BLIND WIDEBAND SPATIAL FILTERING BASED ON HIGHER-ORDER CYCLOSTATIONARITY PROPERTIES

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ABSTRACT

In this paper, a new method for blind spatial signal filtering is proposed. It utilizes the selectivity property of the higher-order cyclostationary statistics exhibited by the signal of interest to accurately estimate its unknown steering vector, also in the presence of strong frequency-overlapped interfering signals, which would render the standard stationarity-based techniques ineffective. The method is useful when the desired signal cannot be accurately extracted by exploiting its second-order cyclostationarity properties, as proposed in [1]. The performance analysis, carried out by computer simulations in the case of QAM-modulated signals with partially overlapping bands, substantiates the effectiveness of the proposed method.

1. INTRODUCTION

The problem of wideband spatial filtering received by a sensor array is relevant in many areas of signal processing and communications systems [2, 3]. Often the direction-of-arrival of the desired signal, and, therefore, its spatial signature (steering vector) cannot be assumed to be known. A first approach to solve this problem is based on the use of a training sequence. In order to avoid the associated waste of bandwidth, one can exploit [1, 4] the cyclostationarity properties [5] of the signal of interest, in order to achieve a blind approximation of the optimum spatial filter, on the basis of the only received data. The peculiar advantage of such an approach is given by its inherent *signal-selectivity*, that is, by the fact that the obtained approximation of the optimum filter is asymptotically (as the observation interval tends to infinity) independent of the power of both the noise and the interfering signals, provided that there exists at least one known cyclostationarity feature of the desired signal that is not shared by any of the interferers.

The method proposed in [1] exploits the signal-selectivity associated with second-order statistics, which is a reasonable choice, since they can be accurately estimated with low computational burden. However, when second-order signal-selectivity does not apply, one must resort to higher-order cyclostationarity (HOCS) properties of the received signal.

In this paper, we propose a method for blind wideband spatial filtering of signals received by an array of sensors, which exploits higher-order signal-selectivity. More precisely, the proposed method, which is an extension of that proposed in [6] for the narrowband spatial filtering case, estimates the steering vector of the desired signal, which is needed to implement the optimal minimum mean-square error (MMSE) spatial filter, on the basis of the cyclic *cross-polyspectra* of the received signals. The paper is organized as follows: in Section 2, we introduce the MMSE non-blind wideband filter; in Section 3, the new method is proposed; in Section 4, the performances of the proposed method are evaluated by computer simulations.

2. MMSE WIDEBAND SPATIAL FILTERING

Let us consider D signals impinging on an array of N sensors. The received signal at the ith sensor is given by:

$$x_i(t) = \sum_{k=1}^{D} s_k(t - d_{ik}) + n_i(t) \, , \tag{1}$$

where $s_k(t)$ is the kth signal, d_{ik} is the delay of the kth signal at the ith sensor, and $n_i(t)$ is the additive (antenna plus thermal) noise at the ith sensor. Therefore, the complex envelope of $x_i(t)$, defined with respect to frequency f_0, can be expressed as:

$$\widetilde{x}_i(t) = \sum_{k=1}^{D} \widetilde{s}_k(t - d_{ik}) e^{-j2\pi f_0 d_{ik}} + \widetilde{n}_i(t) \, , \tag{2}$$

where $\widetilde{s}_k(t)$ and $\widetilde{n}_i(t)$ are the complex envelopes of $s_k(t)$ and $n_i(t)$, respectively.

In the wideband array case, namely, when the usual assumption of approximating $\widetilde{s}_k(t - d_{ik})$ with $\widetilde{s}_k(t)$ does not apply, it is convenient to resort to the finite-time Fourier representation of (2):

$$\widetilde{X}_i(f)_T \simeq \sum_{k=1}^{D} \widetilde{S}_k(f)_T \, e^{-j2\pi(f+f_0)d_{ik}} + \widetilde{N}_i(f)_T \;, \qquad (3)$$

where $\widetilde{X}_i(f)_T$, $\widetilde{S}_k(f)_T$, and $\widetilde{N}_i(f)_T$ are the finite-time Fourier transforms of $\widetilde{x}_i(t)$, $\widetilde{s}_i(t)$, and $\widetilde{n}_i(t)$, respectively, and the approximation holds provided that the observation time $T \gg \max_{i,k}[|d_{ik}|]$. By adopting a vectorial notation, equation (3) can be rewritten more concisely as:

