EIGENMODE ANALYSIS AND DEGREE OF FREEDOM IN CHAOTIC SEMI-STADIUM SOUND FIELDS

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ABSTRACT

This paper describes eigenfrequency spacing statistics including modal patterns and degrees of freedom in semi-stadium type 2-D field. The authors numerically investigated the sound fields surrounded by 2-D semi-stadium type of boundaries as examples of boundaries where chaotic properties are hidden. One limit of the semi-stadium boundaries is a rectangular which gives a regular field, while another limit is a stadium boundary where the chaotic property emerges. The numerical results show that eigenfrequency spacing in all the cases can be expressed as a family of Γ distributions extended to a non-integer degree of freedom. This fractal degree of freedom might be interpreted as the degree of freedom of the sound field. For the regular limit case, that is, a rectangular case, the distribution is the exponential distribution with the freedom of unity, while in the chaotic case, that is, the stadium case, it is the Wigner distribution with a degree of freedom of two. Moreover, modal patterns clearly show breaks of the regular pattern of nodal lines seen in a rectangular case as the boundary is deformed from the rectangular to the stadium condition.

1. INTRODUCTION

Chaotic properties are hidden in a sound ray propagated in a sound space surrounded by irregularly shaped boundaries. A stadium type of boundary is an example of the type of condition where chaotic properties are hidden. This type of chaotic phenomenon was pointed out by Berry[1] who proposed a new research area named by quantum chaology. The main purpose of quantum chaology is to find a new corresponding principle between the classical and quantum systems by studying the "fingerprints of chaos" that remain in linear wave fields. Theoretical study of the fingerprints of waves, which are termd leftover in linear wave theory and are caused by the emergence of chaotic behavior in sound ray propagation, is also important for sound field analysis and soundspace design for irregularly-shaped boundary conditions. Eigenfrequency and modal patterns are significant issues in the study of fingerprints. This paper numerically analyzes eigenfrequency spacing statistics and modal patterns in semi-stadium type 2-D fields as examples of boundaries where chaotic properties are hidden. We define a family of semi-stadium shapes which can be continuously deformed from a rectangular to the chaotic stadium boundary by changing the curvature of the circular portion of the stadium boundary. The eigenfrequency spacing distribution for a stadium boundary condition follows the Wigner distribution[2]. Lyon[3] have already shown that this spacing generally follows the Wigner distribution instead of the exponential distribution, that which spacing follows in a rectangular case. The authors' numerical results show that eigenfrequency spacing in all cases can be expressed as a family of Γ distributions extended to a non-integer (fractal) degree of freedom, which might be interpreted as the degree of freedom of a sound field. Furthermore modal patterns are shown to break the regular pattern of nodal lines seen in a rectangular case as a boundary is deformed from the rectangule to the stadium condition.

2. EIGENFREQUENCY SPACING STATISTICS

The eigenfrequency spacing[2, 3] is from the theoretical wave point of view a significant issue for not only fingerprint analysis but sound field control. Examples of the spacing distributions for the rectangular and stadium cases are shown in Fig. 1(a) and (b), respectively. Here, the eigenfrequencies from the first to 300th for odd-odd modes are counted from the theoretical results for the rectangular case (a) and from the numerical results obtained by FEM(b). The spacing follows the exponential distribution under the rectangular condition, while it follows the Wigner distribution under the stadium condition. Here, the horizontal axis shows the average normalized spacing. We can see that degeneration occurs quite often for the rectangular case, while it is quite unlikely in the stadium. In this paper, we will show a transition from the exponential to the Wigner distribution by comparing examples of the rectangular and stadium boundaries.

