SPACE-TIME BLOCK CODING WITH OPTIMAL ANTENNA SELECTION

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ABSTRACT

Space-time block codes provide maximal diversity advantage over a fading channel. This paper presents a novel technique that provides additional diversity gain by coupling antenna selection with a space-time block code. Specifically, we provide a choice of transmit antenna elements at the transmitter and transmit a space-time code over the optimal antenna pair. We present the optimal selection rule and quantify the improved performance in terms of gain in average SNR. The average SNR gain is calculated as a function of the number of transmit antenna elements and the number of receive antennas. We also investigate the improvement in outage capacity.

1. INTRODUCTION

There is substantial interest in MIMO wireless systems due to their performance enhancing properties. Multiple antennas can be used for increasing capacity [1, 2, 3] or for increasing diversity order [4, 5]. However, multiple antennas typically imply increased cost. There is therefore considerable incentive for low cost, low complexity techniques which improve system performance. Antenna selection techniques where a selection of antenna elements [1] is made available to the transmitter or receiver have been proposed in previous literature [6, 7, 8]. Transmission/reception is performed through the optimal antenna set [9].

This paper proposes a novel technique of antenna selection coupled with the Alamouti scheme, a well-known space-time block code [5, 10, 11]. The case of antenna selection with a general space-time block code has been covered in [12]. The Alamouti scheme provides maximal diversity order over a fading channel. We propose coupling antenna selection with the Alamouti scheme thereby further increasing the diversity order. We use expected SNR as a performance metric and provide results quantifying the gain over Alamouti without antenna selection. The analysis presented in this paper covers transmit antenna selection; but is applicable to receive antenna selection as well. We comment on this issue later on in the paper. This paper also contains analysis of outage capacity behavior with antenna selection and quantifies the improvement in outage probability.

The remainder of the paper is organized as follows. The following section briefly presents the channel model, the Alamouti scheme and the optimal transmit antenna selection technique. Section 3 develops analysis quantifying the gain in expected SNR with antenna selection. Section 4 presents outage capacity analysis. The outage probability is developed as a function of number of transmit antenna elements and the number of receive antennas. Substantial reduction in outage probability is observed. Simulation results are presented in section 5 and we conclude in the last section.

[1] typically much cheaper than RF chains

2. CHANNEL MODEL, ALAMOUTI AND OPTIMAL SELECTION

2.1. Channel Model

Consider a wireless channel with two transmit RF chains and multiple receive antennas, N_R. The channel is assumed to be quasi-static flat fading with the channel remaining constant over several bursts. We assume perfect channel knowledge at the receiver. The channel gains are modeled as i.i.d. zero mean Gaussian random variables with unit variance i.e.

$$h_{i,j} \sim N(0,1) \tag{1}$$

where $h_{i,j}$ is the channel gain from the jth transmit antenna to the ith receive antenna. We have

$$y = Hx + n \tag{2}$$

where y $(N_R \times 1)$ is the received vector, x (2×1) is the transmitted data vector, H $(N_R \times 2)$ is the channel matrix and n $(N_R \times 1)$ is the additive white Gaussian noise vector.

2.2. Alamouti Scheme

Assume that a space-time block code, specifically the Alamouti code, is used for transmission. We reproduce some results from [5] below for the reader's convenience. At a given symbol period, signals s_1 and s_2 are transmitted. In the next symbol period, $-{s_2}^*$ and ${s_1}^*$ is transmitted from antennas one and two respectively. After some receive processing [5] we have two independent streams with the SNR on each stream given by

$$\gamma_i = \gamma_o \sum_{i=1}^{N_R} (|h_{i1}|^2 + |h_{i2}|^2), \quad i = 1,2 \tag{3}$$

where $\gamma_o = \frac{E_s}{N_o}$ is the energy per transmit antenna divided by the noise power. The expected SNR on each stream is

$$E(\gamma) = \gamma_o E \sum_{i=1}^{N_R} (|h_{i1}|^2 + |h_{i2}|^2) = 2N_R\gamma_o \tag{4}$$

since the channel gains are i.i.d. zero mean with unit variance.

