LOW COMPLEXITY ANTI-JAM SPACE-TIME PROCESSING FOR GPS

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ABSTRACT

This paper investigates the performance of reduced rank space-time processors in the context of anti-jam mitigation for an M-Code based GPS receiver utilizing a circular array. Several adaptive processing algorithms are discussed utilizing power minimization techniques. It is assumed an INS (Inertial Navigation System) or direction finding algorithm is incorporated into the receiver for satellite look direction based algorithms. Reduced rank space-time processing is accomplished via the innovative Multistage Wiener filter (MSWF). It is demonstrated that the MSWF does not require matrix inversion, thereby reducing computational complexity. The processing algorithms are compared in terms of available degrees of freedom and distortion of the GPS cross correlation function (CCF).

1. INTRODUCTION

A power minimization processing filter prior to the GPS correlators is one of several methods for suppressing jammers. This filter simply minimizes the output power of the preprocessor (since the satellite signals are well below the noise floor) while having the added advantage not being integrated with the GPS receiver. However, this type of algorithm does not account for DOA (direction-of-arrival) information associated with the satellites in the FOV (field-of-view). A substantial improvement in SINR can be achieved by accounting for satellite direction information with power minimization, but an increase in computational complexity is often incurred for both space and space-time processing. Space-time processing is preferred since the available degrees of freedom to mitigate narrowband jammers increases dramatically relative to space-only processing. However, space-time processing operates in a larger dimension, therefore increasing the dimensionality of the space-time weight vector. This higher dimensionality can translate into a large computational burden and slow convergence. To increase convergence and lower computational complexity, this paper investigates reduced dimension space-time power minimization processor algorithms based on the MSWF [1]. The simulations presented herein reveal the rapid convergence of the MSWF implementation of the power minimization based space-time processors. Furthermore, an analysis of their computational complexity will show their efficacy in adapting to environmental dynamics characterizing a high performance fighter aircraft while minimizing signal processing resources.

2. ADAPTIVE SPACE-TIME POWER MINIMIZATION PROCESSING

Let's define $\mathbf{x}_n$ as an $M \times 1$ vector containing samples across the M antennas at the n-th time instant sampled at a rate above or equal to the Nyquist rate for the M-code.

$$\mathbf{x}_n = [x_1(n), x_2(n), ..., x_M(n)]^T \tag{1}$$

The $MN \times 1$ space-time snapshot, $\tilde{\mathbf{x}}(n)$, is formed from concatenating $\mathbf{x}_n$, $n = 1, 2, ..., N-1$, as

$$\tilde{\mathbf{x}}(n) = [\mathbf{x}_1; \mathbf{x}_2; ...; \mathbf{x}_{N-1}] \tag{2}$$

where ; implies concatenating the vectors into a single column. Similarly, the M tap weights across the M antennas at the n-th time instant are placed as the components of an $M \times 1$ vector as

$$\mathbf{h}_n = [h_1(n), h_2(n), ..., h_M(n)]^T. \tag{3}$$

and the entire set of space-time weights is formed from a concatenation of $\mathbf{h}_n$, $n = 1, ..., N-1$, as

$$\mathbf{h} = [\mathbf{h}_1; \mathbf{h}_2; ...; \mathbf{h}_{N-1}]. \tag{4}$$

The output power of the space-time preprocessor is

$$E\{|\mathbf{h}^H \tilde{\mathbf{x}}(n)|^2\} = \mathbf{h}^H \mathbf{K} \mathbf{h}, \text{ where: } \mathbf{K} = E\{\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n)\}. \tag{5}$$

