OPTIMUM DESIGN OF OVERLAPPING DISCRETE MULTITONE FILTER BANKS FOR DIGITAL SUBSCRIBER LINE CHANNELS

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ABSTRACT

In this paper, a new approach to the design of optimum transmultiplexer filter banks for overlapping discrete multitone (ODMT) multicarrier modulation is presented. ODMT has been proposed for use in high-speed data communications over twisted-pair copper wires such as digital subscriber lines (DSL), due to its ability to cope well with channel degradations and noise. In this study we propose a new design technique which minimizes overall interference. We consider the effect of the time domain overlap between successive symbol waveforms by considering each modulation filter as a series of filter components in time. We explicitly incorporate some knowledge of the digital subscriber line channel into our design technique, to account for the effects of time dispersion and frequency distortion. Numerical simulations show that the optimisation of the components in the mixed time-frequency domain leads to an increase in the signal-to-interference ratio of the order of 0.2–0.3 dB.

1. INTRODUCTION

Overlapped Discrete Multitone (ODMT) is a multichannel modulation technique, which is a member of the family of multicarrier modulation schemes [1,2]. Data is transmitted simultaneously on multiple channels using orthogonal modulation waveforms. In ODMT, modulation waveforms may overlap in time, allowing for longer filters with more design freedom, at the expense of system latency. ODMT systems are robust to noise due to the longer length and good spectral localization of the filters. However interference is a major problem. Orthogonality is lost as the waveforms pass through the channel leading to inter-symbol interference (ISI) and inter-carrier interference (ICI). Figure 1 illustrates the transmitter and receiver used in an ODMT system. ISI is the contribution to the output symbol $z_m[n]$ in time frame n, from all input symbols $x_r[p]$ with $p \neq n$. ICI is the contribution to the output symbol $z_m[n]$, from any input symbol from the same time frame $x_r[n]$ with $r \neq m$. Interference is unavoidable in ODMT systems due to channel distortion. However, careful design of the modulation filter bank, together with equalisation in the time and frequency domain can minimize interference to an acceptable level [1].

One technique to reduce interference is to provide spectral isolation between carrier channels by designing the modulating filters to be spectrally narrow. This technique is near optimum for suppression of ICI, at the expense of ISI.

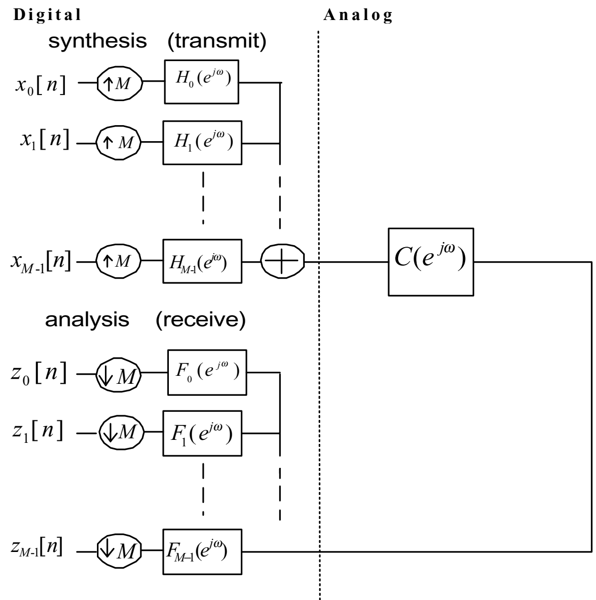

Figure1. O*DMT transceiver with inputs and corresponding outputs having the same index for simplicity. The communication channel is modelled as a filter with Fourier transform* $C(e^{j\omega})$.

This paper presents a novel approach for designing the modulation filters by considering the time-domain overlapping components of the modulating filters, leading to a large reduction in ISI. Numerical simulations indicate that this design approach leads to an increase in overall signal to interference (SIR) ratio.

2. FILTER CONSTRAINTS

Let us first consider the set of desirable features and necessary constraints required of an ODMT system.

2.1 Filter Length: For convenience, all filters should have the same length N, usually selected to be an integer multiple of the number of subchannels M, *i.e.*

$$N = gM. \quad (1)$$

The parameter g is referred to as the degree of overlap.

