# A NEW CRITICALLY SAMPLED NON-UNIFORM SUBBAND ADAPTIVE STRUCTURE


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## ABSTRACT

In this paper, a non-uniform subband adaptive structure with critical sampling of the subband signals is derived. With the assumption of non-overlapping between non-adjacent analysis filters, the resulting structure yields exact modeling of an arbitrary FIR system. An LMS-type adaptation algorithm, which minimizes the sum of the subband squared-errors, is obtained for updating the coefficients of the proposed structure, resulting in significant convergence rate improvement for colored input signals when compared to the fullband LMS algorithm. Computer simulations illustrate the convergence behavior of the proposed non-uniform subband adaptive filter in the applications of system identification and acoustic echo cancellation.


## 1. INTRODUCTION

Adaptive filtering in subbands has been extensively investigated in order to reduce the computational complexity and increase the convergence rate of high-order adaptive FIR filters. It has been mostly applied to echo cancellers for audio teleconferencing, where adaptive FIR filters of very large lengths (1000-4000 coefficients) are used. Subband adaptive structures with critical sampling [1]-[3] and with noncritical sampling [4]-[6] have been proposed in the literature. In order to avoid aliasing problems, the sampling rate of the decomposed signals is reduced by a factor smaller than the number of subbands in the oversampled approach [4], while adaptive cross-filters between adjacent subbands are needed in the critical sampled approach [1]. In a recently proposed critically sampled subband structure [2], the cross-filters are identical to the direct-path adaptive filters and, therefore, do not need to be adapted separately, resulting in an improved adaptation convergence and a reduced computational complexity when compared to other critically sampled subband structures [3].

Most of the subband adaptive filters employ uniform filterbanks. A recent work [6], however, has shown that non-uniform subband adaptive filters might outperform the uniform ones. The non-uniform subband filter presented in [6] used oversampled subband signals. In this paper, we develop a non-uniform subband structure with critical sampling of the subband signals. Such adaptive subband structure is a generalization of the uniform subband filter presented in [2],[3]. Due to the non-uniform frequency decomposition of the input signal, the distinct adaptive subfilters work at different rates, which leads to some particularities in the adaptation algorithm.

## 2. THE CRITICALLY SAMPLED NON-UNIFORM SUBBAND ADAPTIVE STRUCTURE

The non-uniform subband adaptive filter with critical sampling proposed in this paper is derived from an adaptive subband structure which employs an analysis filterbank to decompose the input signal and sparse adaptive filters in the subbands [3]. It is illustrated in Fig. 1, where $x(n)$ is the input signal, $H_k(z)$ are the analysis filters of an M-channel filterbank, $G_k(z^M)$ are the sparse adaptive subfilters, $d(n)$ is the desired signal, $e(n)$ is the error signal used in the adaptation algorithm, and $\Delta$ is the delay introduced in the input signal by the filterbank. It has been shown in [3] that the structure of Fig. 1 is able of modeling an arbitrary FIR system, with the introduction of a delay $\Delta$, by properly choosing the filterbank (perfect or near-perfect reconstruction bank) and the number of adaptive coefficients.

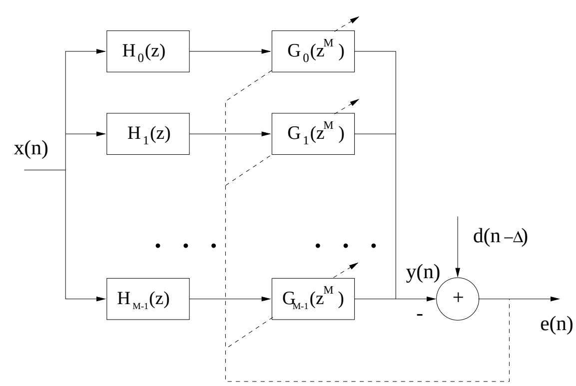


Figure 1: Adaptive structure using an analysis filterbank and sparse subfilters.

