_i})\hat{\mathbf{B}}_{K_i}\mathbf{s}(\hat{\boldsymbol{\omega}}_{K_i},t)\Big)^*\\ &= \ldots = \mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i}) - \tilde{\mathbf{Y}}(\omega)b^*\mathbf{a}^*(\theta)\\ &\quad -\mathbf{a}(\theta)b\tilde{\mathbf{Y}}^*(\omega) + |b|^2\mathbf{a}(\theta)\mathbf{a}^*(\theta)\,, \qquad (8)\end{aligned}$$

where the parameter b is the complex amplitude of the i:th target (the "additional target under K_i") and the quantity $\tilde{\mathbf{Y}}(\omega)$ is defined by

$$\tilde{\mathbf{Y}}(\omega)=\frac{1}{N}\sum_{t=0}^{N-1}\Big(\mathbf{y}(t)-\mathbf{A}(\hat{\boldsymbol{\theta}}_{H_i})\hat{\mathbf{B}}_{H_i}\mathbf{s}(\hat{\boldsymbol{\omega}}_{H_i},t)\Big)\,e^{-j\omega t}\,. \qquad (9)$$

This quantity is the Fourier transform of the residual under H_i evaluated at ω. We have also used the fact that we, under K_i, fix the parameter estimates of the first $i-1$ targets to those obtained under H_i. Completing the square in Equation (8) gives

$$\begin{aligned}\mathbf{C}(\hat{\boldsymbol{\eta}}_{K_i}) &= \mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i}) - \tilde{\mathbf{Y}}(\omega)\tilde{\mathbf{Y}}^*(\omega) + \Big(\tilde{\mathbf{Y}}(\omega)-\mathbf{a}(\theta)b\Big)\\ &\quad\Big(\tilde{\mathbf{Y}}(\omega)-\mathbf{a}(\theta)b\Big)^* \qquad (10)\end{aligned}$$

Defining the matrix $\mathbf{W}=\mathbf{C}(\hat{\boldsymbol{\eta}}_{H_i})-\tilde{\mathbf{Y}}(\omega)\tilde{\mathbf{Y}}^*(\omega)$, using the determinant rule $|\mathbf{I}+\mathbf{AB}|=|\mathbf{I}+\mathbf{BA}|$ for compatible dimensions, we get (dropping the dependence on ω and θ for notational convenience)

$$\ln|\mathbf{C}(\hat{\boldsymbol{\eta}}_{K_i})|=\ln|\mathbf{W}|+\ln\Big(1+\Big(\tilde{\mathbf{Y}}-\mathbf{a}b\Big)^*\mathbf{W}^{-1}\Big(\tilde{\mathbf{Y}}-\mathbf{a}b\Big)\Big)\,. \qquad (11)$$

Using this expression in Equation (6) gives after some algebra

$$\lambda_i=-2N\ln\Big(\big[1-\tilde{\mathbf{Y}}^*\mathbf{C}_{H_i}^{-1}\tilde{\mathbf{Y}}\big]\big[1+(\tilde{\mathbf{Y}}-\mathbf{a}b)^*\mathbf{W}^{-1}(\tilde{\mathbf{Y}}-\mathbf{a}b)\big]\Big). \qquad (12)$$

The quantities $\tilde{\mathbf{Y}}$ and $\tilde{\mathbf{Y}}-\mathbf{a}b$ are $O_p(1/\sqrt{N})$ under H_i and since the matrix $\mathbf{W}$ is equal to $\mathbf{C}_{H_i}+O_p(1/N)$ the expression above tends to

$$\lambda_i=-2N\ln\Big(1+\Big(\tilde{\mathbf{Y}}-\mathbf{a}b\Big)^*\mathbf{C}_{H_i}^{-1}\Big(\tilde{\mathbf{Y}}-\mathbf{a}b\Big)-\tilde{\mathbf{Y}}^*\mathbf{C}_{H_i}^{-1}\tilde{\mathbf{Y}}\Big)\,, \qquad (13)$$

where we have neglected terms of order $O_p(1/N^2)$. Now, the WLS estimate under K_i of the complex amplitude of the i:th signal is given by (using as weighting the matrix $\mathbf{C}_{H_i}^{-1}$)

$$\hat{b}=\frac{\mathbf{a}^*(\theta)\mathbf{C}_{H_i}^{-1}\tilde{\mathbf{Y}}(\omega)}{\mathbf{a}^*(\theta)\mathbf{C}_{H_i}^{-1}\mathbf{a}(\theta)}\,. \qquad (14)$$

