MATRIX FILTERS FOR PASSIVE SONAR

Richard J. Vaccaro

Department of Electrical and Computer Engineering
University of Rhode Island
Kingston, RI 02881 USA
(vaccaro@ele.uri.edu)

Brian F. Harrison

Naval Undersea Warfare Center
Submarine Sonar Department
Newport, RI 02841 USA
(harrison_bf@ieee.org)

ABSTRACT

This paper introduces matrix filters as a tool for localization and detection problems in passive sonar. The outputs of an array of sensors, at some given frequency, can be represented by a vector of complex numbers. A linear filtering operation on the sensor outputs can be expressed as the multiplication of a matrix (called a matrix filter) times this vector. The purpose of a matrix filter is to attenuate unwanted components in the measured sensor data while passing desired components with minimal distortion. Matrix filters are designed by defining an appropriate pass band and stop band and solving a convex optimization problem. This paper formulates the design of matrix filters for passive sonar and gives two examples.

1. INTRODUCTION

We propose the use of matrix filters for passive sonar signal processing. The outputs of an array of N sensors, at some given frequency, can be represented by a vector, $\mathbf{x}$, of complex numbers. A linear filtering operation on the sensor outputs can be expressed as a matrix multiplication, $\mathbf{y} = \mathbf{M}\mathbf{x}$, where $\mathbf{M}$ is an $N \times N$ matrix of complex numbers. The purpose of a matrix filter is to attenuate unwanted components in the measured sensor data, $\mathbf{x}$, while passing desired components with minimal distortion. Matrix filters are designed by defining an appropriate pass band and stop band and solving a convex optimization problem. The design of matrix filters by convex optimization was proposed in [1], which includes examples of frequency selective filters and Hilbert transform filters for short time series. In this paper we show how to design filters that are useful for detection and localization problems in passive sonar.

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2. DATA MODEL

Consider a vertical array of N sensors located at depths $z_1, \cdots, z_N$, and a cw acoustic source at frequency ω_0 located at a depth z and a range r from the array. Under certain assumptions [2], the Fourier transform of the sensor outputs, evaluated at ω_0, can be written as

$$\mathbf{a}(r, z) = s\boldsymbol{\Omega}\boldsymbol{\alpha}(r, z) + \mathbf{n}, \qquad (1)$$

where

$$\boldsymbol{\Omega} = \begin{bmatrix} \phi_1(z_1) & \cdots & \phi_Q(z_1) \\ \vdots & & \vdots \\ \phi_1(z_N) & \cdots & \phi_Q(z_N) \end{bmatrix},$$

$$\boldsymbol{\alpha}(r, z) = \begin{bmatrix} \frac{\sqrt{2\pi}\phi_1(z)}{\sqrt{rk_1}} e^{(jk_1 - \gamma_1)r} \\ \vdots \\ \frac{\sqrt{2\pi}\phi_Q(z)}{\sqrt{rk_Q}} e^{(jk_Q - \gamma_Q)r} \end{bmatrix}, \qquad (2)$$

s is a complex scale factor, and $\mathbf{n}$ is a vector of noise samples. The functions $\phi_i(z)$, $i = 1, \cdots, Q$ are the normal modes associated with the acoustic environment; they can be calculated numerically using a program such as KRAKEN [3]. The numbers $k_1, \cdots, k_Q$ and $\gamma_1, \cdots, \gamma_Q$ are the horizontal wavenumbers and mode attenuation coefficients, respectively, and are also calculated by KRAKEN. The vector $\mathbf{a}(r, z)$ in (1) is referred to as a *replica vector.* In what follows, all replica vectors are normalized to unit length and the dependence on range and depth is not shown explicitly.

3. MATRIX FILTER DESIGN

Matrix filters are designed using a set of replica vectors defined over a region of range and depth. The pass band set, P, consists of replica vectors in a given range/depth region that should "pass" through the filter with only a small, specified, amount of distortion. The stop band set, S, consists of

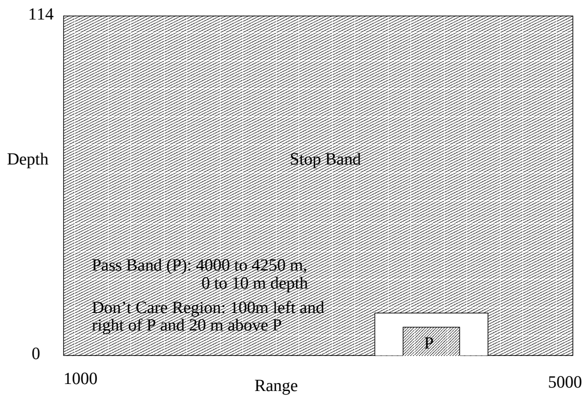

Fig. 1. Pass band and stop band regions for a matrix filter.