Recently, in [7], the subband adaptive filter of Fig. 1 was generalized, such that a non-uniform analysis filterbank (or a discrete wavelet) was used to decompose the input signal, resulting in a structure which employs sparse adaptive subfilters with different lengths and sparsity factors. This non-uniform subband filter is illustrated in Fig. 2, where we consider, for simplicity, a three-channel filterbank implemented by a two-level tree structure. The equivalent analysis filters of the three-channel filterbank of Fig. 2 are

$$H_0(z) \quad = \quad H^0(z)H^0(z^2),$$

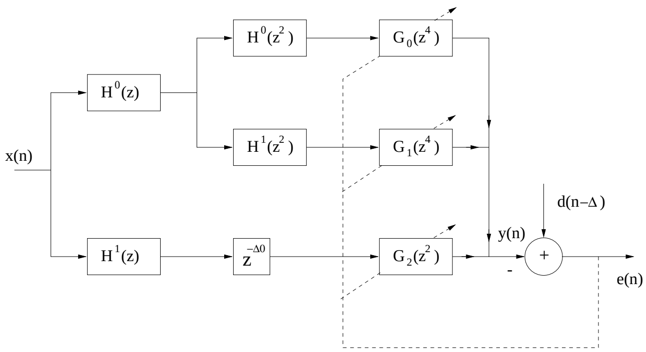


Figure 2: Non-uniform subband filter using a two-level tree structure analysis filterbank and sparse adaptive subfilters.

$$H_1(z) = H^0(z)H^1(z^2),$$
$$H_2(z) = H^1(z), \tag{1}$$

where $H^0(z)$ and $H^1(z)$ represent the analysis filters of a two-channel perfect reconstruction filterbank. The delay $\Delta_0$ in the third band is included to compensate for the smaller delay intoduced in this band by the filterbank. The generalization of the structure of Fig. 2 results in adaptive subfilters with different sparsity factors, that is, $G_i(z^{M_i})$. The length of such subfilters should be at least [7]

$$K_i = N/M_i + N_{F_i}/M_i + 1, \tag{2}$$

where $N$ is the order of the unknown system and $N_{F_i}$ is the order of the $i$-th subfilter $F_i(z)$ of the corresponding synthesis filterbank. For the three-channel structure of Fig. 2, the synthesis filters are

$$F_0(z) = F^0(z^2)F^0(z),$$
$$F_1(z) = F^1(z^2)F^0(z),$$
$$F_2(z) = F^1(z), \tag{3}$$

where $F^0(z)$ and $F^1(z)$ are the corresponding synthesis filters of the two-channel multirate system with analysis filters $H^0(z)$ and $H^1(z)$.

In the sparse subband adaptive structure of Fig. 2, the sampling rates of the subband signals are not reduced. In order to derive a critically sampled subband structure, a non-uniform perfect reconstruction multirate system is included following each adaptive filter of the sparse subband filter of Fig. 2, as illustrated for the $k$-th band in Fig. 3. Considering that the analysis filters are

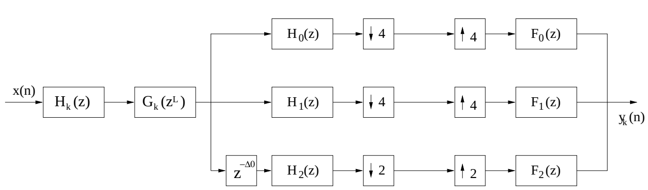


Figure 3: Illustration of the $k$-th band of the structure of Fig. 2 followed by a non-uniform three-channel perfect-reconstruction multirate system.

sufficiently selective, so that it can be assumed that their frequency responses do not overlap with those of non-adjacent subbands, and using the noble identity [8], we obtain the simplified structures of Fig. 4 for the three bands of the non-uniform critically sampled subband filter. The delays $\Delta_1$, $\Delta_2$ and $\Delta_3$ in Fig. 4 are needed in