Inserting this into Equation (13) gives with a little algebra

$$\lambda_i(\omega,\theta)=-2N\ln\left(1-\frac{|\tilde{\mathbf{Y}}(\omega)\mathbf{C}_{H_i}^{-1}\mathbf{a}(\theta)|^2}{\mathbf{a}^*(\theta)\mathbf{C}_{H_i}^{-1}\mathbf{a}(\theta)}\right)\,. \qquad (15)$$

Under H_i, $\tilde{\mathbf{Y}}$ is $O_p(1/\sqrt{N})$, and we may perform a Taylor series expansion giving the final asymptotic expression as

$$\begin{aligned}\lambda_i(\omega,\theta) &= 2N\frac{|\tilde{\mathbf{Y}}(\omega)\mathbf{C}_{H_i}^{-1}\mathbf{a}(\theta)|^2}{\mathbf{a}^*(\theta)\mathbf{C}_{H_i}^{-1}\mathbf{a}(\theta)}\\ &= \tilde{\mathbf{Y}}_w^*(\omega)\mathbf{P}_{\mathbf{a}_w}(\theta)\tilde{\mathbf{Y}}_w(\omega)\,, \qquad (16)\end{aligned}$$

where $\tilde{\mathbf{Y}}_w(\omega)=\sqrt{2N}\mathbf{Q}^{-1/2}\mathbf{Y}(\omega)$ and $\mathbf{a}_w(\theta)=\mathbf{Q}^{-1/2}\mathbf{a}(\theta)$ (note that $\mathbf{C}_{H_i}$ tends to $\mathbf{Q}$ w.p.1 under H_i). The matrix $\mathbf{P}_{\mathbf{a}_w}(\theta)$ is the orthogonal projector onto the range space of $\mathbf{a}_w(\theta)$. Conditioned on a particular grid point, (ω_k,θ_l), the test statistic is χ^2-distributed (under H_i) with two degrees of freedom since, asymptotically,

$$\lambda_i(\omega_k,\theta_l)=\tilde{\mathbf{Y}}_w^*(\omega_k)\mathbf{P}_{\mathbf{a}_w}(\theta_l)\tilde{\mathbf{Y}}_w(\omega_k)=\left|\frac{\mathbf{a}_w(\theta_l)^*\tilde{\mathbf{Y}}_w(\omega_k)}{\sqrt{\mathbf{a}_w(\theta_l)^*\mathbf{a}_w(\theta_l)}}\right|^2, \qquad (17)$$

is the squared absolute value of a complex normal variable with variance equal to 2. We remind that asymptotically, under H_i, $\mathbf{Y}(\omega)\in N_c(0,\frac{1}{N}\mathbf{Q})$, which implies that $\tilde{\mathbf{Y}}_w\in N_c(0,2\mathbf{I})$ (here, $N_c(\cdot,\cdot)$ denotes the multivariate complex normal distribution). Since the real and imaginary parts of the linear combination are independent with unit variance respectively, the χ^2-distribution follows.

We will now maximize $\lambda_i(\omega,\theta)$ over a grid and use the concept of order statistics to derive a useful asymptotic distribution under H_i. In the ω-direction we will use an FFT-grid over which the test statistic will be maximized. Due to the orthogonal properties of the Discrete Fourier transform, the test statistics will be independent for different frequency bins. In the θ-direction, an m-point uniformly spaced grid will be used. We note that, for a particular FFT bin, different test statistic values are not independent since, in general, the whitened steering vectors corresponding to directions θ_k and θ_l are not orthogonal. However, to be able to proceed we will assume that independence applies. We may then use order statistics for deriving the asymptotic distribution under H_i. An important remark in this context is that it can be shown that, using the P_i-values derived subsequently, the multiple level of significance is retained if the test statistics are dependent. We proceed by noting that the χ_2^2-distribution is equal to the exponential distribution with parameter $\gamma=0.5$. Thus, denoting $\lambda_i(\omega_k,\theta_l)$ by $\lambda_{i_{k,l}}$, the probability density function and the cumulative distribution function are given by

$$\begin{aligned}f_{\lambda_{i_{k,l}}}(\lambda) &= \frac{1}{2}e^{-\frac{1}{2}\lambda}\,,\lambda\geqslant 0\,, \qquad (18)\\ F_{\lambda_{i_{k,l}}}(\lambda) &= \Big(1-e^{-\frac{1}{2}\lambda}\Big)\,,\lambda\geqslant 0\,. \qquad (19)\end{aligned}$$

The concept of order statistics for independent variables, see e.g. [9], gives the probability density function of the maximum value of $\lambda_{i_{k,l}}$ over the grid as

$$f_{\lambda_{i_{max}}}(\lambda) = mNF^{mN-1}_{\lambda_{i_{k,l}}}(\lambda) f_{\lambda_{i_{k,l}}}(\lambda)$$
$$= \frac{mN}{2}\left(1 - e^{-\frac{1}{2}\lambda}\right)^{mN-1} e^{-\frac{1}{2}\lambda}, \lambda \geqslant 0\,. \qquad (20)$$

