Novel Inverse Methods in Land Mine Imaging

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ABSTRACT

The imaging of buried land mines continues to present significant signal-processing challenges in the development of inverse methods for the detection of plastic mines buried in soil. To address this difficult problem, recent mathematical advances in the development of the Elliptic Systems Method are used to generate images of the buried land mines. The proposed approach adapts earlier methods, successfully applied in laser tomography of breast tumors using the diffusion equation, to the present problem of land mine imaging using the Helmholtz equation. The images generated by the new method represent electromagnetic properties of underground regions, providing effective differentiation of plastic land mines from surrounding soil. Experimental results are presented to demonstrate the new method.

1 INTRODUCTION

The imaging of buried land mines presents significant challenges in the development of effective signal-processing methods for solving the inverse problem posed by measured ground-penetrating radar returns from plastic mines. A successful practical solution of this difficult problem requires a significant technological advance, rather than marginal improvements. To this end, a novel signal-processing approach is proposed where the ground-penetrating radar system is designed to take advantage of the latest mathematical advances in inverse problems, rather than working around limitations of current radar technology[1-3]. These novel mathematical advances enable direct characterization of the electromagnetic properties of the soil (relative dielectric constant and conductivity) from the radar signals.

In our previous publications, we developed a new approach for the solution of the integro-differential formulation of the inverse problem for the diffusion equation by using a Galerkin-like method. This novel inverse method has been used to solve similar challenging problems in laser tomography [4,5]. More recently, the authors have been investigating adaptation of these earlier successes to imaging underground land mines, which are characterized by a Helmholtz equation [3].

Usually the solution of a linearized inverse problem for the Helmholtz equation is based on the Born or Ryutov approximation as in [7,8]. Other methods which avoid the Born or Ryutov approximation can be divided into two classes: iterative algorithms based on the integral formulation of inverse problem, or optimization approaches [see 9,10,11,12,13]. Both of these types of methods are time consuming because of the huge conditional number of resulting system, even for a very small number of grid points. Thus, the convergence of these methods becomes too slow, except when one assumes a very simple form for the target (i.e., a cylindrical target in [6]) and can construct an analytic approximation formula for the solution of the scattering problem. In these simple cases, the number of independent parameters in optimization procedure is very small, but such algorithms can not accurately handle targets of complex geometry.

In this paper we present a novel approach for the solution of the scattering problem. In this approach, we use an integro-differential form rather than the conventional integral form of the resulting system requiring solution of the overdetermined discretized system at each frequency. The normal equation approach for the solution of such a system leads to the solution of a large, sparse, positive-definite, Hermitian matrix system, rather than the conventional full and ill-conditioned matrix system. This allows us to use an efficient preconditioned technique for the solution of this system.

This new method provides fast and accurate solution of the inverse problem. Both the location and electromagnetic characteristics of targets of interest are accurately determined. An important feature of these methods, for practical purposes, is their rapid convergence for both the forward and inverse problems. In the following, the forward method is first described, followed by discussion of new integro-differential approach for the solution of the inverse problem. Experimental results demonstrating the efficacy of the new method are given.

2 FORWARD METHOD

The development of new methods for solving inverse problems frequently requires the rapid solution of the forward problem for the Hlemholtz equation to generate test data for evaluating the inverse methods. In earlier work, Gryazin et al. [2] presented a novel forward method that provides fast and accurate solution of the forward problem for land mine detection. In this new method, the GMRES (Generalized Minimum Residual) approach is improved by using a carefully chosen preconditioner. This new method overcomes computational difficulties that arise due to the large number of grid points necessary in solving the Helmholtz equations for the land mine problem at high frequencies. A brief summary of the method is given below, and further details are found in [2].

The boundary value problem is governed by the Helmholtz equation with Sommerfeld-like boundary conditions

$$\nabla^2 \overline{E} + g_0{}^2(x,y)\overline{E} = -f(x,y,E_0)$$
$$\overline{E}_n - jg\overline{E} = 0.$$

Where γ(x,y) is the propagation constant as a function of (x,y) coordinates and $\overline{E}$ is the scattered electric field. The source is presumed to lie well inside the spatial region Ω where the solution is computed. The boundary conditions allow reducing the reflection of waves back into the region Ω. The problem is then discretized using second order finite difference equations to compute the solution on a regular mesh of points in Ω. The resulting matrix describing the system of equations has a block tridiagonal structure but is neither positive definite nor Hermitian. Thus, most iterative methods of solution diverge or converge too slowly for the large number of mesh points required at high frequencies.

