OPTIMAL TIME-FREQUENCY SIGNALING FOR RAPIDLY TIME-VARYING CHANNELS

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ABSTRACT

We introduce a new signaling scheme for time- and frequency-selective channels that is a generalization of Multi-Carrier Code Division Multiple Access (MC-CDMA) signaling used in slowly time-varying channels. The Fourier basis functions used in conventional MC-CDMA systems encounter temporal distortion in rapidly time-varying channels resulting in degraded performance. The proposed scheme transmits the data over a set of orthonormal time-frequency basis functions whose time-frequency support is matched to the coherence time and bandwidth of the channel. The time-frequency signaling scheme approximately diagonalizes the time-varying multipath channel and each basis function encounters flat Rayleigh fading. We derive an optimal choice of basis parameters that yield the most accurate diagonalization for given multipath and Doppler spreads of the channel. The proposed system fully exploits the diversity afforded by the channel and delivers improved performance in rapidly time-varying channels in contrast to degraded performance of existing systems under such conditions.

1. INTRODUCTION

MC-CDMA systems operating over slowly time-varying frequency selective channels transmit information over a set of orthogonal subcarriers (Fourier basis functions), each supporting a low rate stream and encountering flat fading. Since Fourier basis functions are eigenfunctions of time-invariant channels, MC signaling diagonalizes the channel and converts the slow fading frequency selective channel into parallel flat fading channels. This reduces the effect of inter symbol interference (ISI) and, in conjunction with a cyclic prefix, eliminates the need for equalization. It is advantageous to increase the symbol duration in each subcarrier as much as possible to reduce the effects of ISI, particularly in high rate applications. However, as the symbol duration increases, the system becomes more prone to temporal distortions in time-varying (fast fading) situations. The subcarriers encounter spectral dispersion under fast fading channels that destroys the orthogonality between the subcarriers and significantly degrades performance [1].

In this paper, we introduce a new signaling scheme based on time-frequency basis functions that approximately diagonalize time-varying frequency selective channels — the basis functions serve as approximate eigenfunctions of the channel and each basis function encounters non-selective fading. The signaling scheme uses time-frequency basis functions whose time-frequency support is matched to the coherence time and bandwidth of the channel. We discuss the necessary conditions for such channel diagonalization to be possible and derive optimal basis parameters that yield the most accurate diagonalization for given multipath and Doppler spreads of the channel. The proposed scheme thus facilitates optimal exploitation of the diversity afforded by the channel.

The next section describes the proposed time-frequency signaling scheme. Optimal choice of basis parameters is derived in Section 3. Section 4 analyzes the performance of a coherent receiver for the proposed signaling scheme. Concluding remarks are provided in Section 5.

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2. TIME-FREQUENCY SIGNALING

Let T denote the duration and B the essential two-sided bandwidth occupied by each symbol waveform. The dimension of the signal space is approximately equal to the time-bandwidth product: $N = BT$. We consider a wide sense stationary uncorrelated scattering (WSSUS) channel model with multipath spread, T_m, and the two-sided Doppler spread, B_d. The multipath spread signifies the maximum delay and the Doppler spread corresponds to the maximum Doppler shift encountered during propagation. The time- and frequency-selectivity of the channel is governed by the *coherence time*, $\Delta t_c \approx 1/B_d$, and the *coherence bandwidth*, $\Delta f_c \approx 1/T_m$. Frequencies separated by Δf_c and times separated by Δt_c encounter independent fading. The level of multipath diversity is $L \approx B/\Delta f_c \approx T_m B$ and level of Doppler diversity is $M \approx T/\Delta t_c \approx TB_d$.

Consider signaling with N basis functions spanning the signal space. All basis functions have the same support in time and frequency: T_o and B_o respectively. Figure 1 illustrates the conventional MC-CDMA (Fourier) signaling for slow fading channels and the proposed time-frequency signaling for fast fading channels. In slowly fading scenarios, the channel response is constant over the symbol duration T. Thus, in conventional MC-CDMA signaling, $T_o = T$ and the information is transmitted over N orthogonal subcarriers in parallel, each with bandwidth $B_o = B/N = 1/T$. Together, the N Fourier basis functions cover the entire bandwidth, as illustrated in Figure 1(a). The symbol duration $T = T_o$ is chosen long enough so that $B_o = 1/T \ll \Delta f_c$ and each carrier encounters flat fading. The transmitted signal in conventional MC-CDMA systems can be written as

$$\tilde{s}(t) = b \sum_{n=0}^{N-1} a_n q_T(t) e^{j\frac{2\pi n}{T}t} \quad (1)$$