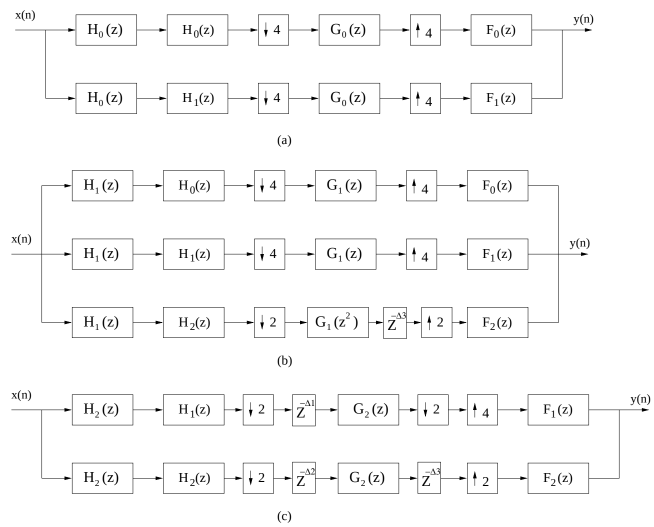


Figure 4: Subbands of the three-channel non-uniform critically sampled structure.

order to match the delays introduced by the different lengths analysis filters. Considering orthogonal filterbanks with analysis and synthesis filters, $H_i(z)$ and $F_i(z)$, of order $N_i$, with $N_0 = N_1$, the delays of Fig. 4 are given by

$$\Delta_1 = \Delta_2 = \frac{N_0 - N_2}{2}, \tag{4}$$
$$\Delta_3 = \frac{\Delta_1}{2} = \frac{N_0 - N_2}{4}. \tag{5}$$

Combining the three bands shown in Fig. 4, and considering the system identification application, we obtain the complete three-channel non-uniform subband adaptive filter of Fig. 5. In this figure, the filters $H_{i,j}(z)$ are related to the analysis filters $H_i(z)$ by

$$H_{i,j}(z) = H_i(z)H_j(z), \tag{6}$$

and the delays $\Delta_4$ and $\Delta_5$ applied to the decomposed desired signals are given by

$$\Delta_4 = \frac{N_0}{4}, \tag{7}$$
$$\Delta_5 = \Delta_2 + \frac{N_2}{4} = \frac{N_0}{2} - \frac{N_2}{4}. \tag{8}$$

The overall input-output delay introduced by the structure of Fig. 5 is $\Delta = 2N_0$.

## 3. ADAPTATION ALGORITHM

We now derive an LMS-type algorithm for updating the subfilters coefficients of the proposed critically sampled structure. For simplicity, we consider here only the three-channel case shown in Fig. 5. The algorithm derivation can be extended in a straigthforward manner to other filter bank configurations.

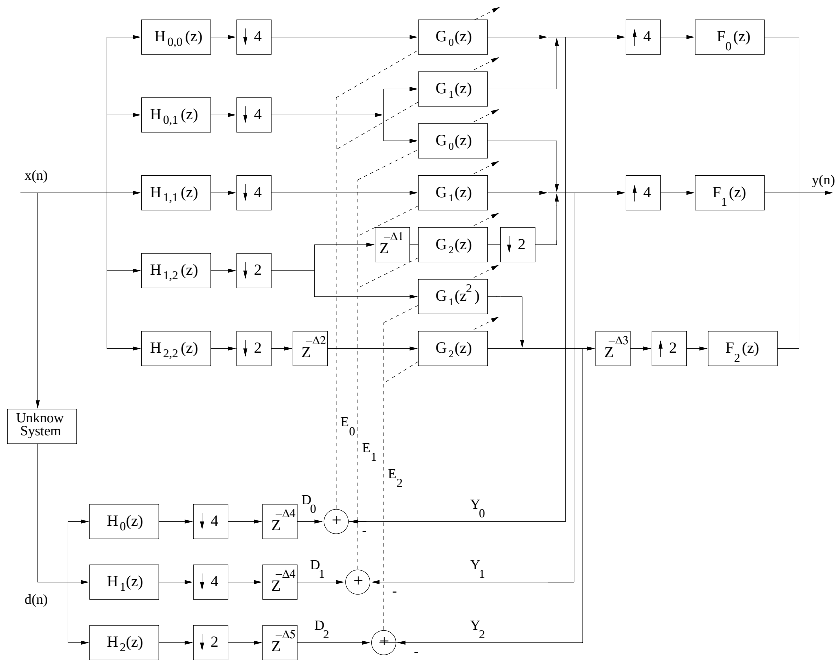


Figure 5: Complete subband adaptive structure considering the identification system problem.