2.1. Per Satellite Power Minimization Space-Time Processor

The Space-Time Power Minimization (ST-PM) based preprocessor utilizing the space-time reference approach does NOT yield maximal SINR for any GPS satellite, but rather attempts to "pass" all GPS satellite signals in the FOV as undistorted as possible while canceling the interference[3]. The shortcoming of this type of ST-PM based preprocessor is that it does not attempt to minimize distortion to any one GPS satellite signal. It is proposed that an estimate of the DOA vector for a given GPS satellite, $\mathbf{a}(\hat{\mathbf{r}}_k^{gps})$, obtained via INS data or DOA algorithm, be used to maximize SINR for a given GPS satellite in the FOV. The array manifold, $\mathbf{a}(\hat{\mathbf{r}}_k^{gps})$, denotes the relative phases across the circular array of the k^{th} satellite in the field of view where $\hat{\mathbf{r}}_k^{gps}$ is a unit vector defined as

$$\hat{\mathbf{r}}_k^{gps} = [cos\phi_k sin\theta_k, sin\phi_k sin\theta_k, cos\theta_k]^T \tag{6}$$

pointing from the origin of the array towards the k^{th} GPS satellite. Note that a different ST-PM preprocessor is required for each of the K GPS satellites in the FOV. Efficient implementation of the K parallel constrained ST-PM preprocessors using the MSWF is discussed in Section 3. Since a different space-time weight vector is formed for each satellite, this is denoted by placing a superscript k on the weight vector $\mathbf{h}$.

For the k-th satellite, constrain the inner product between $\hat{\mathbf{a}}_k$ and the $M \times 1$ vector of weights associated with the **same** time instant but spanning the M antennas, $\tilde{\mathbf{h}}_n^{(k)} = [h_1^{(k)}(n), h_2^{(k)}(n), ..., h_M^{(k)}(n)]^T$ to be unity for each of the N "tap times" comprising the space-time adaptive filter structure. This leads to power minimization with N linear constraints:

$$\underset{\mathbf{h}^{(k)}}{\text{Minimize}} \quad \mathbf{h}^{(k)H}\mathbf{K}\mathbf{h}^{(k)} \tag{7}$$

$$\text{Subject to: } \mathbf{a}(\hat{\mathbf{r}}_k^{gps})^H \mathbf{h}_n^{(k)} = 1, \quad n = 0, 1, ..., N-1$$

Accommodating the N linear constraints consumes N out of MN degrees of freedom. One can rewrite the multiple constrained problem of (7) as

$$\underset{\mathbf{h}^{(k)}}{\text{Minimize}} \quad \mathbf{h}^{(k)H}\mathbf{K}\mathbf{h}^{(k)} \tag{8}$$

$$\text{subject to: } \mathbf{A}_k^H \mathbf{h}^{(k)} = \boldsymbol{\beta}$$

where $\mathbf{A}_k = \mathbf{I} \otimes \mathbf{a}(\hat{\mathbf{r}}_k^{gps})$ and $\boldsymbol{\beta}$ is a $N \times 1$ vector containing all ones, and $\otimes$ is denoted as the Kronecker product operator. The solution to (8) using Lagrange multipliers yields

$$\mathbf{h}^{(k)} = \mathbf{K}^{-1}\mathbf{A}_k\Big[\mathbf{A}_k^H\mathbf{K}^{-1}\mathbf{A}_k\Big]^{-1}\boldsymbol{\beta} \tag{9}$$

If one rewrites $\mathbf{h}^{(k)}$ in terms of an orthogonal decomposition then

$$\mathbf{h}^{(k)} = \mathbf{A}_k\mathbf{c} - \mathbf{B}_k\mathbf{h}_r^{(k)} \tag{10}$$

where $\mathbf{B}_k$ $(MN \times MN-1)$ is chosen such $\mathbf{B}_k^H\mathbf{A}_k = \mathbf{0}$. One can solve for $\mathbf{c}$ (that guarantees the desired constraint) by multiplying both sides of (10) with $\mathbf{A}_k^H$ yielding $\mathbf{c} = \big[\mathbf{A}_k^H\mathbf{A}_k\big]^{-1}\boldsymbol{\beta}$. Let (10) equal (9) and solve for $\mathbf{h}_r^{(k)}$ to get

$$\mathbf{h}^{(k)} = \mathbf{A}_k\mathbf{c} - \mathbf{B}_k\Big[\mathbf{B}_k^H\mathbf{K}\mathbf{B}_k\Big]^{-1}\mathbf{B}_k^H\mathbf{K}\mathbf{A}_k\mathbf{c} \tag{11}$$

