APPLICATION OF STATE SPACE FREQUENCY ESTIMATION TECHNIQUES TO RADAR SYSTEMS

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ABSTRACT

Many applications for FMCW radar systems require the resolution of several closely spaced frequencies. In general the Fast Fourier Transform (FFT) is used for spectrum estimation, despite the inherent resolution problems. To overcome the resolution limitation the state space approach has been proposed in [1-3]. However, most experimental results in [1-3] are based on simulations. This paper points out the differences of the "real world" signal structure to simulated signals, and describes the problems posed by real radar signals. For one of the key steps of the state space algorithm, model order selection, a novel algorithm based on a posteriori analysis is introduced. The feasibility of the new approach is verified with an actual 24 GHz level gauging FMCW radar system. Our approach yields stable and accurate results with a resolution approximately three times higher than the FFT resolution.

1. INTRODUCTION

Precise close distance ranging using radar systems is a very interesting and demanding radar application [4]. Technical problems like multiple echo suppression, signal interference, resolution of closely spaced targets and robustness to clutter noise have to be solved. Additionally, the available bandwidth is limited by technical means and official regulations.

In general FMCW radar systems are well suited for level gauging. They allow the resolution of multiple targets and yield precise, energy efficient measurements [4-5]. In most FMCW systems the measured distance is derived from the spectrum of the received signal. Typically, the spectrum estimation is done with the Fast Fourier Transform (FFT) since it is a well known reliable, stable and fairly fast algorithm. The next step is then the search for maxima in the fourier spectrum. Ideally each maximum corresponds to a reflecting target, and the respective distance is computed from the frequency of the maximum [5].

However, the resolution of closely spaced targets in the FFT spectrum is limited by the bandwidth [6]. The resolution problems associated with the FFT cause strong measurement errors when the desired target, i.e., a liquid level in a tank, is close to another target, i.e. a traverse in a tank. Additionally clutter noise components close to targets are not resolved, causing severe measurement errors. State space methods according to [1-3] have been shown to achieve a far better resolution of closely spaced spectral components than the FFT.

Usually the state space approach consists of two steps: First a singular value decomposition is made, and only the components representing the noise parts are rejected. This requires an accurate and consistent estimation of the number of signal components, the so called model order. The second step is the extraction of the frequencies from the remaining signal components.

The experimental results in [1-3] have been obtained with simulations of 2 to 3 ideal sinuoids superimposed with white noise. The signal structure of applied industrial systems usually differs significantly. For instance FMCW radar system signals compose of many different superimposed sinusoids, some of them with small magnitude forming clutter noise. Additionally, technical signals always inhibit small non-linearity and other systematic distortions [7]. These problems significantly disturb the model order estimation suggested in [1] and require a completely new approach to model order estimation.

To our knowledge, no adaptation of the state space algorithm to typical FMCW radar signals has been made. The goal of this paper is to show the necessary modifications of the state space algorithm to process FMCW radar signals. The new approach is verified with experiments, using a 24 GHz tank level gauging radar system described in [4]. The first actual field measurements yield an increase in resolution by a factor of three over conventional FFT processing.

The paper is structured as follows: First a short review of the applied algorithm according to [1] is given. Then the limitations of conventional model order selection are discussed and the new approach is introduced. The next section contains our experimental results and compares state space and FFT evaluation.

2. STATE SPACE APPROACH

The initial signal consists of the measurement data of superimposed undamped sinusoids, and additional white noise $\xi(n)$. This can be written as:

$$y(n) = \sum_{k=1}^{p} c_k e^{j\cdot\omega_k\cdot n} + \xi(n) \qquad (1)$$

where y(n) is the actual measurement, p represents the model order, ω_k the frequency and c_k is the complex amplitude of the signal. Most technical systems yields only real undamped sinusoidal signals. For such systems the model order p equals two times the number of sinusoidal signal components q. To obtain the desired state space representation we first introduce a state vector X(n) that contains the past p measurements y(n-1)...y(p):

$$X_c(n) = \begin{bmatrix} y(n-1) \\ y(n-2) \\ \vdots \\ y(n-p) \end{bmatrix} \tag{2}$$