replica vectors that should be attenuated as much as possible by the filter. The replica vectors in P and S are calculated at grid points in range and depth and are normalized to have unit length. Replica vectors that are not in either P or S may be thought of as defining a "don't care" region in range and depth. Don't care regions are used to separate the pass and stop bands so that the filter is not asked to simultaneously pass and attenuate very similar replica vectors. A typical definition of pass and stop bands is shown in Fig. 1. The design specifications for a matrix filter, $\mathbf{M}$, are given in terms of convex functions of the filter response. For example, the stop band performance is specified by the function

$$\psi(\mathbf{M}) = \max_{\mathbf{a}\in S} ||\mathbf{Ma}||^2, \quad (3)$$

which measures the maximum energy in the filter stop band response.

We have used two different pass band formulations. The first requires that replica vectors in the pass band pass through the filter with distortion less than a specified tolerance. This requirement is written as

$$||\mathbf{Ma} - \mathbf{a}||^2 < \beta, \;\; \mathbf{a} \in P \quad (4)$$

where β is a user-specified constant.

A second pass band formulation is motivated by a matched-field source localization technique known as the Bartlett processor [4]. Given a data vector $\mathbf{d}$ observed on the sensor array and replica vectors $\mathbf{a}$ from a range/depth search region, R, the Bartlett processor computes

$$\frac{|\mathbf{a}^H\mathbf{d}|^2}{||\mathbf{d}||^2}, \;\; \mathbf{a} \in R. \quad (5)$$

The range and depth associated with the replica vector that produces the maximum value in the above equation is taken as the estimate of the source range and depth.

A pass band formulation motivated by the Bartlett processor is:

$$|1 - \mathbf{a}^H\mathbf{Ma}|^2 \le \epsilon_1 \text{ and } ||\mathbf{M}||^2 \le 1 + \epsilon_2, \;\; \mathbf{a} \in P. \quad (6)$$

The first inequality above can be used to derive the following bound on the Bartlett surface for the matrix filter output in the pass band:

$$(1 - \sqrt{\epsilon_1})^2 < |\mathbf{a}^H\mathbf{Ma}|^2 < (1 + \sqrt{\epsilon_1})^2.$$

Thus, by choosing ϵ_1 appropriately, the Bartlett surface for the filter output in the pass band can be made arbitrarily close to unity. The second inequality in equation (6) keeps the maximum energy gain of the filter close to unity across the pass band.

Matrix filters are calculated by solving an optimization problem; namely, minimize the stop band energy subject to a pass band constraint. The filter design optimization problem using the first pass band formulation is:

$$\min_{\mathbf{M}} \psi(\mathbf{M}), \text{ subject to equation (4)} . \quad (7)$$

The optimization problem using the second pass band formulation is:

$$\min_{\mathbf{M}} \psi(\mathbf{M}), \text{ subject to equation (6)} . \quad (8)$$

It can be shown that the objective and constraint functions in (7) and (8) are convex functions of the elements of $\mathbf{M}$. Attractive features of convex optimization problems such as (7) and (8) are that they can be numerically solved to a guaranteed accuracy, and there are no local minima. We used the ellipsoid algorithm described in [5] to perform the optimization.

As an example, we designed a matrix filter by solving (7) with $\beta = 0.006$. The pass and stop bands were chosen as shown in Fig. 1, with replica vectors calculated every 10 m in range and every meter in depth. The power response of this filter is shown in Fig. 2 as a function of range and depth. The filter power reponse is less than 0.51 in the stop band and greater than 0.9 in the pass band.