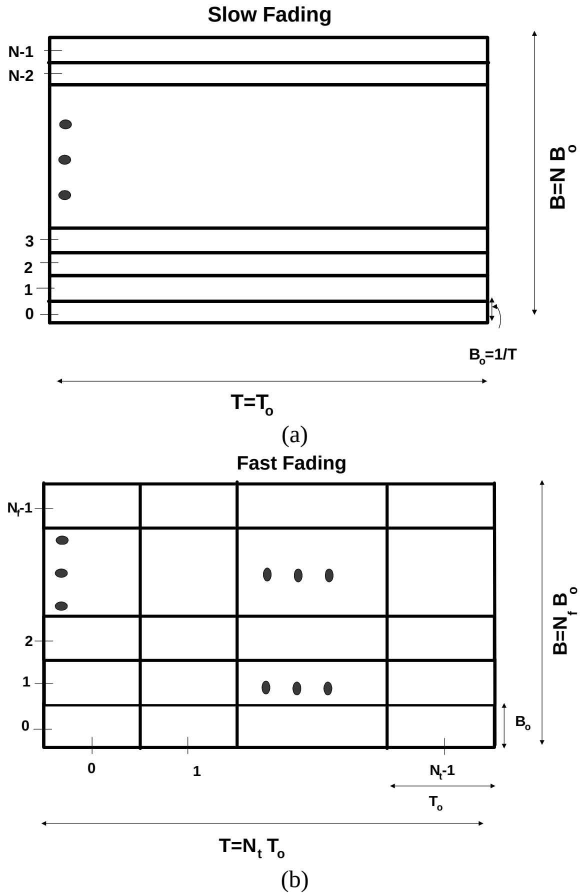


Fig. 1. Time-frequency support of basis signals. (a) Fourier basis used in multicarrier systems in slow fading channels. (b) The proposed short-time Fourier basis in fast fading channels.

where b is the transmitted bit, $\{a_n\}$ is the CDMA signature code, and $q_T(t)$ is the normalized pulse shape of duration T, assumed rectangular for simplicity: $q_T(t) = \frac{1}{\sqrt{T}}, \ -T/2 \le t \le T/2$.

In rapidly time-varying situations, the channel response is no longer constant over the symbol duration and the temporal distortion destroys the orthogonality between subcarriers and degrades performance [1]. We propose signaling with time-frequency basis functions that avoids distortion in both time and frequency by appropriately choosing T_o and B_o according to Δt_c and Δf_c. The transmitted signal using the time-frequency basis functions is

$$s(t) = b \sum_{n=0}^{N_f-1} \sum_{p=0}^{N_t-1} a_{n,p} u_{n,p}(t) \tag{2}$$

where $\{a_{n,p}\}$ is the length N signature code distributed over the $N = N_t N_f$ basis functions

$$u_{n,p}(t) = q_{T_o}(t - pT_o) e^{j2\pi n B_o t}. \tag{3}$$

As illustrated in Figure 1(b), $N = N_t N_f$ time-frequency basis functions in (3) are generated by N_t time shifts and N_f frequency shifts of the prototype signal $q_{T_o}(t)$ of duration $T_o = T/N_t$ and bandwidth $B_o = B/N_f$.

The choice of T_o and B_o is made so that each basis function (3) encounters approximately flat fading. Essentially, T_o has to be sufficiently small compared to Δt_c and B_o has to be sufficiently small compared to Δf_c. More precisely, T_o and B_o need to satisfy the following symmetric conditions:

$$\begin{aligned}
\mathbf{C1}: \ & \max\left(\frac{1}{B}, T_m\right) \ll T_o \ll \min\left(\frac{1}{B_d}, T\right) \quad \text{or, equivalently} \\
& \max\left(TB_d, 1\right) \ll N_t = \frac{T}{T_o} \ll \min\left(\frac{T}{T_m}, N\right) \\
\mathbf{C2}: \ & \max\left(\frac{1}{T}, B_d\right) \ll B_o \ll \min\left(\frac{1}{T_m}, B\right) \quad \text{or, equivalently} \\
& \max\left(T_m B, 1\right) \ll N_f = \frac{B}{B_o} \ll \min\left(\frac{B}{B_d}, N\right)
\end{aligned} \tag{4}$$

Note that the above conditions imply

$$\max\left(\frac{1}{N}, T_m B_d\right) \ll T_o B_o \ll \min\left(\frac{1}{T_m B_d}, N\right) \tag{5}$$

which can be satisfied only for *underspread* channels [4] for which $T_m B_d < 1$. For most practical channels, $T_m B_d \ll 1$. In this paper, we focus on the case where $T_o B_o = 1 \Leftrightarrow N = N_t N_f$, for which **C1** and **C2** become equivalent.