One can show the important fact that $\mathbf{A}_k\mathbf{c} = \alpha\boldsymbol{\beta} \otimes \mathbf{a}(\hat{\mathbf{r}}_k^{gps})$ where α is an arbitrary scaling factor. This implies that our multiple constraint problem for each satellite can be written in terms of a single space-time weight vector. Selecting the $\boldsymbol{\beta}$ to be a vector of ones induces distortion on the GPS signal. However, one can minimize GPS signal distortion and force each filter per satellite to have a known fixed group delay by letting $\boldsymbol{\beta} = \boldsymbol{\delta}_n$ where the $N \times 1$ vector $\boldsymbol{\delta}_n = [0, 1, .., 0, .., 0]^T$ where the 1 is located according to the $n-th$ time instant across antennas that yields a linear-phase filter. Simulations involving this technique will be discussed in Section 4.

2.2. Joint Satellite Power Minimization Space-Time Processor

If one desires to reduce computational complexity by not utilizing a space-time processor for each satellite, one can try to find one set of space-time weights for all satellites in the FOV based on DOA information. This one set of weights will, naturally, not maximize the SINR per satellite but instead maximize the overall SINR of the combined satellites in the FOV. Let us define our space-time constraint $\boldsymbol{\beta} \otimes \mathbf{a}(\hat{\mathbf{r}}_k^{gps}) = \mathbf{a}_k$. Then the joint satellite power minimization problem can be stated as

$$\underset{\mathbf{h}}{\text{Minimize}} \quad \mathbf{h}^H\mathbf{K}\mathbf{h} \tag{12}$$

$$\text{subject to: } \mathbf{h}^H\mathbf{a}_k = 1, k = 1, ..., K$$

This type of power minimization leads to minimization of the beamformer output power subject to a unity gain constraint in the respective directions of each GPS satellite in the FOV. This allows suppression of interference from all other directions. Applying the method of Lagrange Multipliers to (12) yields the solution

$$\mathbf{h} = \mathbf{K}^{-1}\mathbf{A}_{gps}[\mathbf{A}_{gps}^H\mathbf{K}^{-1}\mathbf{A}_{gps}]^{-1}\mathbf{1}_K \tag{13}$$

where $\mathbf{1}_K$ is a $K \times 1$ vector containing all ones and $\mathbf{A}_{gps}$ is an $MN \times K$ matrix where $\mathbf{A}_{gps} = [\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_K]$. It is assumed that the DOA information of the GPS satellites in the FOV is obtained for necessary operation of the algorithm. Again, this information can be gathered from the INS or DOA algorithm.

3. MSWF ALGORITHM FOR SPACE-TIME PROCESSING

The MSWF algorithm is summarized below. The interpretation of the "desired" signal $d_0(n)$ varies amongst the different type of space-time processors.

- *Initialization:* $d_0(n)$ and $\tilde{\mathbf{x}}_0(n) = \tilde{\mathbf{x}}(n)$
- *Forward Recursion:* For $k = 1, 2, ..., D$:

$$\begin{aligned} \mathbf{h}_k &= E\{d_{k-1}^*(n)\tilde{\mathbf{x}}_{k-1}(n)\}/||E\{d_{k-1}^*(n)\tilde{\mathbf{x}}_{k-1}(n)\}|| \\ d_k(n) &= \mathbf{h}_k^H\tilde{\mathbf{x}}_{k-1}(n) \\ \tilde{\mathbf{x}}_k(n) &= \tilde{\mathbf{x}}_{k-1}(n) - \mathbf{h}_k d_k(n) \end{aligned}$$

- *Backward Recursion:* For $k = D, D-1, ..., 1$, with $e_D(n) = d_D(n)$:

$$\begin{aligned} w_k &= E\{d_{k-1}^*(n)e_k(n)\}/E\{|e_k(n)|^2\} \\ e_{k-1}(n) &= d_{k-1}(n) - w_k^* e_k(n) \end{aligned}$$

It follows that the matrix $\mathbf{T}_D = [\mathbf{h}_1 \ \mathbf{h}_2 \ ... \ \mathbf{h}_D]$ contains orthonormal columns and that the reduced dimension $D \times D$ correlation matrix $\mathbf{T}_D^H\mathbf{K}\mathbf{T}_D$ is tri-diagonal [1].