We address these computational difficulties using the GMRES method with a preconditioner using Somerfield boundary conditions at the upper and lower y-axis boundaries and Neuman boundary conditions at the left and right x-axis boundaries, as well as homogenious background. This results in a fast and accurate algorithm for computing the solution of the Helmholtz equation using fast transform algorithms for the inversion of the preconditioner. The effect of using Sommerfeld-like boundary conditions rather than radiation boundary conditions at infinity is minimized by increasing the size of Ω until the solution well within Ω is independent of the size of Ω. This approach works particularly well when the attenuation characteristics of the soil are high, leading to low values of the electric field at the boundaries.

This system is then solved using GMRES and the aforementioned preconditioner and boundary conditions. Further details of this new method can be found in Gryazin et al. [2]. The result is the perturbation in the electric field $\overline{E}$ caused by the presence of a target.

3 INVERSE METHOD

To solve the inverse problem, a second-generation version of the Elliptic Systems Method (ESM) has been developed. The ESM was initially proposed for inverse problems for time-dependent diffusion PDEs (partial differential equations), with applications to, among others, optical medical imaging [6]. More recently, a second-generation version of the ESM has been developed, where the resulting integro-differential PDE is solved directly, rather than using a Galerkin-like approach, as was the case in the first version of ESM. Thus, we approximate the solution of this PDE in its original form, rather than through its first few power moments. The main idea behind this algorithm is to use an integro-differential form rather than the conventional integral form of the resulting equation, which expresses inverse problem. This integro-differential form leads to the solution of a large, sparse, positive-definite, Hermitian matrix system, rather than the conventional full and ill-conditioned matrix system. This allows us to use an efficient preconditioning technique for the solution of this system.

The first step of this imaging algorithm is to derive and reformulate a linearized inverse problem as a boundary value problem for a Volterra-type integro-differential equation of the second order. The integration in this equation is carried out with respect to frequency. The highest value of the frequency in the available frequency band is a quite natural regularization parameter. Therefore, the regularization in this case represents a natural procedure of cutting off high frequencies, which are not available from measurements. Moreover, Volterra-type integral equations are essentially "initial value" problems and we can use efficient "marching" numerical procedures for the solution of such an equation. A difficulty here is that, we don't know the initial distribution of the function at the highest frequency. So to guarantee the uniqueness of such a problem, we need to add a second boundary condition at least over part of the boundary. This leads to an overdetermined boundary-value problem for a Volterra-type integro-differential equation of the second order. The overdetermination is due to the presence of both Dirichlet and Neumann boundary conditions, rather than only one on the surface part of the boundary.

We then approximate this equation by using a second order central finite-difference scheme for the differential part of the operator and a simple trapezoidal rule for the integral part of this equation. The resulting discretized system is overdetermined. To solve it at each step of the marching algorithm (viz. at each frequency), we use the normal equations method. Because of the large computational costs and memory requirements for the direct solution of such problems, iterative methods are preferred. Unlike a discretization of the original second order problem, the normal equation approach produces a positive definite Hermitian system; we use the preconditioned conjugate gradient method for the solution of this system.

However, a central issue in these approaches is the selection of a preconditioner. In [14], Manteuffel et al. show that, to be effective, the preconditioner for conjugate gradient method should use the same boundary conditions as the original operator. We have chosen to use as a preconditioner the exact factorization of the original matrix using the method of nested dissection, but for only a small number of the frequencies. We have found that this selection works well for nearby frequencies where it is an excellent approximate inverse. In this approach we have developed an automatic algorithm for the near optimal choice of frequency ranges, over which we use the same preconditioner. The number of iterations of the conjugate gradient method is usually less than 5 or 6 for all frequencies from the considered interval (from .5GHz to 3GHz). Because the factorized matrix does not depend on the solution of inverse problem, factorization could be effectively parallelized, but this expansion of the presented algorithm is outside the scope of this paper.