2.2 Analysis/Synthesis Relationship: If the analysis bank receives signals directly from the synthesis bank, it is desirable that symbols pass through the system without interference, *i.e.*, the outputs exactly match the inputs. This is known as the perfect reconstruction (PR) property. To achieve this, the analysis and synthesis filters can be chosen to be orthogonal or biorthogonal. Orthogonal filter banks are characterised by having analysis filters, which are the time-reversed mirrors of the synthesis filters:

$$f_m[n] = h_m[N-1-n], \quad 0 \le n \le N-1 \tag{2}$$

This constraint is removed in biorthogonal filter banks, allowing for greater design freedom. An example of biorthogonality being advantageous is the possible trade off between system delay and performance [3]. It has been suggested that relaxing PR constraints may give the benefit of more design freedom to combat channel degradations.

2.3 Fast Implementation: Cosine modulated filter banks (CMFBs) allow an efficient implementation of the structure shown in Figure 1. In a CMFB, all filters are derived from a single prototype filter $p[n]$ [4]. However, the only design freedom is the choice of this prototype.

This study considers orthogonal, perfect reconstruction, cosine modulated filter banks where the filter length is an integer multiple of the number of subchannels. From the discussion above it should be clear that these constraints may compromise overall performance for the sake of design ease and implementability. All filters were derived from a prototype $p[n]$ using the following equation:

$$h_m[n] = 2p[n]\cos\left(\frac{\pi}{M}(m+0.5)(n-\frac{gM-1}{2})-(-1)^n\frac{\pi}{4}\right). \tag{3}$$

The prototype is designed using the lattice approach [4], which gives highly non-linear optimisation problems. The quadratic constrained formulation described in [5] reduces the complexity of the optimization problem.

3. MINIMISING INTERFERENCE

The synthesis and analysis filters are chosen to be of length gM so that each input symbol $x_m[n]$ contributes to a set of N=gM samples transmitted over the channel. Define a sequence of N samples present at the output of the m^{th} filter at the time instance corresponding to the n^{th} data symbol as the waveform $d_m^n[k]$ for $0 \le k \le N-1$. This sequence is composed of contributions from symbols $x_m[q]$ from time frames $n-g+1 \le q \le n+g-1$. All waveforms from time frame n are added together and passed over the channel. Addition of sequences is a linear operation so waveforms form separate carriers, which can be considered independently of each other. The received signal is analysed by each analysis filter f_m. If the channel is ideal and the filter bank is PR, output symbols will exactly match input symbols. The perfect reconstruction conditions can be summarized as follows:

$$\sum_{l=0}^{N-1} f_m[N-1-l]\,d_j^n[l] = a_m x_m[n]\delta(j-m)\ , \tag{4}$$

where a_m is an arbitrary constant, and $\delta[n]$ is the Kronecker delta. These equations will only be approximately obeyed when channel distortion is introduced. Ideally we would like to minimise errors in each of these equations independently by optimising the transmitted waveforms with respect to degradations in the channel. However each waveform will depend on symbols selected from a finite set of fixed values. As the number of fixed values increases the number of possible waveforms to optimise becomes very large. An alternative representation is to consider the modulation waveform in terms of separate contributions from each of the $2g-1$ symbols that affects any set of N samples. Consider the set of $2g-1$ filter components, each padded with leading or trailing zeros to be of length gM=N:

$$h_m^k = [0\ 0\ 0 \ldots 0\ 0\ 0\ h_m[0]\ h_m[1] \ldots h_m[kM-1]],\ 1 \le k \le g$$
$$h_m^k = [h_m[(k-g)M]\ h_m[(k-g)M+1] \ldots h_m[gM-1]\ 0\ 0\ 0 \ldots 0\ 0\ 0\ ],$$
$$g < k \le 2g-1 \tag{5}$$

Using these components the modulation waveforms can be written as:

$$d_m^n[l] = \sum_{k=1-g}^{g-1} x_m[n+k]\,h_m^{g-k}[l]\ ,\ 0 \le l \le gM-1. \tag{6}$$

Eq. (6) allows us to rewrite the orthogonality conditions in terms of filter components:

$$x_j[n+k]\sum_{l=0}^{N-1} f_m[nM-l]\,h_j^{g-k}[l] = a_m x_m[n]\delta(j-m)\delta(k)\ . \tag{7}$$

As the components are passed over the channel orthogonality is destroyed. Errors in any of the conditions involving the time domain components which are not of the form h_m^g contribute to ISI. Optimising the overall spectral localization of the prototype filter does not explicitly take the interference from all these particular components into account. Moreover, to minimise interference at the receiver, it is desirable that distortion of each component due to the channel is limited to a scalar multiplication. The question now arises how do we optimise the components such that this is the case.