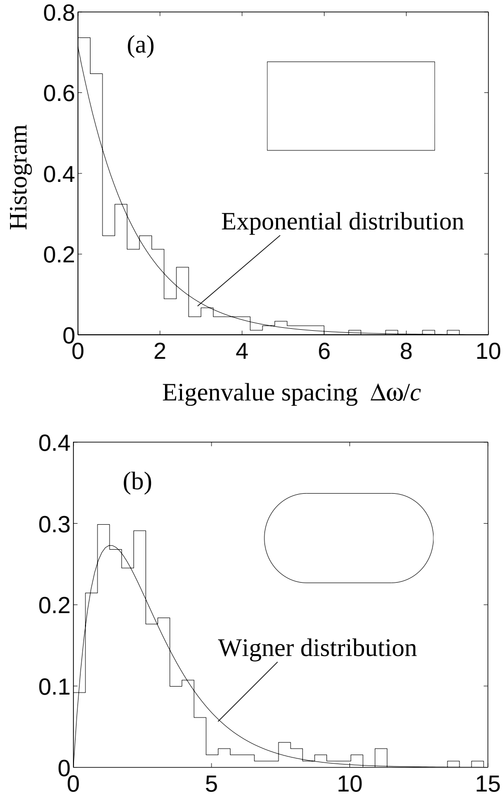

Fig. 1. Eigenvalue spacing statistics for rectangle and stadium boundaries

3. TRANSITION FROM EXPONENTIAL TO WIGNER DISTRIBUTIONS IN SEMI-STADIUM FIELDS AND DEGREE OF FREEDOM OF SOUND FIELD

We define as a family of semi-stadium shapes those which can be continuously deformed from a rectangular to the chaotic stadium boundary by changing the curvature of the circular portion of the stadium boundary that is the curved portion is expressed as $x^i + y^i = r^i$. The shape of a semi-stadium is changed from a stadium to a rectangle as i is increased. Figure 2 shows the spacing distributions of the eigenfrequencies when i=4, 7, 10, and 15. We found that the Γ distribution