$$\widetilde{\boldsymbol{X}}(f)_T \simeq \sum_{k=1}^{D} \widetilde{S}_k(f)_T \, \boldsymbol{a}_k(f) + \widetilde{\boldsymbol{N}}(f)_T \;, \qquad (4)$$

where $\widetilde{\boldsymbol{X}}(f)_T \triangleq [\widetilde{X}_1(f)_T, \ldots, \widetilde{X}_N(f)_T]^T$, $\boldsymbol{a}_k(f) \triangleq [e^{-j2\pi(f+f_0)d_{1k}}, \ldots, e^{-j2\pi(f+f_0)d_{Nk}}]^T$, and $\widetilde{\boldsymbol{N}}(f)_T \triangleq [\widetilde{N}_1(f)_T, \ldots, \widetilde{N}_N(f)_T]^T$.

Let us assume, with no loss of generality, that the signal to be extracted is $s_1(t)$; under the assumption that the signals are zero-mean and independent of each other and of the noise signals, the MMSE filter is [3]:

$$\boldsymbol{W}_{\mathrm{MMSE}}(f) = P_{s_1}(f) \, \boldsymbol{R}_{XX}^{-1}(f) \, \boldsymbol{a}_1(f) \;, \qquad (5)$$

where $P_{s_1}(f)$ is the power spectral density of $\widetilde{s}_1(t)$,

$$\boldsymbol{R}_{XX}(f) \triangleq \lim_{T\to\infty} \frac{1}{T} E[\widetilde{\boldsymbol{X}}(f)_T \widetilde{\boldsymbol{X}}^H(f)_T] \;, \qquad (6)$$

with $E[\cdot]$ denoting ensemble averaging, and $\boldsymbol{a}_1(f)$ is the steering vector of the desired signal.

3. HOCS-BASED BLIND SPATIAL FILTERING

If the delays d_{ik} are assumed to be unknown, an estimate of $\boldsymbol{a}_1(f)$ is needed to implement the MMSE filter. To perform such an estimation, it is convenient to rewrite equation (2) as:

$$\widetilde{x}_i(t) = \sum_{k=1}^{D} h_{ik}(t) \otimes \widetilde{s}_k(t) + \widetilde{n}_i(t) \;, \qquad (7)$$

where $h_{ik}(t) \triangleq \delta(t-d_{ik}) \, e^{-j2\pi f_0 d_{ik}}$ is the impulse response of an ideal delay element, whose Fourier transform $H_{ik}(f) \triangleq e^{-j2\pi(f+f_0)d_{ik}}$ coincides with the ith component of the steering vector $\boldsymbol{a}_k(f)$. Such a formulation allows one to turn the steering vector estimation problem into a system identification one.

We propose here to perform system identification by resorting to an approach based on cyclic polyspectra [7]. Therefore, let n denote the order of the polyspectrum, and define, for a fixed $i \in \{1, 2, \ldots, N\}$, the n-dimensional column vector:

$$\boldsymbol{x}_i(t) \triangleq [\underbrace{\widetilde{x}_1(t)^{(*)_1}, \widetilde{x}_1(t)^{(*)_2}, \ldots, \widetilde{x}_1(t)^{(*)_{n-1}}}_{n-1 \text{ times}}, \widetilde{x}_i(t)^{(*)_n}]^T \;, \qquad (8)$$

where the superscript $(*)_j$ denotes an optional conjugation, and consider the nth-order cyclic polyspectrum with cycle frequency β of the vector $\boldsymbol{x}_i(t)$, which can be shown to be:

$$\overline{P}^{\beta}_{\boldsymbol{x}_i}(\boldsymbol{f}')_n = \sum_{k=1}^{D} \Big\{ H_{ik}^{(*)_n}[(-)_n(\beta - \boldsymbol{1}^T\boldsymbol{f}')] \times \left[\prod_{j=1}^{n-1} H_{1k}^{(*)_j}[(-)_j f_j] \right] \overline{P}^{\beta}_{s_k}(\boldsymbol{f}')_n \Big\} + \overline{P}^{\beta}_{\boldsymbol{n}_i}(\boldsymbol{f}')_n \;, \qquad (9)$$

where $\boldsymbol{f}' = [f_1, f_2, \ldots, f_{n-1}]^T$, the symbol $(-)_j$ denotes an optional minus sign, and $\boldsymbol{n}_i(t)$ is defined analogously to $\boldsymbol{x}_i(t)$. Equation (9) is a straightforward generalization to the case of multiple LTI systems of equation (33) of [7], which refers to the case of a single LTI system.