4. OPTIMISING COMPONENTS

For mathematical convenience, only consider the non-zero samples of each filter component. Each component h_m^k is a sequence of length L^k where

$$\begin{aligned} L^k &= kM, && 1 \le k \le g, \\ L^k &= 2gM - kM, && g < k \le 2g-1. \end{aligned} \tag{8}$$

As a component h_m^k is transmitted over a DSL channel, the channel distorts it to give a received component r_m^k. Since the channel is not flat, the channel impulse response is dispersive. As h_m^k is passed over the channel, it will spread out in the time domain. If the discrete model channel is of length W and h_m^k is of length L^k, the received component r_m^k will be of length $W+L^k-1$. The $W+L^k-1$ samples are rectangular windowed so that L^k samples g_m^k are selected. Ideally, g_m^k is equal to h_m^k. However, even if this was the case, samples of r_m^k outside the window may still contribute to ISI. There are three techniques for reducing the effects of time dispersion:

1. Increase the length of the components.
2. Use a time domain equaliser to reduce the length of the channel.
3. Design the components so that most of the energy of the received component is concentrated inside the rectangular window.

If the effect of dispersion is minimised so as to be insignificant then we can describe the relationship between h_m^k and g_m^k as being approximately linear time invariant i.e.

$$G_m^k(e^{j\omega}) \approx H_m^k(e^{j\omega})C(e^{j\omega})\ . \tag{9}$$

Decompose Eq. (9) into its amplitude and phase components:

$$|G_m^k(e^{j\omega})| \approx |H_m^k(e^{j\omega})||C(e^{j\omega})|\ . \tag{10}$$

$$\angle G_m^k(e^{j\omega}) \approx \angle H_m^k(e^{j\omega}) + \angle C(e^{j\omega})\ . \tag{11}$$

Ideally we would like to limit distortion to a scalar multiplication. Let us consider how amplitude and phase distortion, affect this condition.

4.1 Amplitude Distortion In general the magnitude of the channel frequency response $|C(e^{j\omega})|$ decreases as the frequency ω approaches π. If each component $H_m^k(e^{j\omega})$ is confined to a single centre frequency ω_m, Eq. (10) reduces to

$$|G_m^k(e^{j\omega})| \approx |H_m^k(e^{j\omega})||C(e^{j\omega_m})|, \qquad (12)$$

and distortion is limited to multiplication by a constant. In this case there is zero amplitude distortion. Of course, finite length waveforms can not be restricted to a single frequency. As the different frequencies in the modulation waveform are scaled differently, distortion is no longer limited to a scalar multiplication and there is interference at the receiver. Rewriting Eq. (12) as a sum of the ideal term and an error term we obtain:

$$G_m^k(e^{j\omega}) = \left|H_m^k(e^{j\omega})\right|\left|C(e^{j\omega_m})\right| + \left|H_m^k(e^{j\omega})\right|\left[\left|C(e^{j\omega})\right| - \left|C(e^{j\omega_m})\right|\right] \qquad (13)$$

To minimise amplitude distortion, minimise the error term. Assuming frequency degradation we can say that $\left\|C(e^{j\omega})\right| - \left|C(e^{j\omega_m})\right\|$ increases with $|\omega - \omega_m|$. Thus to limit amplitude distortion we need to compact $\left|H_m^k(e^{j\omega})\right|$, about the centre frequency ω_m. with the degree of compaction dependent upon the channel characteristics.

4.2. Phase Distortion. A DSL channel $C(e^{j\omega})$ can be approximated as having a linear phase response. This implies all frequencies are approximately delayed by a single time delay t_d. As long as timing recovery is performed correctly, there will be negligible phase distortion.