4. SIMULATION EXAMPLES

In this section we give some simulation results on the use of matrix filters in localization and detection problems. We used a realistic simulated environment modeled after a region in the Gulf of Mexico [6]. This is a 3-layer shallow-water environment comprised of water over sediment over rock. A measured sound-velocity profile from the Gulf of Mexico was used in the simulations. The environmental parameters describing this environment are shown in Table 1. The receiving array for these examples is a 20-element ver-

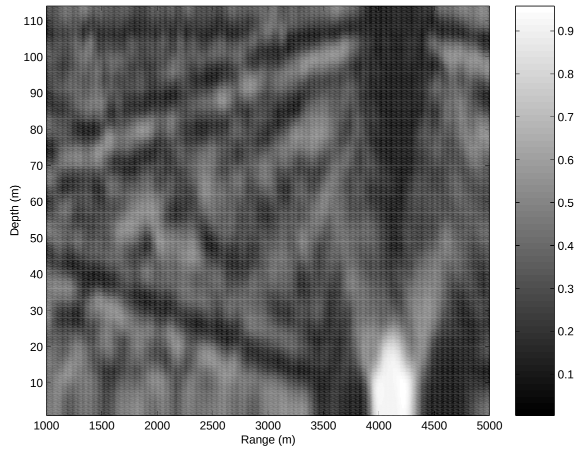

Fig. 2. Power response ($||\mathbf{Ma}||^2$) for a matrix filter designed using (7).

Parameter	Value
water depth	114 m
sediment sound speed	1600 m/s
rock sound speed	1625 m/s
sediment attenuation	0.5 dB/λ
sediment density	1.85
rock density	2.2

Table 1. Environmental parameters for Gulf of Mexico environment

tical array of sensors placed every 5 meters in depth from 10 to 105 m.

In the first example, consider two equal power 250 Hz sources. Source 1 is at a range of 4150 m from the receiving array and a depth of 5 m, and source 2 is at a range of 4250 m and a depth of 98 m. If the received signal consists of source 1 alone, the Bartlett processor produces the output shown in Fig. 3. If the received signal consists of source 2 alone, the Bartlett processor produces the output shown in Fig. 4. Note that the peaks of these plots occur at the ranges and depths corresponding to sources 1 and 2, respectively. When the received signal consists of both source 1 and source 2, the Bartlett processor produces the output shown in Fig. 5. This plot has peaks in the locations corresponding to the two sources but it also has about twenty false peaks!

We now apply a matrix filter to the two-source data before computing the Bartlett processor. The matrix filter was designed by solving the optimization problem in equation (8) with $\epsilon_1 = 2.5 \times 10^{-3}$ and $\epsilon_2 = 1.05$. The pass band for this filter was defined from 4000 to 4250 m in range and 1 to 10 m in depth. Note that source 1 is in the pass band of this filter. The output of the Bartlett processor operating on the

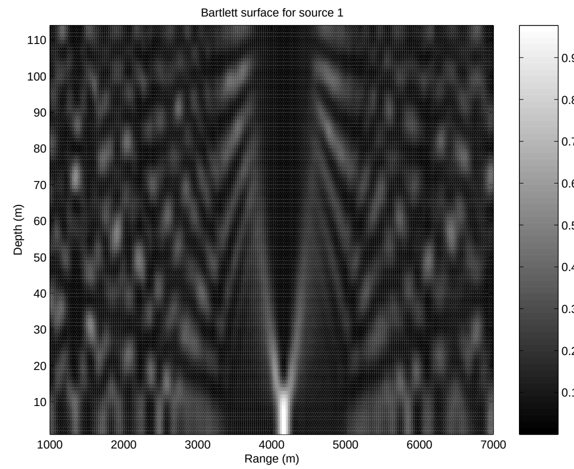

Fig. 3. Bartlett surface for source 1.

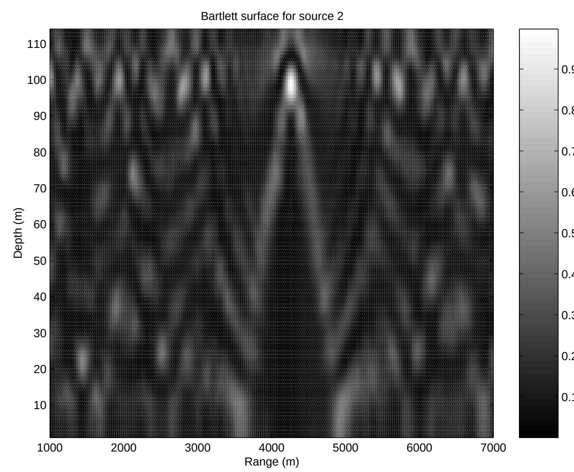

Fig. 4. Bartlett surface for source 2.