A low complexity implementation of the MSWF is depicted in Figure 1 for multiple space-time weight constraints. From our previous analysis, $\mathbf{S} = \mathbf{A}_{gps}$ or $\mathbf{S} = \mathbf{A}_k$. For the single constraint case, replace $\mathbf{S}$ with $\boldsymbol{\beta} \otimes \mathbf{a}(\hat{\mathbf{r}}_k^{gps}) = \mathbf{a}_k$ and replace $\mathbf{c}$ with the scalar 1. This figure clearly displays the multiple stages and modular structure highlighted by the dashed box. Operating in a D-dimensional space is tantamount to "terminating" all stages beyond the D-th stage. It is important to notice that all operations of the MSWF involve complex vector-vector products, not complex matrix-vector products (for the single space-time weight constraint), thereby implying computational complexity $O(MND)$ per snapshot. This particular implementation of the MSWF was first discovered by Ricks and Goldstein [4]. To reduce implementation complexity, they exploited the structure of the full dimension orthogonal projection matrix. Compared to other space-time or space-frequency algorithms having operations of $O(MN)^3$ or $O(QM)^3J$ (where Q=processing order and J=number of bins) respectively, the MSWF is by far more computationally efficient. The impressive capabilities of the MSWF are demonstrated in the next section for the case of space-time power minimization based processing.

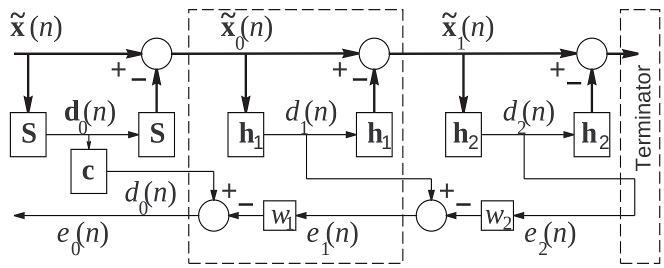

Fig. 1. Efficient implementation of the multiple constrained MSWF based on Correlations Subtractive Structure(CSS).

4. SIMULATIONS

A seven element circular array with isotropic gain and no mutual coupling is used to illustrate the effectiveness of using MSWF based space-time power minimization algorithms to effectively cancel both wideband and narrowband jammers. It is illustrated that a reduction in computational complexity and sample support can be achieved while operating in a reduced-rank mode. Consider the case $M = N = 7$. These definitions imply an $M = 7$ element equi-spaced circular array with $N = 7$ taps at each antenna.

The space-time processors are constrained utilizing the space-time reference constraint, $\boldsymbol{\beta} = \mathbf{1}_N$, $\boldsymbol{\beta} = \boldsymbol{\delta}_n$, and multiple constrained $\boldsymbol{\beta} = \boldsymbol{\delta}_n$ with respect to four GPS satellite DOA vectors. Each of these constrained processors are respectively labeled as Delta Con.,Single Con.(all ones), Single Con.(delta), and Multiple Con.(delta) in the figures. Table 1 summarizes the values used for each space-time processor. The power levels of all thirteen jammers were chosen in such a way to yield a J/S of 88 dB when all jammers are active (Scenario 2). Since we are assuming a $30MHz$ receiver bandwidth at each antenna, the noise floor was determined to be approximately -129 dBW after filtering at each antenna. The satellite locations where chosen assuming position from Billerica, MA at some arbitrary time. All simulations assume Satellite 1 as having the desired look direction.