5. CHOICE OF OBJECTIVE FUNCTION

Having considered sources of inter symbol and inter carrier interference in the DSL environment, let us define objective functions to mitigate these effects. From section 2.4, the only design freedom we have is in the selection of a prototype $p[n]$. If we concern ourselves with the traditional design approach, we need only consider the spectral characteristics of the filter prototype, since these are preserved for all the modulated filters. However cosine modulation does not preserve the frequency characteristics of the filter components defined in section 3. If we optimise the frequency response of the components of the prototype, this does not guarantee the optimisation of components of all filters. Hence the complete set of filters must be considered when optimising the components. In addition cosine modulation does not preserve time domain properties so all filters must be considered individually when optimising time domain properties. Filters were optimised using a combination of two non-linear optimisation techniques: Powell's method and the Nelder-Mead simplex algorithm [6].

5.1 Minimising Stopband Energy: This objective function calculates the ratio of stopband energy to passband energy of the prototype, where the DC gain is set to unity, *i.e.*, $P(0)=1$. Mathematically we can describe this function as follows:

$$O_1(p) = \frac{\int_{\omega=\omega_c}^{\pi} \left|P(e^{j\omega})\right|^2 d\omega}{\int_{\omega=0}^{\omega=\omega_c} \left|P(e^{j\omega})\right|^2 d\omega} \quad with \ \omega_c = \frac{\pi}{M} \qquad (14)$$

5.2 Time Domain Compaction (TDC): This objective function reduces the effects of dispersion in the time domain, for each of the filter components. For an optimal realisation, we need to know the channel impulse response, which will not be known in practice. Instead, we use an "average" channel impulse response $w[n]$, derived by considering five typical DSL impulse responses at the desired sampling frequency. This approximation is reasonable since all DSL impulse responses tend to look similar in shape, starting at zero, rising to a peak before decaying back to zero. Each component is passed through the averaged channel and the ratio of the energy inside the desired window to the energy outside is calculated. To find the position of the desired window, the delay D of the averaged channel was calculated. Mathematically we describe TDC as follows:

$$O_2(p) = \sum_{m=0}^{M-1}\sum_{k=1}^{2g-1} \frac{\sum_{n=-\infty}^{D-1}\left(h_m{}^k * w\right)^2 + \sum_{n=D+L^k}^{\infty}\left(h_m{}^k * w\right)^2}{\sum_{n=D}^{D+L^k-1}\left(h_m{}^k * w\right)^2} \qquad (15)$$

where the numerator represents the energy outside the time window that the received component should ideally occupy, and the denominator is the energy inside this window.

6.3 Amplitude Distortion Reduction (ADR). This objective function mitigates the effects of amplitude distortion (see Section 4.1). Ideally, we would like to know the magnitude of the channel frequency response. As an approximation we use an averaged channel frequency response $A(e^{j\omega})$ derived from five standard channels. This is a reasonable approximation, as generally DSL channels will degrade with frequency with the greatest variations at lower frequencies. Using the averaged channel we estimate $\left\|C(e^{j\omega_m})\right| - \left|C(e^{j\omega_n})\right\|$ where ω_m is the centre frequency of the component and ω_n is a set of P equally spaced frequencies. Mathematically we can define ADR as follows:

$$O_3(p) = \sum_{k=1}^{2g-1}\sum_{m=0}^{M-1}\sum_{n=0}^{P-1}\left|H_m^k(e^{j\omega_n})\right|\left[\left\|A(e^{j\omega_m})\right| - \left|A(e^{j\omega_n})\right\|\right] \qquad (16)$$

6. SIMULATIONS AND RESULTS

For comparison, we designed two sets of modulation filters using two different objective functions.

Criterion 1: This is the proposed new design with the components optimised in the mixed time frequency domain. In this case the objective function was chosen as being a weighted sum of the $O_2(p)$ and $O_3(p)$ objective functions. The selection of weights depends on the length of the channel impulse response as compared to the length of the components.

Criterion 2: This is the original design technique of [3] with the prototype designed using objective function $O_1(p)$.