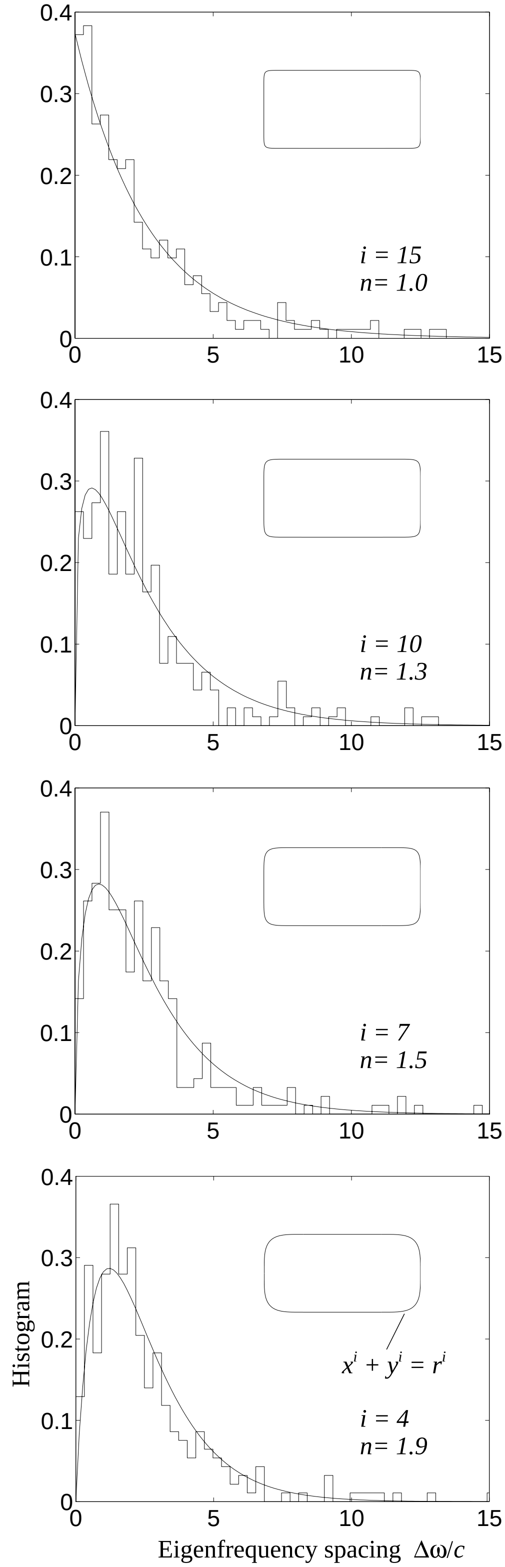

Fig. 2. Eigenvalue spacing statistics for semi-stadium boundaries,(n: degree of freedom for Γ distribution)

$$P(x,n) \equiv \frac{1}{\beta^n \Gamma(n)} x^{n-1} e^{-\frac{x}{\beta}}, x > 0 \quad (1)$$

($\beta = \mu/n$, μ indicates the mean value of x) could be fitted to the distribution with the fractal degree of freedom n, 1.9, 1.5, 1.3, and 1.0 for each boundary, respectively. Here the exponential distribution degree of freedom is unity, and the Wigner distribution degree of freedom is two. The authors already have found that the correlation dimension is about two for the history of sound ray reflection points in a stadium; on the other hand the dimension stays at unity for the rectangular case[4]. Consequently, the authors surmised that this fractal degree of freedom could be interpreted as the freedom of the 2-D sound fields between the rectangular and regular cases.

4. MODAL PATTERNS FOR THE SEMI-STADIA

Spatial distributions of modal patterns are also significant features used for the analysis of fingerprints and sound fields. The modal patterns of irregularly shaped rooms are of particular interest for evaluating the perturbed effects coused by the boundaries that affect on the sound field, particularly in regard to the acoustics of a reverberation room. The authors analyzed these modal patterns using the point matching method for 2-D fields[5]. Figure 3 shows examples of lower modes which correspond to (2,2) modes in the rectangular portion. We can clearly see that the regular pattern of nodal lines in the rectangular case becomes greatly deformed as the room shape changes to the stadium condition. Figure 4 illustrates the accumulated distribution functions for the squared quantities sampled from the modal patterns. The low values of the samples are more likely to be observed in the semi-stadium cases rather than in the rectangular case[6, 7]. However, a full evaluation of the perturbed effects from the point of view of the degree of freedom of the sound field has not been done.

5. CONCLUSION

The authors numerically identified that eigenfrequency spacing in 2-D semi-stadium boundaries can be expressed as a family of Γ distributions extended to a non-integer degree of freedom. This fractal degree of freedom might be surmised as the degree of freedom of the sound field. In a rectangular case, the degree of freedom was unity, while the degree of freedom was two in the chaotic(stadium) case. Moreover, the modal patterns observed clearly showed breaks of the regular pattern of nodal lines seen in a rectangular case as a boundary was deformed. The relationship between the break of the modal pattern and the degree of freedom is a problem to be studied in the future.