Reduced Dimension Performance. Figure 2 illustrate the SINR of the space-time power minimization processing algorithms before decorrelation based on the MSWF as a function of subspace dimension. The maximum achievable ideal SINR assumes no interference with prior knowledge of the DOA associated with the desired satellite in the FOV. Scenario 1 (1 wideband(WB) and 1 narrowband(NB) jammer) provided a non-saturated interfering environment in terms of not using all the available degrees of freedom associated with the filter to suppress the jammers. The single DOA constrained space-time processing filter outperformed the other processors as expected for scenario 1, but Figure 2 illustrates that in a saturated jamming environment where all spatial degrees of freedom have been used, the single DOA constraint and space-time reference based algorithm (Delta Con.) yield similar performances. In the saturated jammer environment notice that the algorithms are able to operate at rank 22 (out of 49). Both scenarios illustrate a reduction in computational complexity using the MSWF since the processor can operate in a reduced dimension rather than the full dimension by exploiting the filter structure of Figure 1.

Convergence Performance/Low Sample Support/Complexity. The previous simulations associated with the power minimization algorithms gave insight into what rank would yield a desired ideal SINR. It is necessary to consider the effects of sample support and understand the tradeoff between sample support needed and desired SINR for a specific algorithm. In a non-saturated jammer environment at a lower rank versus the full rank solution, Figure 3 illustrates that a higher SINR can actually be achieved. For scenario 1, Figure 4 illustrates that all MSWF based filtering only requires 50 samples at rank 5 to yield the best SINR for each particular power minimization algorithm. This implies that in order to completely null the jammers in Scenario 1 the MSWF would need $O(49 * 5 * 50)$ operations where $MN = 49$,$D = 5$, and $samples = 50$.

Cross Correlation Function (CCF) Distortion. It was discussed previously that a proper choice of $\boldsymbol{\beta}$ when forming the space-time weight vector can ensure that the distortion of the GPS signal will be minimized with a known fixed group delay, i.e. linear phase filter. It can be shown that if the effective filter response for the space-time processor has a linear phase then the CCF associated with a GPS signal passed through such a filter will have minimal distortion. Figure 5 illustrates the effects of using the different space-time weight vectors on the ideal CCF. The ideal CCF was generated based on a formulation from [2]. Notice that choosing the space-time weight vector such that $\boldsymbol{\beta} = \boldsymbol{\delta}_n$ (Single Con.(delta)) yielded minimal distortion to the ideal CCF.

Table 1. Simulation Parameters for Circular Array

Code Type	SNR	(ϕ, θ)
Satellite 1,2,3&4	-157 dBW	$(51°, 35°)$ $(53°, 75°)$ $(247°, 27°)$ $(307°, 39°)$
Jammer Type	SNR	(ϕ, θ)
WB Jammers	-80 dBW	$(15°, 80°)$ $(45°, 80°)$ $(90°, 80°)$ $(135°, 80°)$ $(180°, 80°)$$(225°, 80°)$ $(270°, 80°)$$(315°, 80°)$
Jammer Type	SNR	(ϕ, θ)
NB Jammers	-80 dBW	$(5°, 80°)$ $(240°, 80°)$ $(120°, 80°)$ $(216°, 80°)$ $(288°, 80°)$

5. CONCLUSION

The MSWF based space-time processor was shown to exhibit exceptional nulling performance for both wideband and narrowband jammers at low rank while maintaining reduced complexity. The reduced dimension subspace selected by the MSWF exhibits rapid convergence in SINR implying adaptive null tracking in a dynamic jamming environment. Distortion of the ideal CCF using a properly constrained power minimization algorithm was shown to be

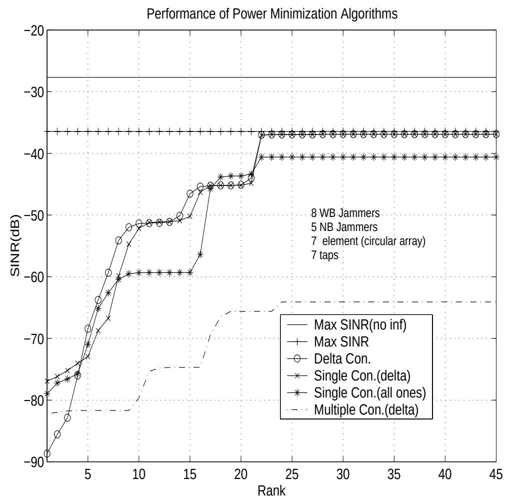

Fig. 2. Reduced Rank Performance of Power Minimization Algorithms for Scenario 2.