Using the state space vector and the linear predictability of sinusoidal signals, the state propagation equation is obtained:

$$X_c(n+1) = F_c \cdot X_c(n) \tag{3}$$

with F_c being the state space transition matrix. It can be shown that the eigenvalues λ_i of the state space transition matrix F then contain the frequencies ω_k, or more exact [1]:

$$\lambda_k = e^{j \cdot \omega_k} \; k = 1...p \tag{4}$$

The state space transition matrix can be obtained from any factorization of the covariance matrix F [1], the most common approach is the use of the singular value decomposition. With $\hat{R}$ being an estimator of the Toeplitz Covariance Matrix with size $L \times L$ and L > p one can compute the SVD:

$$\hat{R} = U \cdot S \cdot V^H \tag{5}$$

Then, the noise reduction by eliminating the last L-p elements and the corresponding eigenvectors is performed. The state space matrix F is computed from U using the least squares approach. At first, one sets up a matrix Θ_1 consisting of U without the first row, and then another matrix Θ_2, consisting of U without the last row. F is then given by:

$$F = \Theta_1 \cdot \Theta_2^+ \tag{6}$$

where Θ_2^+ denotes the pseudoinverse of Θ_2 .

Obviously the correct estimation of the model order p is the crucial point for the success of the noise reduction. In most practical applications the estimation of the model order becomes very complicated, due to the large number of sinusoids present.

3. MODEL ORDER SELECTION

The simplest approach to the determination of the model order is the singular value spectrum [1]. An ideal input signal for the state space system consists only of p superimposed exponentials. For such a system the rank of the covariance matrix equals the number of independent exponentials. Clearly the SVD of R (5) then yields p non-zero eigenvalues of the covariance matrix as shown in Fig. 1 a). The fact that the eigenvalues are not exactly zero but have a very small magnitude is caused by quantization noise and round off errors. The gap between those eigenvalues representing the signals and those caused by quantization is more than 120 dB. However, it is important to note that each eigenvalue represents a superposition of the underlying

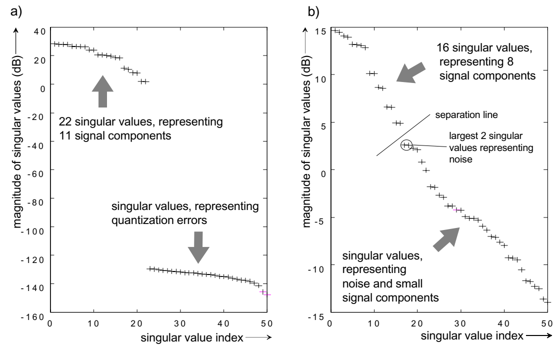

Fig. 1: singular value spectrum of the proposed spectrum
a) simulated signals without noise b) radar signals

exponential signals. Therefor a pair of eigenvalues does not necessarily represent a specific sinusoid.

The initial system proposed in (1) consists of sinusoids with additive white noise. Then the covariance matrix R in (5) is of full rank, and consequently all eigenvalues are greater then zero. The algorithm proposed in [1] assumes that the magnitude of the singular values due to noise are significantly smaller. It works as follows: The eigenvalue spectrum is searched for a gap between two singular values σ that is greater than a certain threshold Δσ:

$$\sigma_j - \sigma_{j-1} \geq \Delta\sigma \quad j = Q, Q-1, \ldots, 1 \tag{7}$$

Starting from a small value singular value, the singular value spectrum is searched towards the larger singular values. The singular values above the first gap exceeding the threshold are considered as signal components, the ones below are rejected as noise components. Fig. 1 a) shows the eigenvalues of a simulated frequency spectrum and b) the eigenvalues of a real radar spectrum. In case a) where simulated signals without additive white noise are shown, the separation can easily be done. The singular values caused by quantization errors are rather small. Case b), the true radar spectrum is different. No clear gap is visible, and no obvious decision boundary can be found.

The underlying assumptions of the state space model are strictly sinusoidal signals with superimposed white noise. Hence the attempt to fit a state space model to real radar signals is prone to yield systematic misfits. Problem is that our radar signals contain both clutter noise and slight non-linearity [7]. These distortions of the signals add additional signal components. Furthermore these components can be fairly small. Hence the noise and signal space are no longer completely separable.

Now recall that each eigenvalue represents a superposition of different underlying signals. As soon as one or several components belonging to the signal space are cut off in the model order selection process all underlying signals will be affected. Depending on the specific situation the rejection of eigenvalues representing parts of the signal space can even lead to a spectrum estimation that has no physical meaning. Consequently, our first implementations of the state space algorithm performed very poor, lacking stability and consistency.