2.3. Antenna Selection with Alamouti

We now couple antenna selection with the space-time block code. A selection of N_T transmit antenna elements is made available at the transmitter (see figure 1). The channel matrix H is now of size $N_R \times N_T$. Transmission is allowed through only two antennas. There are $\binom{N_T}{2}$ possible pairs of transmit antennas that can be used for transmission. The goal is to transmit on the optimal transmit antenna pair. The receiver estimates the channel in the training period. It computes the the optimal transmit antenna pair and feeds it to the transmitter through the feedback link. The space-time code is transmitted on this selected optimal set. Assume that the m^{th} and n^{th} transmit antenna elements are chosen. The received SNR on either data stream is

$$\gamma_i = \gamma_o \sum_{i=1}^{N_R} (|h_{im}|^2 + |h_{in}|^2) \tag{5}$$

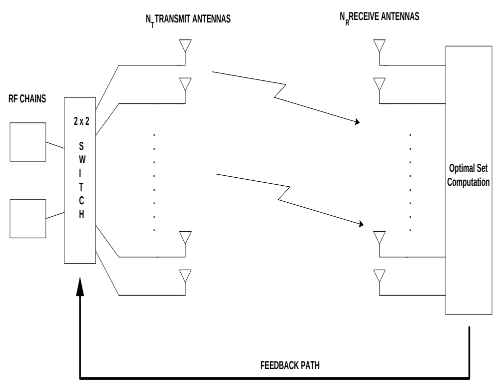

Fig. 1. Antenna Selection Schematic

The optimal selection rule is to choose the columns of H that maximize equation 5. Clearly, we should choose the columns with the highest and second highest norms. This automatically ensures that the resulting sum of norms is maximized. Therefore, the optimal transmission scheme is to transmit on the antennas with the highest and second highest norms.

3. AVERAGE SNR GAIN

As stated previously, we use the gain in expected SNR as a metric of performance. The SNR with antenna selection coupled with the space-time code is

$$\gamma = \gamma_o \sum_{i=1}^{N_R} (|h_{i,m}|^2 + |h_{i,n}|^2) \tag{6}$$

where m and n are the columns of H with the highest and second highest norms respectively. The expected SNR, $E(\gamma)$, with optimal antenna selection is

$$\begin{aligned} E(\gamma) &= \gamma_o E \sum_{i=1}^{N_R} (|h_{i,m}|^2 + |h_{i,n}|^2) \\ &= \gamma_o E \sum_{i=1}^{N_R} |h_{i,m}|^2 + \gamma_o E \sum_{i=1}^{N_R} |h_{i,n}|^2 \end{aligned} \tag{7}$$

The squared column norms are i.i.d. gamma distributed variables. i.e.

$$f_X(c_i) = \frac{1}{(N_R - 1)!} {c_i}^{N_R - 1} e^{-c_i} \tag{8}$$

where c_i is the square of the i^{th} column norm. We are interested in the statistics of the largest and second largest of these random variables. We use the 'order statistics' approach to solve the problem. The squared column norms are ordered in increasing order of magnitude i.e. we generate a new set of ordered random variables, X_i, such that $X_1 \leq X_2 \leq ... \leq X_i ... \leq X_{N_T - 1} \leq X_{N_T}$ where X_i is the i^{th} largest squared column norm.
The expected SNR is now

$$E(\gamma) = E\left(\gamma_o \left(X_{N_T} + X_{N_T - 1}\right)\right) \tag{9}$$

where X_{N_T} and $X_{N_T - 1}$ are the squares of the highest and second highest column norms respectively.
The probability density function of the i^{th} order statistic of a set of N_T, i.i.d. gamma distributed variables is [13]

$$\begin{aligned} p_X(x_i) = \frac{N_T!}{(N_T - i)!(i-1)!} \left(1 - F_X(x_i)\right)^{(N_T - i)} \\ * \left(F_X(x_i)\right)^{(i-1)} f_X(x_i) \end{aligned} \tag{10}$$

where f_X is as defined in equation 8 and F_X is the cumulative distribution function of a gamma distributed random variable

$$F_X(x) = 1 - e^{-x} \sum_{i=0}^{N_R - 1} \frac{x^i}{i!}$$

Therefore, the expected SNR is characterized by the first moment of the highest and second highest order statistic of N_T, i.i.d. gamma distributed variables. The moments of the ordered set of i.i.d. gamma variables have been well studied in literature [13, 14]. We are interested in the first moment only which is given by [13]

$$\begin{aligned} E(X_{N_T}) = \frac{N_T}{(N_R - 1)!} \sum_{r=0}^{N_T - 1} (-1)^r \binom{N_T - 1}{r} \\ \sum_{s=0}^{(N_R - 1)r} a_s\left(N_R, r\right) \frac{(N_R + s)!}{(r+1)^{(N_R + s + 1)}} \end{aligned} \tag{11}$$