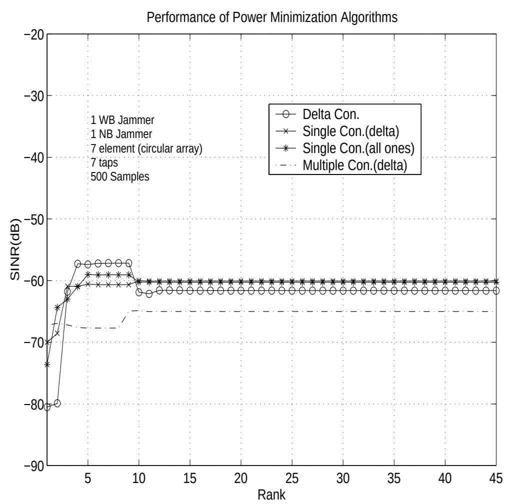

Fig. 3. Reduced Rank Performance of Power Minimization Algorithms for Scenario 1 with 500 Samples.

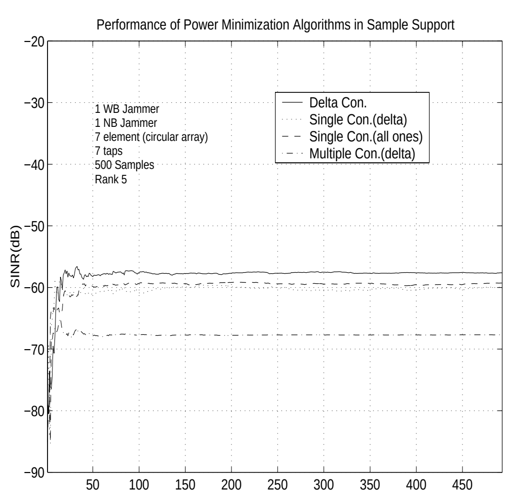

Fig. 4. Reduced Rank Performance of Power Minimization Algorithms for Scenario 1 with varying Sample Support .

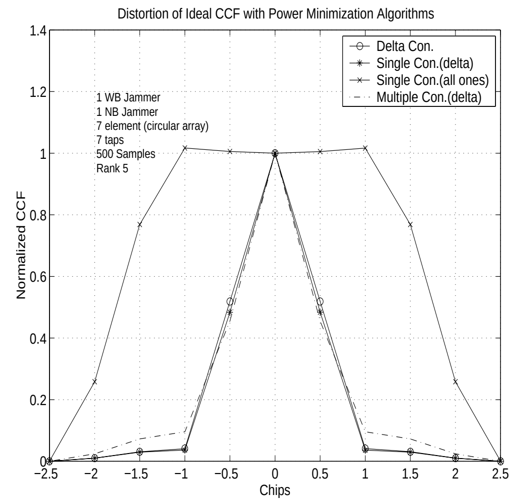

Fig. 5. CCF Performance of Power Minimization Algorithms for Scenario 1.

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6. REFERENCES

[1] J. Scott Goldstein, I. S. Reed, and L. L. Scharf, "A multistage representation of the wiener filter based on orthogonal Projections," *IEEE Trans. on Information Theory*, vol. 44, pp. 2943-2959, Nov. 1998.

[2] R. L. Fante and J. J. Vaccaro, "Wideband cancellation of interference in a GPS receive array," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 36, pp. 549-564, April 2000.

[3] W.L. Myrick, M.D. Zoltowski and J.Scott Goldstein, "Exploiting conjugate symmetry in power minimization based pre-processing for GPS: reduced complexity and smoothness," *Proc. of 2000 IEEE Int'l Conf. on Acoustics, Speech, and Signal Processing*, Istanbul, Turkey, 5-9 June 2000.

[4] D. C. Ricks and J. S. Goldstein, "Efficient architectures for implementing adaptive algorithms," *Proceedings of the 2000 Antenna Applications Symposium*,pp. 29-41, Allerton Park, Monticello, Illinois, September 20-22,2000.