The measured signals impose new requirements on model order selection: An application oriented algorithm has to cope with the systematic misfits and the fact that no clear decision boundary between eigenvalues representing signal components and those representing noise components exists. It has to resolve as many distorted frequencies in the best possible way while at the same time guaranteeing stability by suppressing not resolvable signal components. This approach contradicts classic model order estimators, that are designed to detect the total number of exponentials. Hence the threshold based algorithm, and other algorithms like the Akaike Information Criteria (AIC) in [8] fail to account for the structure of our radar signals and consequently yield poor overall performance.

To develop a new model order algorithm one has to find a model order selection process that attempts to resolve as many signal components as possible while not resolving any components induced by noise or unresolvable systematic distortions. At first, we observed the effects of mal-selected model orders. They can be devided into the effects of choosing a too large model order (overmodeling) and of choosing a model order that is too small (undermodeling).

Let us first consider the observed effects of overmodeling:

- One signal will be represented by two closely spaced signals with very large amplitudes. These are called spurious signals [3].

- Undamped signals can accidentally be resolved as damped signals, represented by complex frequencies [9].

- The last effect is the resolution of noise frequencies, peaks that occur in the spectrum due to noise. However, these noise frequencies are limited in magnitude.

The effects of undermodeling are different:

- Two different signals are represented as a single signal component, that is some average of the two components. The underlying averaging principle is unknown, and appears to be highly phase and magnitude dependant. The estimated spectrum no longer displays the physical properties of the underlying signal.

- A single frequency with no neighboring signals might not be resolved at all.

Additionally, there are application specific characteristics that can be used. Most near field ranging applications show only small changes in between two adjacent measurements. For instance the frequencies of systematic disturbances will vary only by small amounts and the targets are fairly steady over a few seconds. Therefore the model order will only change slowly and by small amounts, i.e. there will be at maximum one additional reflector in each new measurement. Furthermore the underlying signals are real undamped sinusoids. Hence the model order is an even number, that has to be changed by even numbers only.

The proposed approach to the determination of the model order p is to first assume an initial number of q sinuoids and compute the frequencies with the corresponding model order p. Then, in step two, overmodeling is detected by checking for complex frequencies, peaks below the noise level and peaks that are too closely spaced. If any of the above mentioned is detected, the assumption about the number of independent sinusoids, q is reduced by one or respectively, the model order p by 2. The

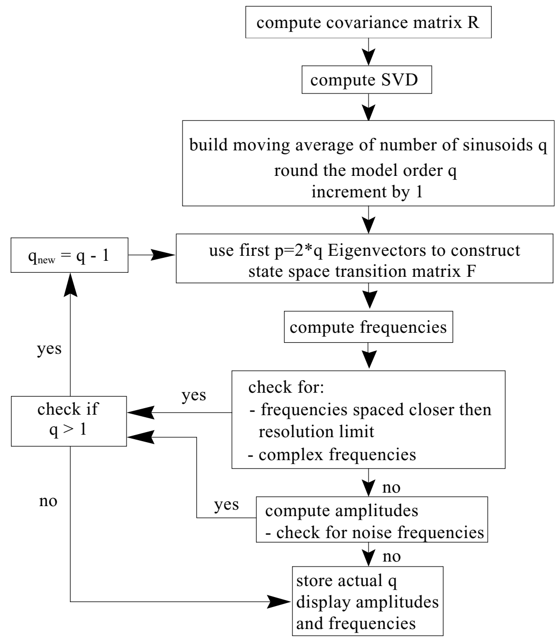

Figure 2 Flow chart of the adaptive model order criteria

frequencies are computed with the new model order. The checking for overmodeling is done, and the procedure is repeated until the estimated frequencies exhibit no signs of overmodeling. The algorithm is summarized in Figure 2.

The setting of the noise threshold strongly depends on the SNR, hence it becomes critical in those cases where the SNR changes between the measurements. Our experiments have shown that the resolution of small noise frequencies is fairly harmless, so one might as well allow their occurrence. Another way to detect noise frequencies is to recall that the frequencies only vary by a small Δf, and that there will be no abrupt changes in the model order. Then, one can detect noise frequencies by the fact that their relative frequencies vary by a strong Δf between two measurements.