where $a_s\left(N_R, r\right)$ is the coefficient of x^s in the expansion of

$$\left(\sum_{l=0}^{N_R - 1} \frac{x^l}{l!} \right)^r$$

and

$$\begin{aligned} E(X_{N_T - 1}) = \frac{N_T(N_T - 1)}{(N_R - 1)!} \sum_{r=0}^{N_T - 2} (-1)^r \binom{N_T - 2}{r} \\ \sum_{s=0}^{(N_R - 1)(r+1)} a_s\left(N_R, r+1\right) \frac{(N_R + s)!}{(r+2)^{(N_R + s + 1)}} \end{aligned} \tag{12}$$

where $a_s\left(N_R, r+1\right)$ is the coefficient of x^s in the expansion of

$$\left(\sum_{l=0}^{N_R - 1} \frac{x^l}{l!} \right)^{(r+1)}$$

These values have been tabulated in [14] for the case of $1 \leq N_R \leq 4$ and for $1 \leq N_T \leq 40$. Table 1 indicates the average snr for various transmit receive antenna configurations for $\gamma_o = 0\ dB$. The columns indicate the number of transmit antennas available to choose from and the rows are the number of receive antennas in the system. The gain is better depicted by figure 2.

We define the gain from the antenna selection technique as

$$g(dB) = 10log_{10}\left(\frac{E(X_{N_T} + X_{N_T-1})}{2N_R}\right) \tag{13}$$

which is the gain in expected SNR of the technique over the standard Alamouti scheme. Each curve in figure 2 depicts the gain (dB) in average SNR for a fixed number of receive antennas and a variable number of transmit antenna elements. We have gains of 2-3 dB for a reasonable number of transmit antenna elements and for two receive antennas. One point worth noting is that the average SNR gain is higher for fewer number of receive antennas which makes intuitive sense. As the number of receive antennas increases, the column squared norms, which are sums of squares of Gaussian, look increasingly similar. Not much is gained by choosing the highest and second highest of a set of similar looking variables. We also note that though the analysis so far treats transmit antenna selection, it is applicable to receive antenna selection as well with transmission of a space-time block code on two antennas (alamouti). The optimal selection rule is to choose the rows of H with highest and second highest norm. The gains are the same as those depicted in the top-most curve in figure 2 with the x-axis now corresponding to the number of available receive antennas to choose from and with the number of transmit antennas fixed at two.

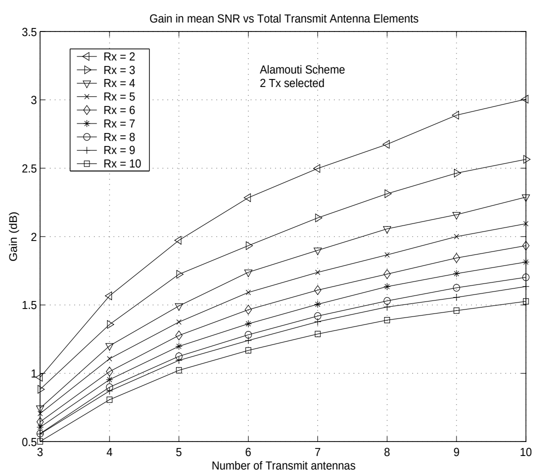

Fig. 2. Gain in Expected SNR for Alamouti Scheme with Antenna Selection

4. OUTAGE CAPACITY ANALYSIS

Since the channel gains are random variables, the instantaneous capacity is also a random variable. In such a scenario, the outage capacity is a parameter of interest [2, 3]. It is defined as that value of capacity, C_{out}, for which the outage probability is p_{out}. i.e

$$Prob(C \leq C_{out}) = p_{out} \tag{14}$$

This section analyzes antenna selection from an outage capacity viewpoint and quantifies the reduction in outage probability for a specified capacity. The capacity for antenna selection coupled with the space-time code is

$$C = log_2\left(1 + \gamma_o(X_{N_T} + X_{N_T-1})\right)$$

where X_{N_T} and X_{N_T-1} are as defined previously. We have,

$$\begin{aligned} P(C \leq C_{out}) &= P\left((X_{N_T} + X_{N_T-1}) \leq \frac{2^{C_{out}} - 1}{\gamma_o} = k\right) \\ &= \int_{X=0}^{k/2}\int_{Y=0}^{x} f_{XY}(x,y)dydx \\ &+ \int_{X=k/2}^{k}\int_{Y=0}^{k-x} f_{XY}(x,y)dydx \end{aligned} \tag{15}$$

