IMPROVING THE NEAR-PERFECT HYBRID FILTER BANK PERFORMANCE IN THE PRESENCE OF REALIZATION ERRORS

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ABSTRACT

Hybrid filter banks have received increasing attention in the literature, for applications such as high-speed, high-resolution A/D and D/A converter design. In the manufacturing process, however, the filter coefficients of a hybrid filter bank are plagued with some errors due to technological limitations, particularly those of the analog filters, leading to degradation of the system performance. This work presents a novel method for improving the mean signal-to-noise-ratio of near-perfect reconstruction filter banks, taking into account such realization errors. The method consists in minimizing the total noise energy derived in an accurate way by a theoretical expression.

I. INTRODUCTION

Many valuable filter bank design methods have been proposed in the literature: the perfect reconstruction (PR) ones try to completely eliminate amplitude, phase and aliasing distortions, while the near-perfect reconstruction (NPR) ones allow some aliasing [1]. However, regardless of the chosen method, the filter bank performance is usually assessed by parameters, such as the maximum *reconstruction* error, which is the difference between the input and the output of the filter bank [1], that do not take into account (and are not intended to consider) imperfections presented by the system after it is manufactured. These imperfections, usually referred to as *realization* errors, are determined by technological limitations and spurious elements, and impose errors to the filter coefficients. It has been shown that in practical implementations, realization errors in digital filter banks can generally be neglected [2]. While realization error effects in digital filters can be virtually eliminated at the cost of increasing the number of bits to represent the filter coefficients, in the analog counterpart the achievable accuracy is limited by errors due to the fabrication process, degrading the performance of the resulting system. This is a more and more defying problem due to the growing interest in hybrid filter banks in applications such as A/D and D/A conversion [3]-[7].

Considering the stochastic process of the realization errors, simulations have revealed that the filter banks signal-to-(reconstruction)noise ratio (SNR) histograms follow a gaussian distribution. For PR prototype filters, such histograms have approximately the same mean value, which is lower than those obtained when all coefficients are correct. On the other hand, the histograms corresponding to NPR filter banks exhibit still lower, and quite different, mean values. The design method proposed here increases the SNR mean values of the NPR filter banks, so that the SNR obtained by PR designs can be achieved. This task is accomplished by minimizing an expression, developed in Section III, that accurately predicts the histogram mean values.

The following investigations consider uniform, maximally-decimated, M-channel, length N FIR filter banks. It is assumed that the analog filters are implemented by discrete-time switched-capacitor networks, so that they can be directly designed in the z-domain [3]. Uniform, zero mean white-noise sequences, with variance σ_x^2 are applied as inputs. The use of uniform sequences does not cause any lack of generality, for the results are not based on the probability density function type. Ergodicity is assumed.

II. FILTER BANK ERRORS

Letting $\hat{h}_k(\cdot)$ and $h_k(\cdot)$ represent, respectively, the effective and the correct impulse responses of the k-th analog analysis filter, $k = 0,\cdots, M-1$, then

$$\hat{h}_k(n) = h_k(n) + \Delta h_k(n) = h_k(n) + \varepsilon_{n,h_k} h_k(n), \qquad (1)$$

where $\Delta h_k(\cdot)$ is the associated realization error sequence. Similarly, in the case of analog synthesis filters,

$$\hat{f}_k(n) = f_k(n) + \Delta f_k(n) = f_k(n) + \varepsilon_{n,f_k} f_k(n), \qquad (2)$$

for $k = 0,\cdots, M-1$. The error factors ε_{n,h_k} and ε_{n,f_k} are zero mean random numbers with variances $\sigma^2_{\varepsilon,h}$ and $\sigma^2_{\varepsilon,f}$, respectively. They are also assumed independent, whether taken from the same or different filters. After the filter bank circuit is built and has its operation conditions fixed, the error factors are no longer unpredictable, allowing the z-transform to be applied to Eqs. (1) and (2):

$$\hat{H}_k(z) = H_k(z) + \Delta H_k(z), \qquad (3)$$

$$\hat{F}_k(z) = F_k(z) + \Delta F_k(z). \qquad (4)$$

The filter bank effective transfer function [1] can be written as:

$$\begin{aligned}\hat{\mathbf{A}}(z) &= \left[\hat{A}_0(z) \;\; \cdots \;\; \hat{A}_r(z) \;\; \cdots \;\; \hat{A}_{M-1}(z)\right]^T \\ &= \mathbf{A}(z) + \Delta\mathbf{A}(z) = (1/M)\hat{\mathbf{H}}^T(z)\hat{\mathbf{F}}(z),\end{aligned} \qquad (5)$$

where

$$\hat{\mathbf{H}}^T(z) = \mathbf{H}^T(z) + \Delta\mathbf{H}^T(z)$$
$$= \begin{bmatrix} \hat{H}_0(z) & \cdots & \hat{H}_k(z) & \cdots & \hat{H}_{M-1}(z) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{H}_0(zW^r) & \cdots & \hat{H}_k(zW^r) & \cdots & \hat{H}_{M-1}(zW^r) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{H}_0(zW^{M-1}) & \cdots & \hat{H}_k(zW^{M-1}) & \cdots & \hat{H}_{M-1}(zW^{M-1}) \end{bmatrix}, \qquad (6)$$

$$\Delta\mathbf{H}^T(z)=$$
$$=\begin{bmatrix} \Delta H_0(z) & \cdots & \Delta H_k(z) & \cdots & \Delta H_{M-1}(z) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \Delta H_0(zW^r) & \cdots & \Delta H_k(zW^r) & \cdots & \Delta H_{M-1}(zW^r) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \Delta H_0(zW^{M-1}) & \cdots & \Delta H_k(zW^{M-1}) & \cdots & \Delta H_{M-1}(zW^{M-1}) \end{bmatrix}, \quad (7)$$

$$\hat{\mathbf{F}}(z)=\mathbf{F}(z)+\Delta\mathbf{F}(z)=M\left[\hat{F}_0(z) \ \cdots \ \hat{F}_k(z) \ \cdots \ \hat{F}_{M-1}(z)\right]^T, \quad (8)$$

$$\Delta\mathbf{F}(z)=M\left[\Delta F_0(z) \ \cdots \ \Delta F_k(z) \ \cdots \ \Delta F_{M-1}(z)\right]^T, \quad (9)$$

and $W=e^{-j2\pi/M}$. In Eq. (5), $\hat{A}_0(z)=A_0(z)+\Delta A_0(z)$ is the actual filter bank transfer function, while $\hat{A}_r(z)=A_r(z)+\Delta A_r(z)$, $r=1,\cdots,M-1$, are the aliasing functions. The PR design methods do suppress $A_r(z)$. However, such functions will be maintained in the following analysis in order to encompass the NPR methods. Substituting Eqs. (6)-(9) in Eq. (5) yields

$$\mathbf{A}(z)=\left[A_0(z) \ \cdots \ A_r(z) \ \cdots \ A_{M-1}(z)\right]^T=(1/M)\mathbf{H}^T(z)\mathbf{F}(z), \quad (10)$$

and

$$\begin{aligned}\Delta\mathbf{A}(z)&=\left[\Delta A_0(z) \ \cdots \ \Delta A_r(z) \ \cdots \ \Delta A_{M-1}(z)\right]^T \\ &=(1/M)\left(\mathbf{H}^T(z)\Delta\mathbf{F}(z)+\Delta\mathbf{H}^T(z)\mathbf{F}(z)+\Delta\mathbf{H}^T(z)\Delta\mathbf{F}(z)\right).\end{aligned} \quad (11)$$