Recognizing the inner derivative in (20) produces the cumulative distribution function, $F_{\lambda_{i_{max}}}(\lambda)$ of $\lambda_{i_{max}}$ as

$$F_{\lambda_{i_{max}}}(\lambda) = \left(1 - e^{-\frac{1}{2}\lambda}\right)^{mN}, \qquad (21)$$

so that the P_i-value is given by, denoting the observed value of $\lambda_{i_{max}}$ by $\hat{\lambda}_{i_{max}}$,

$$P_i = 1 - F_{\lambda_{i_{max}}}(\hat{\lambda}_{i_{max}}) = 1 - \left(1 - e^{-\frac{1}{2}\hat{\lambda}_{i_{max}}}\right)^{mN}. \quad (22)$$

7. COMPUTER SIMULATIONS

In the simulations, we use a uniform linear array with 10 elements placed at a distance of $\lambda/2$ apart. The number of snapshots is 256 and the number of independent Monte Carlo runs is 100. In the detection scheme, the multiple level of significance is set to $\alpha = 0.1$ and the maximum number of sources is set to $p_{max} = 4$. Two jammers, each one having a jammer-to-noise-ratio, JNR, of 20 dB, are present at -10 and -20 degrees relative the broadside of the array. The sources are located at $0°$ and $20°$ with frequencies $\omega_{01} = 0.793$ and $\omega_{02} = 1.578$ (chosen such that they do not correspond to a grid point in the FFT). The source at $20°$ is 10 dB weaker in SNR than the one at $0°$. In Figure 1, the probability of detecting the correct number of sources and the false alarm probability are plotted as a function of the SNR for the sources. The SNR is varied by varying the SNR of the source at $0°$ from -30 to 20 dB in steps of 5 dB (the source at $20°$, having 10 dB lower SNR, is thus swept from -40 dB to 10 dB). As a comparison, the performance of the MDL, [10], scheme is plotted in the same figure. We see that the MDL scheme has a larger probability of overestimating the number of signals while the same probability of the proposed scheme is below 0.1. A second simulation was run (not shown here due to limited space), using the same parameters except that θ_{02} was set to $5.7°$, which corresponds to half the beamwidth of the array. Also in this case the method worked well in comparison to the MDL method with about the same threshold as in Figure 1 and a limited false alarm probability (below 0.1). However, due to the fact that the inverse of the estimated covariance matrix assuming H_i is used when we search for the i:th signal, there is a risk that the i:th signal cancels for high SNR:s. This phenomenon was observed for an SNR of 20 dB in this second simulation. This is the price paid for fixing the parameter estimates of the $i-1$ first signals to those obtained under H_i. If all the parameters assuming K_i are estimated "from scratch", a consistent estimate of the covariance matrix under K_i can be computed, and the cancellation problem can be avoided.

8. CONCLUSIONS

A detection scheme for determining the number of sinusoidal signals impinging on a sensor array in spatially colored noise environments was presented. The scheme limits the probability of overestimating the number of sources, or more precisely, exhibits a controllable multiple level of significance. In connection to this procedure, a two step weighted lest squares procedure which yields consistent and efficient parameter estimates was presented and used in place of the maximum likelihood estimates in the generalized likelihood ratio tests. Computer simulations showed good performance of the test procedure in comparison to the MDL method.

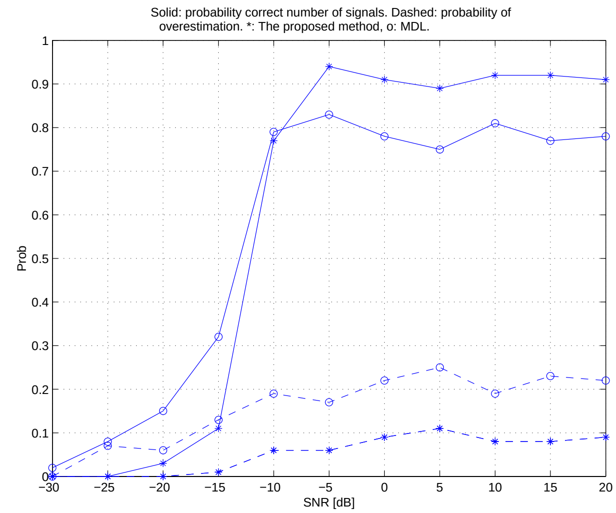

Fig. 1. Solid: probability of correct decision, dashed: false alarm probability. *: proposed method, o: MDL.

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