The adaptation is performed at the lowest rate (one fourth of the input rate for the three-channel structure). Denoting as $X_{i,j}(m)$ the downsampled signal at the output of the filters $H_{i,j}(z)$ (see Fig. 5), as $\mathbf{X}_{i,j}(m)$ the vector containing the most recent $K_i$ samples of $X_{i,j}(m)$, and as $\mathbf{G}_i(m)$ the vector containing the coefficients of the subfilter $G_i(z)$ at iteration $m$, and considering as cost function the sum of the instantaneous squared-errors of the first two subbands and of the average of the two samples of the squared-error of the third subband (which is working at twice the adaptation rate), that is,

$$J(m) = E_0^2(m) + E_1^2(m) + \frac{1}{2}\left[E_2^2(2m-1) + E_2^2(2m)\right], \quad (9)$$

the general form for the LMS-type adaptation algorithm that minimizes $J(m)$ is given by

$$\begin{aligned} \boldsymbol{G}_0(m+1) &= \boldsymbol{G}_0(m) + \mu_0[\boldsymbol{X}_{0,0}(m)E_0(m) \\ &\quad + \boldsymbol{X}_{0,1}(m)E_1(m)], \end{aligned} \quad (10)$$

$$\begin{aligned} \boldsymbol{G}_1(m+1) = \boldsymbol{G}_1(m) + \mu_1[&\boldsymbol{X}_{0,1}(m)E_0(m) + \boldsymbol{X}_{1,1}(m)E_1(m) \\ &+ \frac{1}{2}(\boldsymbol{X}^e_{1,2}(2m-1)E_2(2m-1) \\ &+ \boldsymbol{X}^e_{1,2}(2m)E_2(2m))], \end{aligned} \quad (11)$$

$$\begin{aligned} \boldsymbol{G}_2(m+1) &= \boldsymbol{G}_2(m) + \mu_2[\boldsymbol{X}_{1,2}(m)E_1(m) \\ &\quad + \frac{1}{2}(\boldsymbol{X}_{2,2}(2m-1-\Delta_2)E_2(2m-1) \\ &\quad + \boldsymbol{X}_{2,2}(2m-\Delta_2)E_2(2m))], \end{aligned} \quad (12)$$

where the vector $\boldsymbol{X}^e_{1,2}$ contains the most recent $K_1$ even samples of the signal $X_{1,2}$. In the last equations, the error signals are given by

$$E_0(m) = D_0(m-\Delta_4) - [\boldsymbol{X}^T_{0,0}(m)\boldsymbol{G}_0(m) + \boldsymbol{X}^T_{0,1}(m)\boldsymbol{G}_1(m)], \quad (13)$$

$$\begin{aligned} E_1(m) = D_1(m-\Delta_4) - [&\boldsymbol{X}^T_{0,1}(m)\boldsymbol{G}_0(m) + \boldsymbol{X}^T_{1,1}(m)\boldsymbol{G}_1(m) \\ &+ \boldsymbol{X}^T_{1,2}(2m-\Delta_1)\boldsymbol{G}_2(m-\Delta_1)], \end{aligned} \quad (14)$$

$$\begin{aligned} E_2(m') = D_2(m'-\Delta_5) - [&\boldsymbol{X}^{e^T}_{1,2}(m')\boldsymbol{G}_1(m) \\ &+ \boldsymbol{X}^T_{2,2}(m'-\Delta_2)\boldsymbol{G}_2(m-\Delta_2)], \end{aligned} \quad (15)$$

for $m' = 2m$ and $m' = 2m-1$. In order to improve the convergence rate of the adaptation algorithm for colored noise input signals, the step-sizes are made inversely proportional to the sum of the powers of the signals involved in the adaptation of the coefficients, that is:

$$\begin{aligned} \mu_0 &= \frac{\mu}{P_{0,0} + P_{0,1}}, \\ \mu_1 &= \frac{\mu}{P_{0,1} + P_{1,1} + P_{1,2}}, \\ \mu_2 &= \frac{\mu}{P_{1,2} + P_{2,2}}, \end{aligned} \quad (16)$$

where $P_{i,j}$ is the power estimate of the signal $X_{i,j}$.