Applying the inverse z-transform to Eqs. (10) and (11) one obtains the sequences

$$a_r(n)=\sum_{k=0}^{M-1}\sum_{u=0}^{N-1}e^{j2\pi ru/M}h_k(u)f_k(n-u), \quad (12)$$

and

$$\begin{aligned}\Delta a_r(n)=\sum_{k=0}^{M-1}\sum_{u=0}^{N-1}e^{j2\pi ru/M}&\left[h_k(u)\Delta f_k(n-u)\right. \\ &\left.+\Delta h_k(u)f_k(n-u)+\Delta h_k(u)\Delta f_k(n-u)\right]\end{aligned} \quad (13)$$

for $r=0,\cdots,M-1$. Both $a_r(\cdot)$ and $\Delta a_r(\cdot)$ are of length $C=2N-1$. Using Eqs. (5), (12) and (13), the filter bank output in the presence of realization errors can be written in the z and time domains, respectively, as

$$\hat{X}(z)=\sum_{r=0}^{M-1}\left(A_r(z)+\Delta A_r(z)\right)X\left(zW^r\right), \quad (14)$$

$$\hat{x}(n)=\sum_{r=0}^{M-1}\sum_{v=0}^{C-1}\left(a_r(v)+\Delta a_r(v)\right)e^{j2\pi r(n-v)/M}x(n-v). \quad (15)$$

Eq. (15) shows that the non-suppression of the realization errors leads to amplitude and phase distortions, besides those due to $A_0(z)$, which cause reconstruction errors. Moreover, there is aliasing because $X\left(zW^r\right)$ is transmitted to the output by the non-zero terms $A_r(z)$ and $\Delta A_r(z)$, $r=1,\cdots,M-1$. As a result, the factors $e^{j2\pi rn/M}$ appear in the time domain, not only turning the output sequence complex, but also forcing the system to be cyclostationary. This last property is taken into account by decomposing $\hat{x}(\cdot)$ as

$$\hat{x}_\ell(n)=\begin{cases}\hat{x}(n), & \text{for } n=mM+\ell \\ 0, & \text{for } n\neq mM+\ell\end{cases} \quad (16)$$

$\ell=0,\cdots,M-1$, so that

$$\hat{\mathbf{x}}(n)=\left[\hat{x}_0(n) \ \cdots \ \hat{x}_\ell(n) \ \cdots \ \hat{x}_{M-1}(n)\right]^T. \quad (17)$$

Similar decomposition can be applied to the input sequence $x(\cdot)$:

$$\mathbf{x}(n)=\left[x_0(n) \ \cdots \ x_\ell(n) \ \cdots \ x_{M-1}(n)\right]^T. \quad (18)$$

III. SIGNAL-TO-NOISE RATIO

A. Total noise power

The filter bank total noise vector defined by

$$\mathbf{r}(n)=\hat{\mathbf{x}}(n)-\mathbf{x}(n-N+1), \quad (19)$$

accumulates reconstruction and realization errors. Using Eqs. (12) and (13), it follows that the variance matrix of $\mathbf{r}(\cdot)$ can be written as

$$\begin{aligned}\boldsymbol{\sigma}_{\mathbf{r}}^2(\boldsymbol{x},\boldsymbol{\varepsilon})=\Bigg\{1-\sum_{r=0}^{M-1}&\left[a_r(N-1)+a_r^*(N-1)+\Delta a_r(N-1)\right. \\ &\left.+\Delta a_r^*(N-1)\right]+\sum_{n=0}^{C-1}\sum_{r=0}^{M-1}\sum_{s=0}^{M-1}\left[a_r(n)a_s^*(n)\right. \\ &\left.+a_r(n)\Delta a_s^*(n)+\Delta a_r(n)a_s^*(n)+\Delta a_r(n)\Delta a_s^*(n)\right]\Bigg\}\sigma_x^2\mathbf{I},\end{aligned} \quad (20)$$

where $\mathbf{I}$ is the identity matrix, and $\boldsymbol{x}$ and $\boldsymbol{\varepsilon}$ represent, respectively, the filter bank input and the realization error stochastic processes. Due to lack of space, Eq. (20) is not demonstrated here.