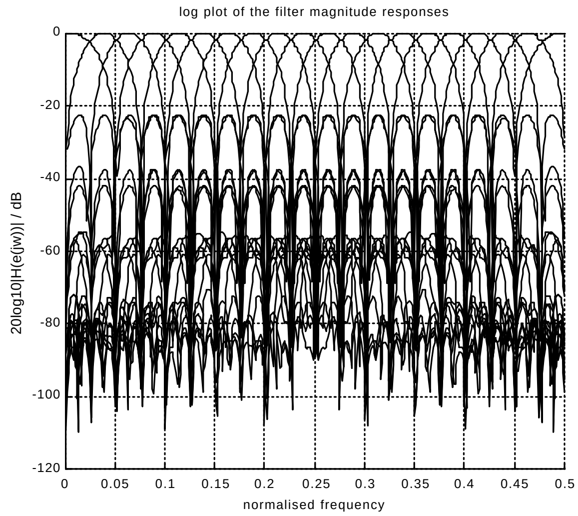

Figure 2: *Modulation filters designed under Criterion 1. The magnitude responses of twenty orthogonal filters are shown here, with each filter having 120 taps. The sidelobes of each filter are relatively large in magnitude (approximately only –22 dB down on the main lobe).*

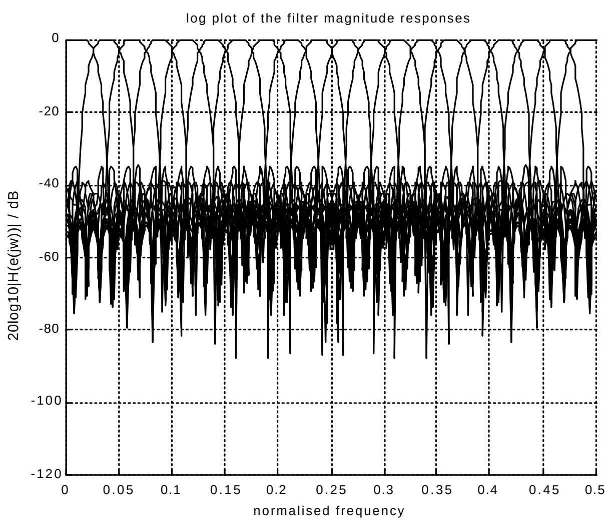

Figure 3: *Modulation filters designed under Criterion 2. The magnitude responses of twenty orthogonal filters are shown here, with each filter having 120 taps. The sidelobes of each filter are far lower in magnitude than those in Fig. 2 (approximately –40 dB down relative to main lobe).*

Simulations were performed for 2 ADSL test channels at a sampling frequency of 100 kHz giving a nominal channel spacing of 2.5kHz. The value of M was taken as 20. Pulses were modulated at 16 different frequencies only, *i.e* only 16 out of the possible 20 channels were used, and various values of g ranging from 2 to 6 were considered. Table 1 shows the differences in the ratio of stopband energy to passband energy for the two proposed methods. Not surprisingly, Criterion 2 produces filters with better spectral localization by an order of two approximately.

Filter Length	**Criterion1**	**Criterion2**
40	4.23×10^{-2}	2.59×10^{-2}
80	4.28×10^{-2}	3.45×10^{-3}
120	4.81×10^{-2}	5.84×10^{-4}

Table 1: *The ratio of stopband energy to passband energy for the prototype lowpass filter designed under Criteria 1 and 2.*

To assess the effect of our designs on system performance, we calculated measures of signal-to-interference ratio (SIR) for various configurations. This is the ratio of observed power due to each correctly received symbol to power introduced by other symbols. The simulations employed a 1-tap frequency equaliser at the output of each receive filter. Using a set of 10000 random binary symbols (±1), we calculated SIR for each channel and calculated the average SIR. External noise was not included in the simulation.

Channel	**Filter Length**	**Criterion 1 SIR (dB)**	**Criterion 2 SIR (dB)**
t1_601_no13	**40**	**14.76**	**14.20**
t1_601_no13	**80**	**14.76**	**14.48**
t1_601_no13	**120**	**14.77**	**14.47**
csa_no4	**40**	**17.30**	**17.09**
csa_no4	**80**	**17.32**	**17.18**
csa_no4	**120**	**17.34**	**17.14**

Table 2: *Comparison of observed signal-to-interference ratios for different filter designs.*

7. CONCLUSION

Our approach to optimisation, which considers time and frequency domain interference, gives an average SIR gain of 0.24 dB as compared to the minimisation of stopband energy approach. However, optimisation of multiple time-domain components is a highly challenging non-linear optimisation problem. Future work will concentrate on exploring more powerful non-linear optimisation techniques and the choice of a more flexible set of design constraints. Further improvements would include employing an adaptive training scheme of the filters to replace the averaged channel.

8. REFERENCES

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