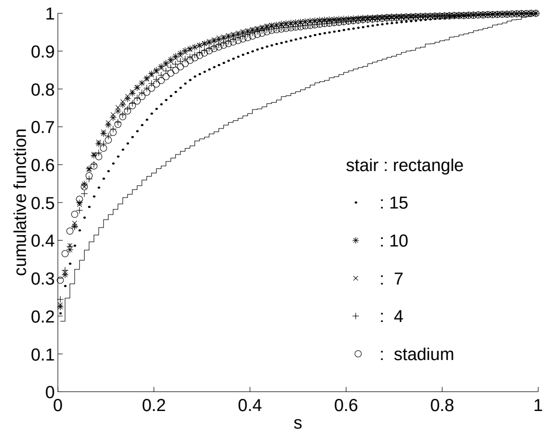

Fig. 4. Cumulative function for each shape

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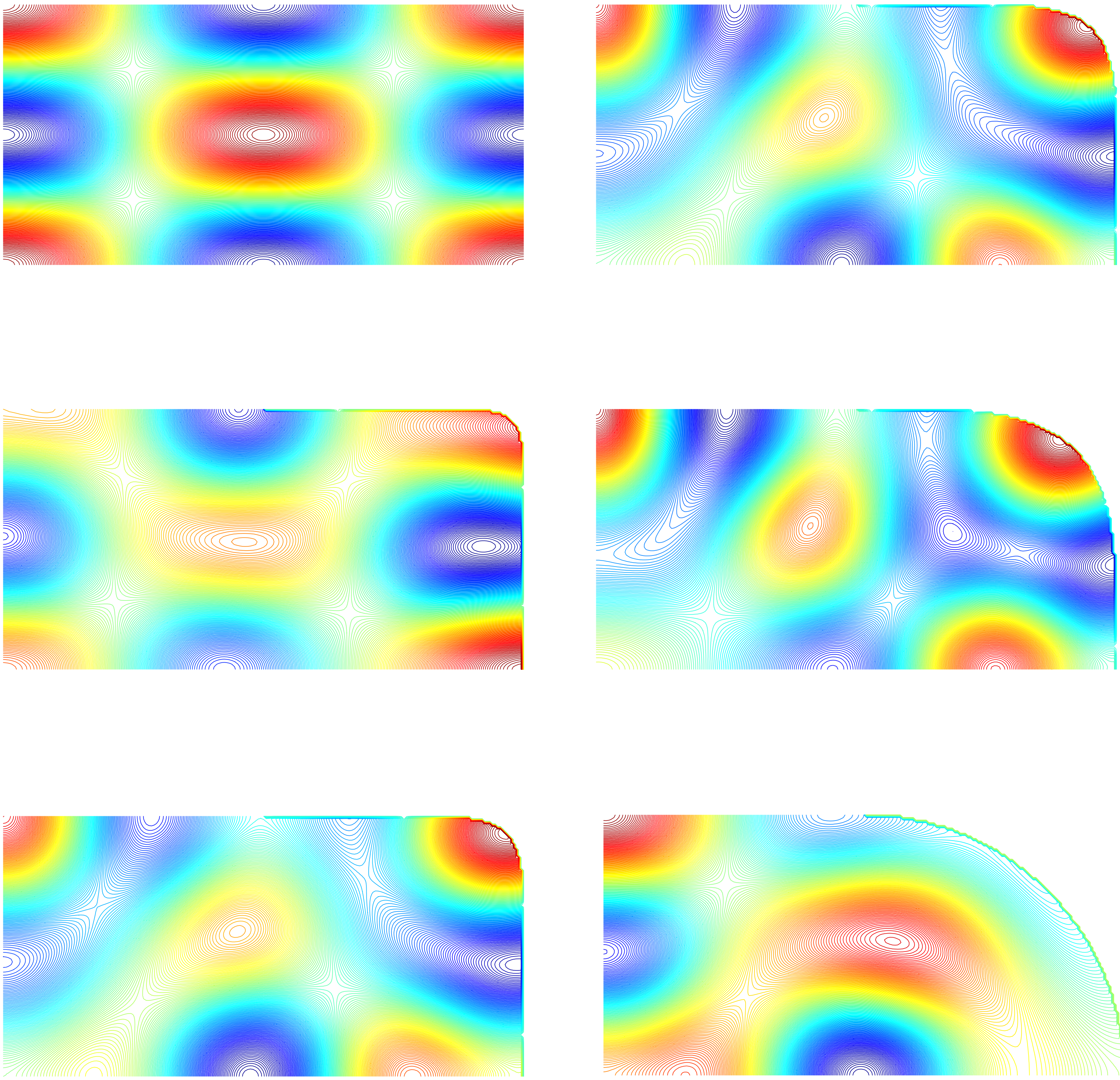

Fig. 3. Corresponding (2,2) rectangle mode for semi-stadia