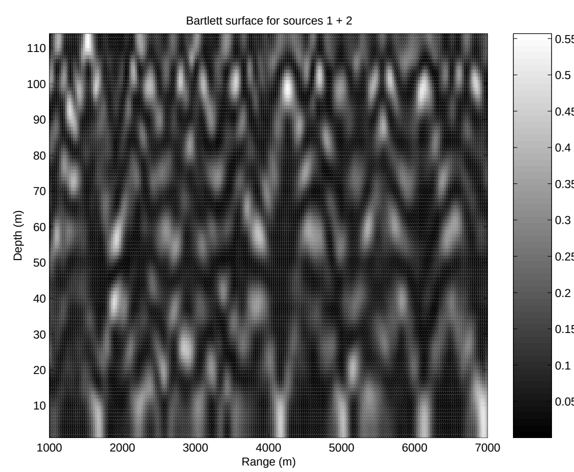

Fig. 5. Bartlett surface for sources 1 and 2.

matrix filtered two-source data is shown in Fig. 6. The effect of source 2 has been removed and the Bartlett surface has a peak at the location of source 1.

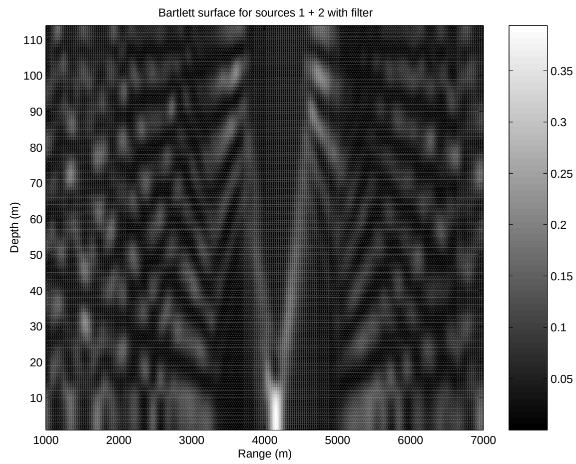

Fig. 6. Bartlett surface for sources 1 and 2 with matrix filtering.

In the second example we use the matrix filter designed using (7), whose power response is shown in Fig. 2. This filter is used as a "trip wire" to detect when a shallow source moves through the filter pass band (4000-4250 m). The source is at 5 m depth and moving at 5 m/s. Fig. 7 shows the range of the source as a function of time as well as the matrix filter power output, $||\mathbf{Md}||^2/||\mathbf{d}||^2$, as a function of time. The filter output is computed every 25 seconds, as indicated by circles in the plot. The output stays below 0.51

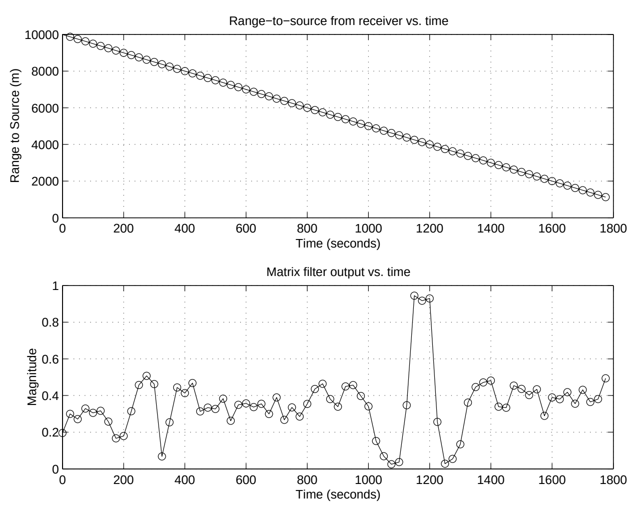

Fig. 7. Range to source and matrix filter power output versus time.

while the source is in the stop band, and exceeds 0.9 when the source is in the pass band. The three samples at which the matrix filter output exceeds 0.9 correspond to source ranges of 4000, 4125, and 4250 m.

5. CONCLUSION

This paper introduced matrix filters as a tool for localization and detection problems in passive sonar and gave two examples. The first example showed that matrix filters can simplify the localization problem by effectively removing a source from the received data. In this example the shallow source was in the pass band of the matrix filter. In practice, one could use a bank of matrix filters whose pass bands cover a region of range. Filters whose outputs exceeded a threshold would indicate the location of shallow sources. These sources could be subtracted from the observed data before attempting to localize the submerged source(s).

The second example showed that a shallow, moving source could be detected when it reached the filter pass band of a fixed matrix filter. Future work will include further development of these applications as well as the extension of matrix filter design to broadband signals.

6. REFERENCES

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