We note that the proposed time-frequency signaling can be efficiently implemented digitally by dividing the N samples in each symbol into N_t segments, each containing N_f samples, and applying an N_f-point discrete Fourier transform to each segment.

3. OPTIMAL CHOICE OF T_O AND B_O

For given pulse shape, the time-frequency basis functions (3) are completely determined by T_o and B_o. In this section, we discuss the optimal choice of T_o and B_o for the case $T_o B_o = 1$ so that the time-frequency basis functions yield the most accurate approximate diagonalization of the channel with each basis function encountering approximately flat fading. The problem boils down to finding the optimal T_o subject to the condition **C1**. We first discuss some assumptions on channel statistics and then derive a representation for the received signal in terms of the time-frequency basis functions which is then used to derive optimal T_o and B_o.

The received signal can be written as

$$r(t) = \int h(t,f) S(f) e^{j2\pi f t} df + n(t) \tag{6}$$

where $S(f)$ is the Fourier transform of the transmitted signal, $h(t,f)$ denotes the time-varying channel frequency response and $n(t)$ is AWGN with power spectral density σ^2. Under the WSSUS assumption, the correlation function of $h(t,f)$ is given by

$$R(\Delta t, \Delta f) = \mathrm{E}\left[h\left(t + \Delta t, f + \Delta f\right) h^*\left(t, f\right)\right] \tag{7}$$

which is separable if all multipaths have the same *spaced-time* correlation function, $R_1(\Delta t) = \mathrm{E}\left[h(t + \Delta t, .) h^*(t, .)\right]$ [2]. That is,

$$R(\Delta t, \Delta f) = R_1(\Delta t) R_2(\Delta f) \tag{8}$$

where $R_2(\Delta f) = \mathrm{E}\left[h(., f + \Delta f) h^*(., f)\right]$ is the *spaced-frequency* correlation function. We also assume a flat Doppler and multipath power profile: $\psi(\theta) = FT\{R_1(\Delta t)\} = \frac{1}{B_d}, \ -\frac{B_d}{2} \le \theta \le \frac{B_d}{2}$

and $\phi(\tau) = FT\{R_2(\Delta f)\} = \frac{1}{T_m},\ 0 \leq \tau \leq T_m$, where FT denotes the Fourier transform. Under this assumption

$$R_1(\Delta t) = \text{sinc}\,(B_d \Delta t) \quad , \quad R_2(\Delta f) = \text{sinc}\,(T_m \Delta f)\, e^{-j\pi T_m \Delta f} \tag{9}$$

For the proposed signaling scheme (2), the received signal in (6) can be expressed as

$$\begin{aligned} r(t) &= b \sum_{n=0}^{N_f-1} \sum_{p=0}^{N_t-1} a_{n,p} \int_{(n-1/2)B_o}^{(n+1/2)B_o} \tilde{h}_{n,p}(t,f) \times \\ &\quad U_{n,p}(f) e^{j2\pi f t} df + n(t) \end{aligned} \tag{10}$$

where $U_{n,p}(f) = FT\{u_{n,p}(t)\}$ and

$$\begin{aligned} \tilde{h}_{n,p}(t,f) &= h(t,f) I_{[(p-1/2)T_o,(p+1/2)T_o]}(t) \times \\ &\quad I_{[(n-1/2)B_o,(n+1/2)B_o]}(f) \end{aligned} \tag{11}$$

denotes the part of $h(t,f)$ affecting $u_{n,p}(t) = q(t - pT_o)e^{j2\pi n B_o t}$ and $I_{[x,y]}(t)$ is the indicator function of the interval $[x, y]$. $\tilde{h}_{n,p}(t,f)$ admits the Fourier series

$$\tilde{h}_{n,p}(t,f) \approx \sum_{m=-M_o}^{M_o} \sum_{l=0}^{L_o} c_{n,p}^{(m,l)} e^{j\frac{2\pi m t}{T_o}} e^{-j\frac{2\pi l f}{B_o}} \tag{12}$$