Under the assumption that, for a given choice of the optional conjugations, only the signal to be extracted exhibits nth-order cyclostationarity with cycle frequency β, equation (9) reduces to:

$$\overline{P}^{\beta}_{\boldsymbol{x}_i}(\boldsymbol{f}')_n = H_{i1}^{(*)_n}[(-)_n(\beta - \boldsymbol{1}^T\boldsymbol{f}')] \times \left[\prod_{j=1}^{n-1} H_{11}^{(*)_j}[(-)_j f_j] \right] \overline{P}^{\beta}_{s_1}(\boldsymbol{f}')_n \,. \qquad (10)$$

By collecting the polyspectra $\overline{P}^{\beta}_{\boldsymbol{x}_i}(\boldsymbol{f}')_n$ for $i = 1, 2, \ldots, N$ in the vector $\overline{\boldsymbol{P}}^{\beta}_{\boldsymbol{x}}(\boldsymbol{f}')_n = [\overline{P}^{\beta}_{\boldsymbol{x}_1}(\boldsymbol{f}')_n, \ldots, \overline{P}^{\beta}_{\boldsymbol{x}_N}(\boldsymbol{f}')_n]^T$, one has:

$$\begin{aligned} \overline{\boldsymbol{P}}^{\beta}_{\boldsymbol{x}}(\boldsymbol{f}')_n &= \overline{P}^{\beta}_{s_1}(\boldsymbol{f}')_n \left[\prod_{j=1}^{n-1} a_{11}^{(*)_j}[(-)_j f_j] \right] \times \boldsymbol{a}_1^{(*)_n}[(-)_n(\beta - \boldsymbol{1}^T\boldsymbol{f}')] \\ &= \overline{P}^{\beta}_{s_1}(\boldsymbol{f}')_n \left[\prod_{j=1}^{n-1} a_{11}(f_j) \right] \boldsymbol{a}_1(\beta - \boldsymbol{1}^T\boldsymbol{f}') \;, \end{aligned} \qquad (11)$$

where we have taken into account that the elements of $\boldsymbol{a}_1(f)$ are complex exponentials. Note that we can assume $d_{11} = 0$ with no loss of generality, since this merely amounts to a shift of the time origin. Therefore, in this case, $a_{11}(f) = 1$ and, therefore, equation (11) simplifies to:

$$\overline{\boldsymbol{P}}^{\beta}_{\boldsymbol{x}}(\boldsymbol{f}')_n = \overline{P}^{\beta}_{s_1}(\boldsymbol{f}')_n \, \boldsymbol{a}_1(\beta - \boldsymbol{1}^T\boldsymbol{f}') \,. \qquad (12)$$

Equation (12) is the key result upon which our method is based. Indeed, it shows that the vector $\overline{\boldsymbol{P}}_{\boldsymbol{x}}^{\beta}(\boldsymbol{f}')_n$ is proportional to the steering vector $\boldsymbol{a}_1(\beta - \mathbf{1}^T\boldsymbol{f}')$, provided that $\overline{P}_{s_1}^{\beta}(\boldsymbol{f}')_n \neq 0$. Therefore, for any frequency value f of interest, by appropriately choosing $\boldsymbol{f}' = \boldsymbol{g}(f)$ so that $f = \beta - \mathbf{1}^T\boldsymbol{f}'$ and $\overline{P}_{s_1}^{\beta}(\boldsymbol{f}')_n \neq 0$, an estimate of the the polyspectra slices $\overline{\boldsymbol{P}}_{\boldsymbol{x}}^{\beta}[\boldsymbol{g}(f)]_n$ can be utilized instead of the unknown steering vector $\boldsymbol{a}_1(f)$ to implement a blind approximation of the MMSE filter (5). To further investigate this point, replace $\boldsymbol{a}_1(f)$ with $\overline{\boldsymbol{P}}_{\boldsymbol{x}}^{\beta}[\boldsymbol{g}(f)]_n$ in (5), obtaining

$$\begin{aligned}\boldsymbol{W}_{\mathrm{BLIND}}(f) &\triangleq P_{s_1}(f)\boldsymbol{R}_{XX}^{-1}(f)\overline{\boldsymbol{P}}_{\boldsymbol{x}}^{\beta}[\boldsymbol{g}(f)]_n \\ &= P_{s_1}(f)\,\boldsymbol{R}_{XX}^{-1}(f)\,\overline{P}_{s_1}^{\beta}[\boldsymbol{g}(f)]_n\,\boldsymbol{a}_1(f) \\ &= \overline{P}_{s_1}^{\beta}[\boldsymbol{g}(f)]_n\,\boldsymbol{W}_{\mathrm{MMSE}}(f)\,. \quad (13)\end{aligned}$$