## 4. SIMULATION RESULTS

Computer simulations are presented in order to illustrate the convergence behavior of the non-uniform subband adaptive filter proposed in this paper. In the first experiment, the identification of a length $N = 400$ FIR system is considered. The input signal was a colored noise sequence generated by passing a white noise sequence by a first-order IIR filter with its pole located at $z = 0.9$. The non-uniform subband filter of Fig. 5 was implemented by a two-level tree structure (as in Fig. 2) with near perfect-reconstruction prototype filters ($H^0(z)$ and $H^1(z)$) of order 31 [8] and adaptation step-size $\mu = 0.5/K_0$. Figure 6 presents the mean-square error (MSE) evolution for the non-uniform subband structure (NUSB) and for the fullband LMS algorithm (LMS). We can see from Fig. 6 that the non-uniform subband filter has significantly better performance than the fullband LMS for colored-noise input. The steady-state MSE of the non-uniform subband structure is determined by the stopband attenuation of the analysis filters, since it was assumed that there was no overlap among non-adjacent subbands.

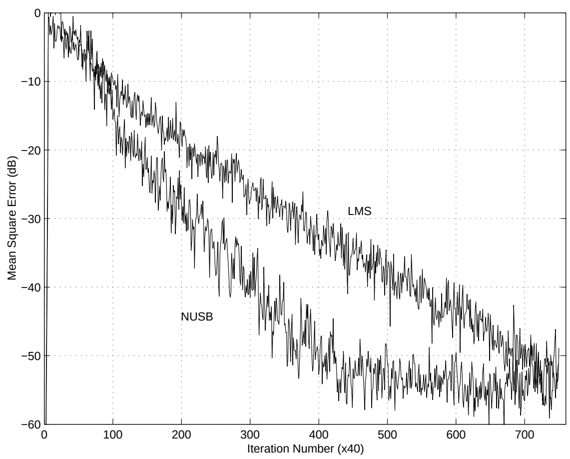


Figure 6: Results for the system identification simulations.

Another experiment with the non-uniform subband adaptive filter and with the fullband LMS algorithm was carried out for echo cancelling in a teleconference room. The microphone and loudspeaker signals were sampled at 8KHz. In order to implement a length 1200 impulse response, the number of coefficients of the subfilter were $K_0 = K_1 = 323$ and $K_2 = 646$. To visualize the improvement obtained with the non-uniform subband structure, the residual echo was decomposed into two subbands. In the low frequency band, the LMS and the NUSB algorithms had the same performance. However, in the high frequency band, the residual echo of the NUSB was significantly smaller, as can be seen in Fig. 7.

## 5. CONCLUSIONS

In this paper, we have developed a new critically sampled subband adaptive filter, which employs a non-uniform filterbank (or a discrete wavelet) to decompose the input signal. The adaptation is performed at the lowest sampling rate, using an LMS-type algorithm. Simulation results in system identification and echo cancellation applications were presented. In the system identification simulation, it was shown that, besides exactly modeling, significant convergence improvement can be achieved for colored input signals when compared to the conventional LMS algorithm. In the acoustic echo cancellation simulation, the high frequency residual echo of the proposed non-uniform subband structure was significantly smaller than that of the fullband LMS algorithm.

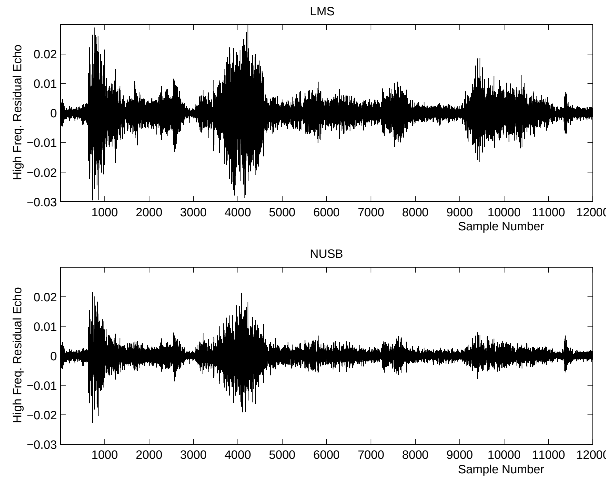


Figure 7: Results for the acoustic echo cancelling simulations.

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