In our numerical experiments we take wet soil with 5% moisture as a background medium. We introduce multiplicative 10% Gaussian noise in the data at the surface. The mathematical expectation of this noise is zero. Figure 1 displays the original, noisy and smooth data at the surface just above the target, as a function of frequency. The solid, represents the "exact" value of this function obtained through the solution of the forward problem. Stars represent noisy data. Circles show result of the above smoothing procedure through splines. Figures 3 and 4 show results for the scenario illustrated in Fig. 2. The image of Fig. 3 shows the physical model of a plastic land mine buried in soil. The image in Fig. 4 shows the recovered image using the presented algorithm on a forward data set generated using preconditioned GMRES. In the figure, varying shades of gray in the reconstructed image represent different material characteristics (conductivity and dielectric constant).

Numerical experiments for TNT-filled mine-like targets, given in Fig. 4, show that locations, sizes, and real parts of $f(x,y,E_0)$ within targets are imaged with good accuracy. Somewhat lower quality images of $Im[f(x,y,E_0)]$ in recent experiments may be improved by introduction of Newton-like updates in order to take into account a non-linear dependence of the function $\overline{E}$ from the perturbation term $f(x,y,E_0)$.

4 ACKNOWLEDGMENT

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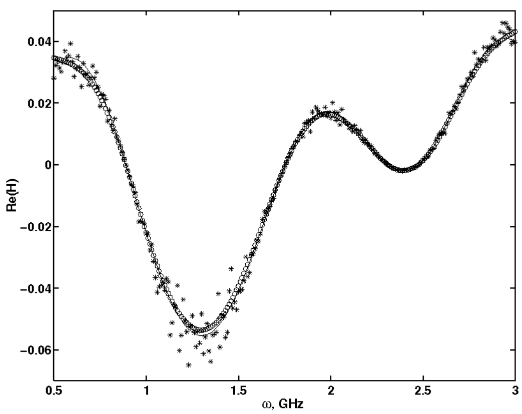

Figure 1. The original, noisy, and smoothed data at the surface just above the target, as a function of frequency.

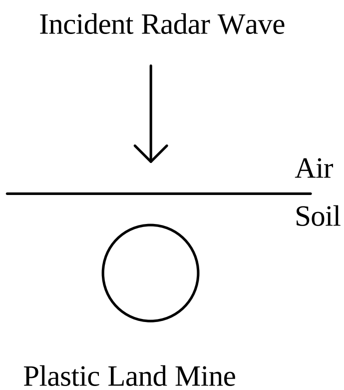

Figure 2. Illustration of scenario under consideration with plastic land mine target buried in the soil and incident ground-penetrating radar signal.

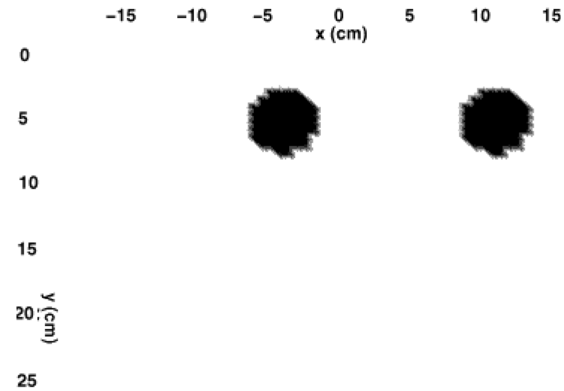

Figure 3. Model of two plastic land mines buried in soil.

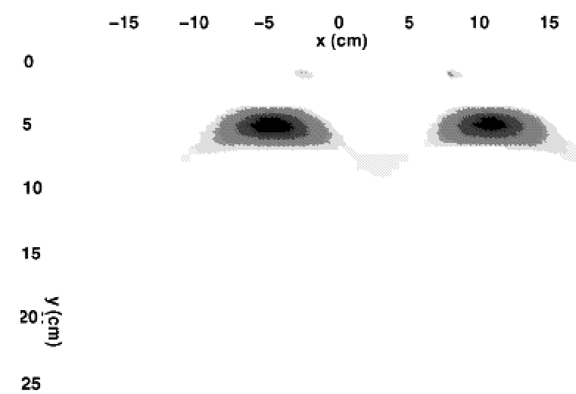

Figure 4. Recovered image using the proposed inverse method.