$$\begin{aligned} c_{n,p}^{(m,l)} &= \frac{1}{T_o B_o} \int_{(p-1/2)T_o}^{(p+1/2)T_o} \int_{(n-1/2)B_o}^{(n+1/2)B_o} h_{n,p}(t,f) \\ &\quad e^{-j\frac{2\pi m t}{T_o}} e^{j\frac{2\pi l f}{B_o}} df dt \end{aligned} \tag{13}$$

where $M_o \approx \lceil \frac{B_d T_o}{2} \rceil$ and $L_o \approx \lceil T_m B_o \rceil$. Substituting (12) in (10), we get

$$\begin{aligned} r(t) &\approx b \sum_{n=0}^{N_f-1} \sum_{p=0}^{N_t-1} a_{n,p} \sum_{m=-M_o}^{M_o} \sum_{l=0}^{L_o} c_{n,p}^{(m,l)} \times \\ &\quad u_{n,p}\left(t - \frac{l}{B_o}\right) e^{j\frac{2\pi m t}{T_o}} + n(t) \end{aligned} \tag{14}$$

If **C1** and **C2** are satisfied then $h(t,f)$ is approximately constant over any time-frequency region with support $T_o \times B_o$. Thus, $c_{n,p}^{(m,l)} \approx 0$ for $l \neq 0$ or $m \neq 0$ in (13). This reduces (14) to

$$r(t) \approx b \sum_{n=0}^{N_f-1} \sum_{p=0}^{N_t-1} a_{n,p} c_{n,p} u_{n,p}(t) + n(t) \tag{15}$$

where $c_{n,p} = c_{n,p}^{(0,0)}$ and we have dropped the superscript $(0,0)$ for convenience of notation. The approximate diagonalization of the channel by the time-frequency basis functions is evident from (15).

The main source of error in the diagonal approximation (15) is the contribution due to the $c_{n,p}^{(m,l)}$'s in (14) corresponding to the $m \neq 0$ and $l \neq 0$ terms. We now discuss the optimal choice of T_o (and $B_o = 1/T_o$) that minimizes this contribution. Under the WSSUS assumption, the statistics of $c_{n,p}^{(m,l)}$ are independent of n or p. Hence, without loss of generality, we derive optimum T_o for $n = p = 0$. We define the approximation error ρ to be the ratio of the energy in $\tilde{h}_{0,0}(t,f)$ not captured by $c_{0,0}$ to the total energy

$$\rho = \frac{\text{E}\left[\int_{-T_o/2}^{T_o/2} \int_{-B_0/2}^{B_0/2} |h(t,f)|^2 dt df\right] - \text{E}[|c_{0,0}|^2]}{\text{E}\left[\int_{-T_o/2}^{T_o/2} \int_{-B_0/2}^{B_0/2} |h(t,f)|^2 dt df\right]} \tag{16}$$

Proposition 1: Given a flat multipath and Doppler power profile, ρ in (16) is minimized when $\frac{T_o}{B_o} = \frac{T_m}{B_d}$.

Proof: See the Appendix.

Hence, under the assumption $T_o B_o = 1$, the optimal choice of T_o is $T_o = \sqrt{T_m/B_d}$.

4. SIGNAL RECEPTION AND PERFORMANCE

Let $z_{n,p}$ denote the projection of the received signal in (15) onto $u_{n,p}(t)$

$$z_{n,p} = \int r(t) q_{T_o}^*(t - pT_o) e^{-j2\pi n B_o t} dt = b_1 a_{n,p} c_{n,p} + v_{n,p} \tag{17}$$