As shown in (13), the blind filter can be regarded as the cascade of the MMSE filter and the LTI filter with transfer function $\overline{P}_{s_1}^{\beta}[\boldsymbol{g}(f)]_n$. If the signal polyspectrum slice can be reasonably assumed flat within the band of interest, the filtering effect of $\overline{P}_{s_1}^{\beta}[\boldsymbol{g}(f)]_n$ can be neglected, otherwise it must be equalized (to within a scale factor) in order not to introduce a significant amount of distortion in the recovered signal. Note that if the signal modulation format is known, the signal polyspectrum can be analytically evaluated (to within a scale factor).

Since polyspectrum estimates exhibit slow convergence, a more effective strategy can be based on the estimation of the unknown delays d_{ik} as an intermediate step for constructing an approximate steering vector. This is motivated by the fact that the algorithms for delay estimation converge relatively faster (with respect to the observation time) than the algorithm for polyspectrum estimation. For this reason, in the simulation experiments we have adopted such a two-step strategy.

4. SIMULATION RESULTS

In this section we analyze the performance of the proposed method operating in a digital communications scenario.

We assume that the D signals are QAM-modulated with root raised cosine pulses with 100 % excess bandwidth, identical baud-rates T_S^{-1} and partially overlapping bands, so that they cannot be separated with classical time-filtering techniques. Moreover, we assume that the lowest radio frequency is 0.6 times the highest, which renders the bandwidth of the impinging signals wide enough to make the standard narrowband techniques largely ineffective. The noise is modeled as stationary complex white Gaussian, uncorrelated from sensor to sensor. Finally, observe that the choice of QAM modulation with identical baud-rates for all the signals precludes the utilization of second-order cyclic features [1].

In the simulations, we set $D = 2$ and we consider a non-uniform linear array of three sensors, where the distance between the first and the second one is γ (set to one-half of the wavelenght corresponding to the highest frequency in the useful signal) and the distance between the first and the third is $b\gamma$. The directions-of-arrival of the desired signal and the interfering one are 40^o and 36^o, respectively, evaluated with respect to array broadside. The separation between the center frequencies of the two signals is set to one quarter of their bandwidth and the offset f_1 of the desired signal is 2.3 times this separation.

At each sensor, we first perform matched filtering of the received signal and, then, oversampling with period $T_c = T_s/p$. Then, the discrete-time version of the complex envelope of the desired signal at the output of the matched filter of the first sensor can be written as

$$s_1[n] = \widetilde{s}_{m,1}(nT_c)e^{j2\pi f_1 nT_c}\,, \quad (14)$$

where $\widetilde{s}_{m,1}(t)$ denotes the complex envelope of the continuous-time desired signal at the output of the matched filter and f_1 is the carrier frequency offset. Thus, the complex envelope of the third-sensor useful output can be written as

$$s_3[n] = \widetilde{s}_{m,1}(nT_c + \Delta)e^{j2\pi f_1(nT_c+\Delta)}e^{j2\pi f_0\Delta}\,, \quad (15)$$

where $\Delta = -d_{31}$ is the delay between the third and the first sensor. Denoting with $x_1[n]$ and $x_3[n]$ the discrete-time outputs of the first and third sensors, it can be shown, under simple conditions on the offset frequencies of the useful signal and the interfering one, that

$$|r_{x_1x_1x_1x_3}^{4\nu_1}[m+L]| \simeq |r_{x_1x_1x_1x_1}^{4\nu_1}[m]| \quad (16)$$

where $r_{x_1x_1x_1x_k}^{4\nu_1}[m]$ denotes the fourth-order moment among $x_1[n]$, $x_1[n]$, $x_1[n]$ and $x_k[n-m]$ at cycle frequency $4\nu_1$, $\nu_1 = f_1T_c$, and L is such that $\Delta = LT_c + \Delta_1$ with $\Delta_1 \in (-0.5T_c, 0.5T_c)$. Moreover, since

$$\frac{\arg\left\{r_{x_1x_1x_1x_3}^{4\nu_1}[L]\right\} - \arg\left\{r_{x_1x_1x_1x_1}^{4\nu_1}[0]\right\}}{2\pi} = (f_0\Delta + f_1\Delta_1)_{\mathrm{mod1}}\,, \quad (17)$$

it is possible to obtain an estimate $\hat{\Delta}$ of Δ, and, therefore, of the steering vector of the desired signal, by utilizing an estimate of the moment function slices.