In order to express Eq. (20) as a function of the filters coefficients, it is noted that the summations in r obey to:

$$\begin{aligned}\sum_{s=0}^{M-1}\Delta a_s(n)&=M\sum_{k=0}^{M-1}\sum_{m=0}^{\lfloor (N-1)/M\rfloor}\left[h_k(mM)\Delta f_k(n-mM)\right. \\ &\quad\left.+\Delta h_k(mM)f_k(n-mM)+\Delta h_k(mM)\Delta f_k(n-mM)\right] \\ &=\sum_{r=0}^{M-1}\Delta a_r^*(n),\end{aligned} \quad (21)$$

$$\sum_{r=0}^{M-1}a_r(n)=M\sum_{k=0}^{M-1}\sum_{m=0}^{\lfloor (N-1)/M\rfloor}h_k(mM)f_k(n-mM)=\sum_{r=0}^{M-1}a_r^*(n), \quad (22)$$

where $\lfloor\cdot\rfloor$ denotes the largest integer less than or equal to its argument. Since Eq. (21) is a diagonal matrix with identical diagonal elements, the components $\hat{x}_\ell(\cdot)$, $\ell=1,\cdots,M-1$, of the output vector, $\hat{\mathbf{x}}(n)$, have identical noise power. Defining

$$\alpha(n) = \sum_{r=0}^{M-1} a_r(n) = \sum_{r=0}^{M-1} a_r^*(n), \quad (23)$$

$$\Delta\alpha(n) = \sum_{s=0}^{M-1} \Delta a_s(n) = \sum_{s=0}^{M-1} \Delta a_s^*(n), \quad (24)$$

then Eq. (20) can be rewritten as:

$$\sigma_r^2(\boldsymbol{x},\boldsymbol{\varepsilon})\big|\varepsilon = \sigma_x^2\Big[1-2\big(\alpha(N-1)+\Delta\alpha(N-1)\big) + \sum_{n=0}^{C-1}\big(\alpha^2(n)+\alpha(n)\Delta\alpha(n)+\Delta\alpha^2(n)\big)\Big], \quad (25)$$

which determines the filter bank total noise power for a given set of error factors. Considering the whole process $\boldsymbol{\varepsilon}$, it follows that $\sigma_r^2(\boldsymbol{x},\boldsymbol{\varepsilon})$ becomes a random variable with mean value

$$\mathrm{E}\left\{\sigma_r^2(\boldsymbol{x},\boldsymbol{\varepsilon})\right\} = \Gamma\sigma_x^2 \quad (26)$$

where

$$\Gamma = 1-2\big(\alpha(N-1)+\mathrm{E}\{\Delta\alpha(N-1)\}\big) + \sum_{n=0}^{C-1}\left[\alpha^2(n)+\alpha(n)\mathrm{E}\{\Delta\alpha(n)\}+\mathrm{E}\left\{\Delta\alpha^2(n)\right\}\right] \quad (27)$$

In the absence of realization errors $(\Delta\alpha(n)=0,\forall n)$, Eq. (26) reduces to

$$\sigma_{rec}^2(\boldsymbol{x},\boldsymbol{\varepsilon}) = \sigma_x^2\left(1-2\alpha(N-1)+\sum_{n=0}^{C-1}\alpha^2(n)\right), \quad (28)$$

which gives noise power due to the reconstruction error alone.

B. Signal-to-noise ratio mean value

Assuming that the variance of $\sigma_r^2(\boldsymbol{x},\boldsymbol{\varepsilon})$ is much smaller than its mean value, which is a realistic assumption, and using Eq. (26), the mean value of the filter bank SNR can be expressed as [8]:

$$\overline{SNR} = \frac{\sigma_x^2}{\mathrm{E}\left\{\sigma_r^2(\boldsymbol{x},\boldsymbol{\varepsilon})\right\}} = \Gamma^{-1}, \quad (29)$$

whose value depends on whether the filters are analog or digital. Let us assume initially that the analysis and synthesis filters are analog. Since the error factors ε_{n,h_k} and ε_{n,f_k} in Eqs. (1) and (2) are zero mean random variables, then from Eqs. (23) and (24):

$$\mathrm{E}\{\Delta\alpha(n)\} = 0, \forall n \quad (30a)$$

$$\mathrm{E}\left\{\Delta\alpha^2(n)\right\} = \left(\sigma_{\varepsilon,h}^2+\sigma_{\varepsilon,f}^2+\sigma_{\varepsilon,h}^2\sigma_{\varepsilon,f}^2\right)\xi(n), \quad (30b)$$

where:

$$\xi(n) = M^2\sum_{k=0}^{M-1}\sum_{m=0}^{\lfloor (N-1)/M \rfloor} h_k^2(mM) f_k^2(n-mM). \quad (31)$$