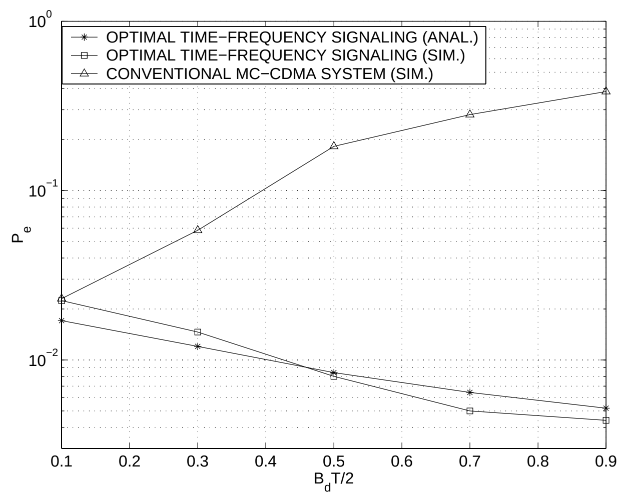
where $\{v_{n,p}\}$ are independent Gaussian random variables with variance σ^2. For convenience of notation, we stack the test statistics in one vector, $\mathbf{z} = b\mathbf{A}\mathbf{c} + \mathbf{v}$, where $\mathbf{A}$ is a diagonal matrix whose entries are the signature code, $\mathbf{c}$ is the Gaussian vector of channel coefficients $\{c_{n,p}\}$, and $\mathbf{v}$ is the noise vector. Given the knowledge of $\mathbf{c}$ at the receiver, the optimum coherent bit decision is given by $\hat{b} = \text{sign}[\text{real}(\mathbf{c}^H \mathbf{A}^H \mathbf{z})]$. The bit error probability conditioned on $\mathbf{c}$ is $P_e(\mathbf{c}) = Q(\sqrt{2\mathbf{c}^H \mathbf{c}/\sigma^2})$ where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-x^2}{2}} dx$. The unconditional $P_e = \text{E}[P_e(\mathbf{c})]$ where the averaging is over the distribution of $\mathbf{c}$. Since $\mathbf{c}$ is complex Gaussian random vector, P_e is determined by the eigenvalues of $\mathbf{R}_{\mathbf{c},\mathbf{c}} = \text{E}[\mathbf{c}\mathbf{c}^H]$ and a convenient closed form expression for P_e can be found in [3]. $\mathbf{R}_{\mathbf{c},\mathbf{c}}$ is the correlation matrix of the channel coefficients $\{c_{n,p} = c_{n,p}^{(0,0)}\}$ in (13). Since the channel is almost constant over each region of support $T_o \times B_o$, we have $c_{n,p} \approx h(pT_o, nB_0)$ and $\text{E}\left[c_{n,p} c_{m,l}^*\right] = R((p-l)T_o, (n-m)B_o) = R_1((p-l)T_o)\, R_2((n-m)B_o)$.

We note that for a given frequency index n, the different time samples of the channel have a covariance matrix of rank M — the Doppler diversity in the channel. Similarly, for a given time slot p, the different frequency samples of the channel have a covariance matrix of rank L — the multipath diversity in the channel. Consequently, the rank of $\mathbf{R}_{\mathbf{c},\mathbf{c}}$ is LM and the proposed system exploits full LM-fold diversity.

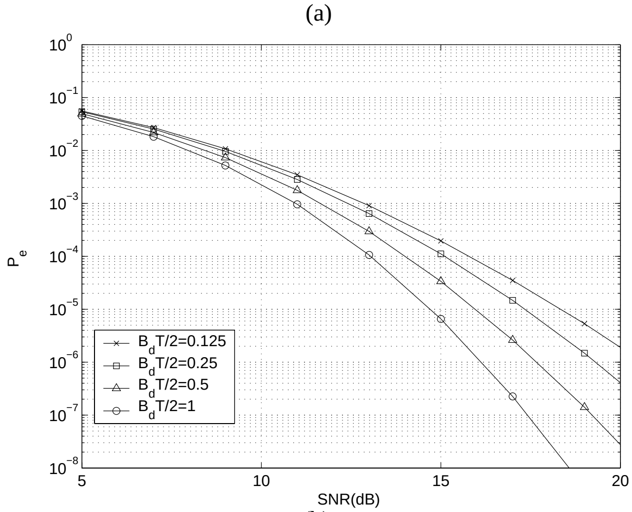
Figure 2(a) compares the performance of the conventional MC-CDMA system with one based on the proposed time-frequency signaling. The SNR is 10dB and the underlying channel has 2 resolvable paths ($L = 2$). The figure shows the P_e of the two systems as a function of the fast fading parameter $TB_d/2$. Perfect channel estimates are available at the receiver. $T_o B_d$ is kept fixed corresponding to the optimum choice of T_o which results in $T_o B_d = \sqrt{T_m B_d} = 0.2$ in this case. TB_d is increased by increasing T from T_o to $9T_o$ which corresponds to increasing N from 10 to 90 in this case. This is akin to using longer symbol duration for given bandwidth and thus increasing the signal space dimension. The simulations results are generated via Monte-Carlo simulation of 100,000 symbols using Jakes Model. The channel estimates for the simulation results are generated via a noise-free pilot signal. The analytical P_e computation for the proposed system uses the separability assumption (8) and assumes uniform multipath and Doppler power profiles. Furthermore, the interference between different time-frequency basis functions is ignored in the analytical computation. The close agreement between the analytical and simulated results for the proposed system demonstrates the accuracy of the diagonalization at optimum choice of T_o.