The undermodeled case is harder to detect. The frequencies estimated with the small model order p do not exhibit any special behavior. Our algorithm avoids undermodeling by starting with a relatively high model order, and by increasing the model order in-between two measurements. This approach is confined to applications with only slow variations in between measurements, or respectively a fast measurement rate.

4. RESULTS

Our experimental setup consists of the 24 GHz radar system described in [7]. Several obstacles are placed at various distances from 1 to 10 m. The surrounding of the experiment has many potential reflectors for clutter noise and multi-path echoes in between the objects and between the antenna and the objects occur. Hence it is not possible to create a situation were all the frequencies of the signals are known a posteriori. Therefore verification of the theoretical claims in experiments is relatively

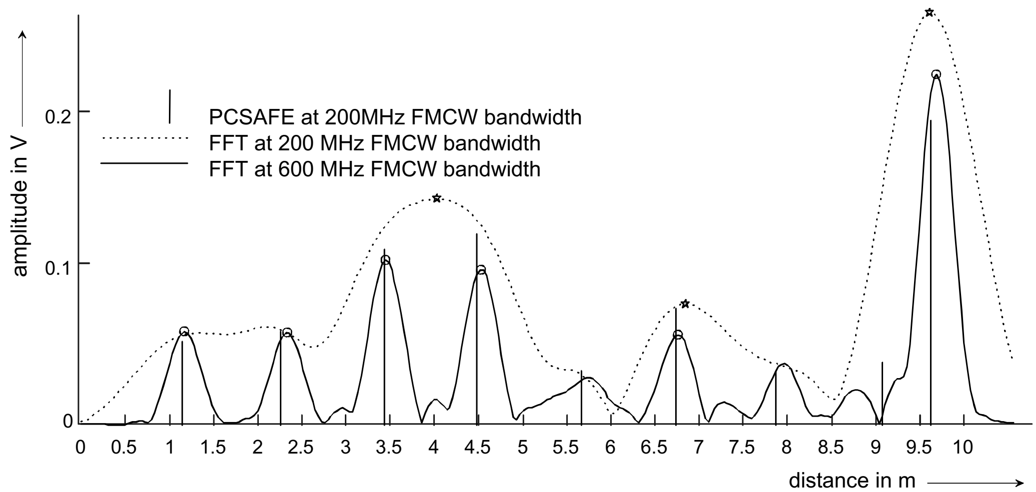

Figure 3: multi-target spectrum

complicated. One cannot judge whether one peak in the spectrum equals a multi path frequency or the superposition of targets. The only measurable goal is the accurate resolution of the known objects.

However, the measurement time, or more accurate, the length of the measured signals are proportional the bandwidth of FMCW-radar systems. Now the peak width of the fourier spectrum is proportional to the employed signal length [6]. Therefore the fourier resolution is proportional to the FMCW bandwidth. Hence the resolution of a reference measurement with FFT evaluation at higher bandwidth will be a good reference about the spectrum obtained in a specific measurement situation. Once the reference measurement is made a second measurement with lower bandwidth is evaluated with both FFT and state space approach. The results are compared to those of the reference measurement, and one is able to see if all signal components are resolved at the lower bandwidth or not.

The reference measurement is made with a bandwidth of 600 MHz, the actual measurement with approximately on third of the bandwidth at 200 MHz. Fig. 3 shows the FFT spectrum for both 600 MHz and 200 MHz, plus the state space approach evaluation for 200 MHz. It is obvious that the state space approach evaluation at 200 MHz resolves all signal components resolved by the FFT evaluation of the 600 MHz signal. At the same time performance of the FFT with the 200 MHz signal is very poor, resolving only 3 peaks, with one of them being a combination of 2 peaks. The FFT evaluation at the lower bandwidth does not yield reasonable estimates of the physical nature of the signal. At the same time the state space approach is capable of resolving the same target at one third of the Radar bandwidth, equaling a signal length of one third. In other words, the resolution has been increased by a factor of three.

5. CONCLUSIONS

A novel model order selection algorithm for state space spectrum estimation is derived from the practical observations made with a radar system. The new approach accounts for the typical properties of technical radar signals. Our experiments with a commercial 24 GHz radar unit show that the adapted state space algorithm increases the resolution by a factor of three over the Fast Fourier Transform. At the same time the new algorithm yields stable and consistent results. For the first time the benefits associated with state space frequency estimation have been brought to use for industrial radar based distance measurement.

6. REFERENCES

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