where we have replaced X_{N_T} and X_{N_T-1} with X and Y respectively for notational convenience. f_{XY} is the joint distribution of the highest and second highest order statistic and is given by [13]

$$f_{XY} = N_T(N_T - 1)F_Y(y)^{N_T-2}fxfy \tag{16}$$

where F_Y, fx and fy are as defined previously. Equation 15 works out to

$$P(C \leq C_{out}) = F_X(\frac{k}{2})^{N_T} + N_T\int_{X=0}^{k/2} f_{k-X}(x)F_X(x)^{N_T-1}dx \tag{17}$$

which for low values of k is well approximated by

$$P(C \leq C_{out}) = 2 * F_X(\frac{k}{2})^{N_T} \tag{18}$$

Figure 3 compares the approximated theoretical outage probability versus the exact value for low values of p_{out} [2]. We note that the approximation is fairly accurate. The improvement in outage probability is therefore,

$$\Delta p_{out} = 2(F(\frac{k}{2})^2 - F(\frac{k}{2})^{N_T}) \tag{19}$$

which is substantial as indicated in figure 3. For example, the outage probability for $C_{out} = 4.2\ bits/Hz$ reduces drastically from 10% with no selection to about 2.5% with $N_T = 3$ to about 0.5% for $N_T = 4$.

5. SIMULATIONS

This section investigates the effect of antenna selection on the probability of symbol error. The number of receive antennas is kept fixed at two. The number of transmit antenna elements is increased from two (no selection) to six (select two out of six). Perfect channel estimation with symbol by symbol decoding is assumed at the receiver. The alamouti space-time code is transmitted on the optimal two antennas, selected as per section 2.3. The channel gains are modeled as i.i.d. Gaussian random variables with zero mean and unit variance. Figure 4 is a plot of the symbol error rate versus the SNR, γ_o. The diversity gain is clearly visible from the increase in the slope of the curves.

[2]which is generally the region of interest

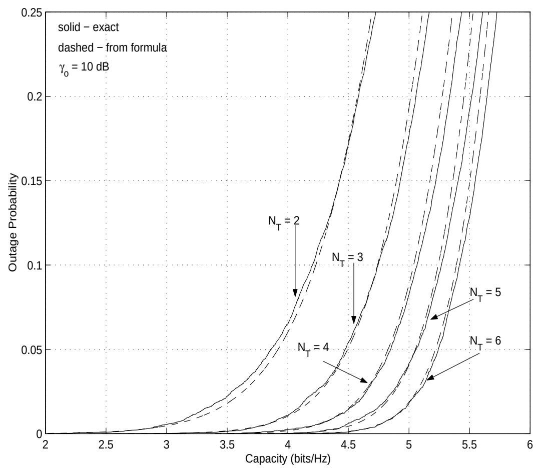

Fig. 3. Comparison of Outage Probability, Exact vs Approximated, N_R = 2

	N_T	2	3	4	5	6
N_R						
2		4.00	5.03	5.75	6.30	6.75
3		6.00	7.31	8.19	8.85	9.39
4		8.00	9.54	10.56	11.32	11.92

Table 1. Expected SNR for Antenna Selection With Alamouti

6. CONCLUSIONS

This paper presented a novel scheme of antenna selection coupled with a popular space-time code. We have demonstrated gains in average SNR of almost 3 dB for the case of 2 receive antennas and several transmit antenna elements. We reiterate that the analysis presented in this paper can be trivially extended to the case of fixed number (two) of transmit antennas with optimal antenna selection at the receiver. We have also demonstrated considerable improvement in outage capacity through this technique.

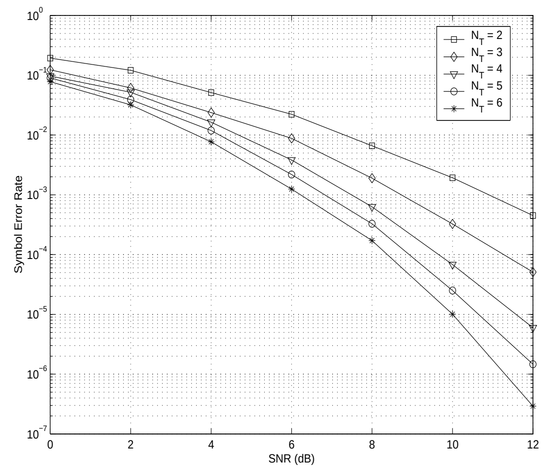

Fig. 4. SER Curves for Alamouti Scheme with Antenna Selection. 2 Rx antennas, 4 QAM

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