Figure 2(a) clearly shows the significant degradation in the performance of conventional MC-CDMA systems under fast fading, consistent with results reported by other authors (see, e.g., [1]). On the other hand, the performance of the proposed time-frequency signaling scheme improves significantly under faster fading due to Doppler diversity. Thus, the proposed scheme not only facilitates longer symbol durations to reduce the effects of ISI but also yields improved performance with longer symbols.

Figure 2(b) shows the analytically computed P_e of the proposed system as a function of SNR for different values of $TB_d/2$. As evident, the gain in performance with increasing TB_d is more pronounced at higher SNRs. Furthermore, the slope of the P_e curve gets sharper with increasing TB_d due to Doppler diversity.



(a)



(b)

Fig. 2. (a) Comparison between the P_e of a conventional MC-CDMA system and the proposed system as a function of $TB_d/2$. There are two resolvable multipaths ($L = 2$) and SNR=10dB. The simulation results are based on Monte-Carlo simulation of 100,000 symbols via Jakes Model. The performance of the proposed system improves with faster fading, in contrast to the degradation in performance of the conventional system. (b) The P_e of the proposed system as a function of SNR for different values of $TB_d/2$ ($L = 2$). The increase in the slope of the P_e curve with increasing TB_d (higher Doppler diversity) is evident.

5. CONCLUSION

We have introduced a novel signaling scheme for CDMA communication systems operating over time-and frequency-selective channels. Information is transmitted over time-frequency basis functions whose time-frequency support is matched to the coherence time and bandwidth of the channel. The time-frequency basis functions serve as approximate eigenfunctions of underspread channels with each basis signal encountering flat fading. We derive optimum duration and bandwidth of the basis functions that yields the most accurate diagonalization for given channel spread parameters. The proposed system fully exploits channel diversity and our results demonstrate that it delivers significantly improved performance under fast fading by exploiting Doppler diversity. This is in stark contrast to existing systems whose performance deteriorates under fast fading. While the focus of this paper was on orthogonal basis functions, we are currently exploring the use of non-orthogonal basis functions [5] that may be more appropriate for non-separable channels.

Appendix

Using (8) in (16) we get $\rho = 1 - xy$ where

$$x = \frac{1}{(T_0)^2} \int_{-T_o/2}^{T_0/2} \int_{-T_o/2}^{T_o/2} R_1(t_1 - t_2) dt_1 dt_2 \quad (18)$$

$$y = \frac{1}{(B_0)^2} \int_{-B_0/2}^{B_0/2} \int_{-B_0/2}^{B_0/2} R_2(f_1 - f_2) df_1 df_2. \quad (19)$$

Using (9), (18) becomes $x \approx 2\text{sinc}(B_d T_0/2) - \text{sinc}^2(B_d T_0/2)$ by using the approximation $\frac{1}{a}\int_0^a \text{sinc}(l) dl = \text{sinc}(a/2)$ that is justified by **C1**. A similar expression for y can be obtained by replacing B_d with T_m and T_o with B_o and using the fact that $e^{-j\pi T_m \Delta f} \approx 1$ in (9) for $|\Delta f| < B_o$ (due to **C2**). Thus, $\rho = 1 - \left[1 - (1 - \text{sinc}(B_d T_0/2))^2\right]\left[1 - (1 - \text{sinc}(T_m B_0/2))^2\right]$ which can be further simplified to $\rho = K((B_d T_0)^6 + (T_m B_0)^6) - K^2(B_d T_m)^6$ by using the fact that $1 - \text{sinc}(a) \approx \frac{\pi}{3!}a^3$ for $x \ll 1$, where $K = \frac{\pi^2}{(3!)^2 2^6}$. Since $(B_d T_0)^6 (T_m B_0)^6 = (T_m B_d)^6$ is a constant, we deduce that ρ has a minimum at $B_d T_o = T_m B_o$ by using the inequality $\frac{a+b}{2} \geq \sqrt{ab}$ for $a > 0$ and $b > 0$.

6. REFERENCES

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