With $\alpha(\cdot)$ given by Eq. (24) and by virtue of Eqs. (30), it follows from Eq. (29) that:

$$\overline{SNR}_{AA} = \left(1-2\alpha(N-1)+\sum_{n=0}^{C-1}\alpha^2(n)+\left(\sigma_{\varepsilon,h}^2+\sigma_{\varepsilon,f}^2+\sigma_{\varepsilon,h}^2\sigma_{\varepsilon,f}^2\right)\sum_{n=0}^{C-1}\xi(n)\right)^{-1}. \quad (32)$$

The terms

$$E_{real} = \left(\sigma_{\varepsilon,h}^2+\sigma_{\varepsilon,f}^2+\sigma_{\varepsilon,h}^2\sigma_{\varepsilon,f}^2\right)\sum_{n=0}^{C-1}\xi(n), \quad (33a)$$

$$E_{rec} = 1-2\alpha(N-1)+\sum_{n=0}^{C-1}\alpha^2(n), \quad (33b)$$

represent, respectively, the contributions of the realization and reconstruction errors to the filter bank SNR.

For analog analysis filters and digital synthesis filters, which is the case in A/D conversion applications [3], [4], [6], we have $\sigma_{\varepsilon,f}^2 = 0$, resulting in

$$\overline{SNR}_{AD} = \left(1-2\alpha(N-1)+\sum_{n=0}^{C-1}\alpha^2(n)+\sigma_{\varepsilon,h}^2\sum_{n=0}^{C-1}\xi(n)\right)^{-1}. \quad (34)$$

IV. FILTER BANK DESIGN

This section describes an optimization method for improving the SNR of cosine-modulated NPR hybrid filter banks. Assuming that $p(\cdot)$ is the length N impulse response of a prototype filter $P(z)$, the remaining filters of an M band NPR bank are determined by [1]:

$$h_k(n) = 2p(n)\cos\left(\left(k-\frac{1}{2}\right)\left(n-1-\frac{(N-1)}{2}\right)\frac{\pi}{M}-(-1)^k\frac{\pi}{4}\right), \quad (35)$$

$$f_k(n) = 2p(n)\cos\left(\left(k-\frac{1}{2}\right)\left(n-1-\frac{(N-1)}{2}\right)\frac{\pi}{M}+(-1)^k\frac{\pi}{4}\right), \quad (36)$$

for $0 \le k \le M-1$ and $0 \le n \le N-1$.

In order to obtain the optimized prototype filter that maximizes the filter bank SNR, the energy in the stopband of the NPR prototype filter is reduced by minimizing

$$E_s = \sum_{\omega=\omega_s}^{\pi}\left|P\left(e^{j\omega}\right)\right|^2, \quad (37)$$

where $\omega_s = \pi/M$ is the stopband edge frequency of the NPR prototype filter. Including the terms corresponding to the energies of the realization error (Eq. 33a) and reconstruction error (Eq. 33b), the cost function becomes

$$E_{total} = \alpha_1 E_s + \alpha_2 E_{real} + \alpha_3 E_{rec}, \quad (38)$$

where α_1, α_2 and α_3 are fixed weights that control each term during the optimization process.

The *SNR* results for different numbers of channels, *M*, and prototype filter lengths, *N*, are shown in Table 1 for the NPR prototype filters obtained from [9] (old) and for the optimized filters obtained by minimizing Eq. (38) (new). These examples are for A/D conversion. The analysis filter coefficients are analog, subject to independent gaussian errors with $\sigma_{\varepsilon,h} = 0.01$ and the synthesis filter coefficients are digital, and do not suffer from errors, for they are represented in floating-point. Comparisons between the *SNR* values show improvement in the system performance with the new NPR design in the presence of analysis realization errors, therefore confirming the validity of the proposed method. Fig. 1 displays the magnitude of the Fourier transform of the sequence formed by taking the difference between the filter bank input and output, using the new 4-channel, 16-coefficient NPR prototype filter. The coefficients and main frequency characteristics of this filter are given in Tables 2 and 3, respectively.