Note that we utilize the third sensor only to obtain an estimate $\frac{\hat{\Delta}}{b}$ of the delay between the second sensor and the first one, under the assumption that b has been calibrated with sufficient accuracy; then, we utilize only the first two sensors to perform spatial filtering.

We consider a scenario in which there is a strong interfering signal (with signal-to-interference ratio equal to

M	1	4	8	16	32	128
MSE	1.28	0.90	5.5e-2	2.3e-2	6e-3	5e-3

Table 1. MSE of the optimum MMSE filter versus the number M of taps of the FIR filters.

0 dB) and a high signal-to-noise ratio (40dB); we also set $b = 100$ and $p = 5$. As performance measure, we utilize the MSE (obtained by averaging the results of 20 independent trials) between the reconstructed signal and the desired one. The following methods are compared: (i) the optimum MMSE spatial filtering; (ii) the non-blind MMSE filter, where the steering vector $\boldsymbol{a}_1(f)$ is known and the matrix $\boldsymbol{R}_{XX}(f)$ in (5) is estimated from the received data; (iii) the proposed blind MMSE filter, where both $\boldsymbol{R}_{XX}(f)$ and $\boldsymbol{a}_1(f)$ are estimated from the received data.

As a preliminary step, Tab. 1 shows the results for the optimum MMSE filter versus the number of taps M of the FIR filters at the first two sensors. Let us note that the narrowband approach ($M = 1$) is not able to separate the two signals [see also Fig. 1 (a)]; moreover, the performances improve with increasing values of M [compare the results in Fig. 1 (b)]. Therefore, in the next experiment, we choose $M = 128$ for all the methods under comparison. Then, we evaluate the performance of both non-blind and blind MMSE methods for a sample size equal to 25600. We found for the two methods the same value of MSE, equal to 2.2e-2, which compares favorably with the value 5e-3 of Tab. 1 corresponding to the optimum MMSE filter for $M = 128$. To show how effective is the proposed method, we also present in Fig. 1 the eye diagrams of the considered methods. The similarity of the eye diagrams of Figs. 1(c) and 1(d) confirms that, in this scenario, delay estimation converges much faster than $\boldsymbol{R}_{XX}(f)$ estimation and, hence, does not suffer of the typically slow convergence of sample higher-order statistics, making the proposed algorithm well suited for real-time implementation.

5. CONCLUSIONS

In this paper we have proposed a new method for blind wideband spatial filtering, which exploits the higher-order cyclostationarity properties of the received signals. In particular, under the assumption that the desired signal exhibits higher-order cyclostationarity at a cycle frequency different from those of the interfering signals, we have proposed to utilize the cross-polyspectra of the received signals to estimate the unknown steering vector. The performance analysis, carried out by computer simulations in the case of QAM-modulated signals with partially overlapping bands, shows the effectiveness of the proposed blind method in situations where the existing second-order techniques (such as the one proposed in [1]) cannot work.

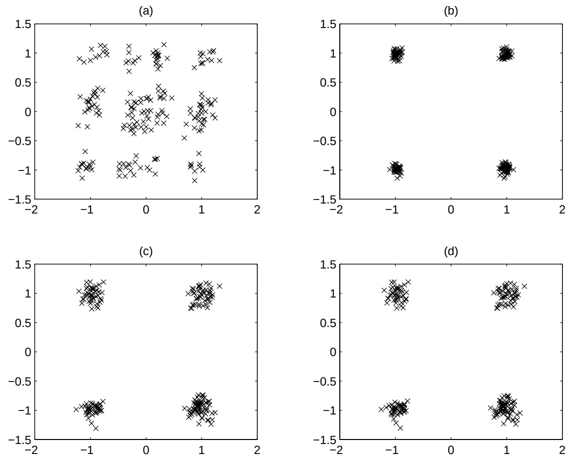

Fig. 1. The eye diagrams relative to the following methods: (a) optimum MMSE single-tap (narrowband) filter; (b) optimum MMSE filter ($M = 128$); (c) non-blind MMSE filter ($M = 128$); (d) proposed blind MMSE filter ($M = 128$).

6. REFERENCES

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