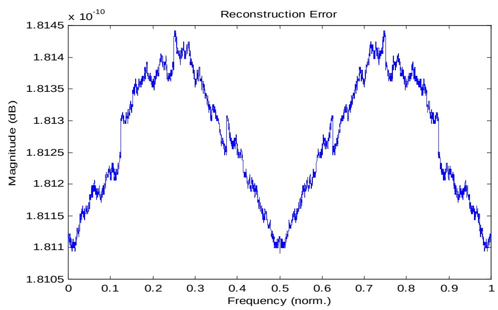

Fig. 1 – New 4 channel NPR filter reconstruction error.

Table 1 - Optimization results for NPR *M* channel filter banks.

		NPR - old			NPR - new		
M	*N*	E_{rec} (10^{-4})	E_{real} (10^{-4})	*SNR* (dB)	E_{rec} (10^{-9})	E_{real} (10^{-4})	*SNR* (dB)
4	16	3.83	0.99	33.15	0.62	1.00	40.00
4	32	0.17	1.00	39.30	76.56	1.00	39.99
8	32	10.34	1.00	29.45	0.021	1.00	40.00
16	64	24.85	1.02	25.87	2.35	0.99	40.00

Table 2 – New 4 channel NPR filter impulse response.

$p(0)$	-1.354611903524573e-002
$p(1)$	-6.091918578065508e-003
$p(2)$	7.922632493321435e-003
$p(3)$	3.227002151444836e-002
$p(4)$	6.708255743790741e-002
$p(5)$	1.075398756215563e-001
$p(6)$	1.399473570044523e-001
$p(7)$	1.597658026608897e-001
$p(8)$	1.597658026608897e-001
$p(9)$	1.399473570044523e-001
$p(10)$	1.075398756215563e-001
$p(11)$	6.708255743790741e-002
$p(12)$	3.227002151444836e-002
$p(13)$	7.922632493321435e-003
$p(14)$	-6.091918578065508e-003
$p(15)$	-1.354611903524573e-002

Table 3 – New 4 channel NPR frequency characteristics.

Filter	3 dB frequency (norm.)	Transition band width (norm.)	Bandstop attenuation (dB)
new	$\pi/4$	7.65e-002	-30.5
old	$\pi/4$	7.33e-002	-29.0

In the presence of realization errors, it has been verified that E_{real} prevails over E_{rec} (Eqs. (33)), determining the SNR value given by Eq. (32). Besides, for PR methods, it can be found that $\sum_{n=0}^{C-1} \xi(n) \cong 1$, and, in such cases, it is possible to approximate Eq. (32) and (34), in dB, respectively by:

$$\overline{SNR}_{AA}(dB) \cong -10\log_{10}\left(\sigma_{\varepsilon,h}^2 + \sigma_{\varepsilon,f}^2\right) \quad (39)$$

$$\overline{SNR}_{AD}(dB) \cong -10\log_{10}\left(\sigma_{\varepsilon,h}^2\right) \quad (40)$$

In fact, for the A/D (floating-point) case, where $\sigma_{\varepsilon,h} = 0.01$ and $\sigma_{\varepsilon,f} = 0$, Eq. (40) gives about 40 dB, which is the maximum possible value in this situation. Therefore, the results presented in Table 1 show that the proposed method increases the SNR of NPR filter banks to its maximum achievable value by minimizing, mainly, the realization error noise energy.

VI. CONCLUSIONS

A new analytical expression for the signal-to-noise ratio of hybrid near-perfect reconstruction filter banks subject to realization errors was derived. An optimization procedure was then described to improve the performance of such filter banks, by minimizing reconstruction and realization error energies. An illustrative design example was shown to give support to the theory. The proposed approach can be applied to the design of analog-digital, digital-analog and analog-analog filter banks.

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