DESIGN OF A TIME-FREQUENCY DOMAIN MATCHED FILTER FOR DETECTION OF NON-STATIONARY SIGNALS

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ABSTRACT

In this paper, a practical and effective approach is proposed to detect a *transient* or *nonstationary* signal component of interest from a composite signal waveform. The detection problem has been re-formulated in terms of time-frequency analysis, and, thus, the conventional 1-D (i.e., time-domain) matched filter approach is extended to the 2-D (here, time-frequency domain) optimal filtering. For that purpose, the reduced interference distribution (RID) algorithm, the outer product expansion of the time-frequency distribution, the singular value decomposition (SVD), and *a priori* available time-frequency information of a signal part of interest are employed to derive a time-frequency domain matched filter by utilizing the singular values of the sampled time-frequency distribution and the corresponding fractions of signal energy. Finally, one real problem of detecting the snare drum sound event from a measured musical signal is considered to demonstrate the performance of the proposed approach.

1. INTRODUCTION

The classical formulation for the detection of a signal $s(t)$ with additive noise $n(t)$ can be given as follows [1]:

$$\begin{aligned} H_0 : z(t) &= n(t) \\ H_1 : z(t) &= s(t) + n(t) \end{aligned} \quad (1)$$

where $t \in T$, and it is assumed that the statistical information of the noise, i.e., the mean $\mathrm{E}\{n(t)\}$ and the covariance $\phi_n(\tau, \sigma)$, are known. The detection problem has been well-established and the optimal detection filter $h(t)$ can be obtained by solving the Fredholm integral equation[2]. In this paper, we address a new time-frequency formulation of detecting a transient or non-stationary signal event from an observed composite signal. Such time-frequency formulation of optimum detection has been addressed by several references [3] [4]. Previous work on random transient or non-stationary signal detection has been designed to distinguish transient or non-stationary signals from stationary noise background. In this paper, a new time-frequency domain filtering approach is proposed to detect a certain non-stationary signal component mixed with non-stationary noise signals, where *a priori* time-frequency information of a signal part of interest is assumed to be available in part. The conventional 1-D (i.e., time-domain) matched filter approach is extended to the 2-D (here, time-frequency domain) optimal filtering. Here, a time-varying optimal filter design has been approached in the sense of Wiener filter [5], and the optimal time-frequency filter is to be identified with the filtered output having the maximum signal-to-noise ratio (SNR) by utilizing the limited *a priori* information of the object signal. In particular, to detect a specific object signal component from a mixed signal, we employ the reduced interference distribution (RID), and singular value decomposition (SVD) for the outer product expansion of the sampled time-frequency distribution. More specifically, the RID reduces, in the time-frequency distribution, the effect of the interference between multi-components in a given signal, and the SVD enables one to express the characteristics of signals in terms of the optimum outer product expansion in the framework of finite-dimensional vector space.

This paper is organized as follows: In Sec. 2, the detection problem is re-formulated in terms of time-frequency analysis, and an optimal time-frequency filter design is discussed. In Sec. 3, the derived time-frequency filtering approach has been tested by detecting a specific sound component from a real-world musical signal. Finally, the conclusion of the paper is given in Sec.4.

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2. DETECTION PROBLEM SETTING VIA TIME-FREQUENCY ANALYSIS

Let $x_1(t)$ and $x_0(t)$ denote an object non-stationary signal to be detected and non-stationary noise, respectively. We want to detect the event time of the specific signal component $x_1(t)$. If the observation signal $z(t)$ with a short time interval $t_1 \leq t \leq t_2$ is measured, the detection problem can be re-formulated in terms of the following time-frequency distributions:

$$\begin{aligned} H_0 : C_z(t, \omega; \phi) &= C_{x_0}(t, \omega; \phi) \\ H_1 : C_z(t, \omega; \phi) &= C_{x_0+x_1}(t, \omega; \phi) \end{aligned} \quad (2)$$

where $C_x(t, \omega; \phi)$ is a Cohen's class time-frequency distribution of $x(t)$ with kernel $\phi(\theta, \tau)$ [6]. Note that $C_{x_0+x_1}(t, \omega; \phi) \neq C_{x_0}(t, \omega; \phi) + C_{x_1}(t, \omega; \phi)$. In particular, it is assumed that a limited time-frequency *a priori* information on the object signal $x_1(t)$ is available in terms of the frequency bandwidth B_{x_1} and the time duration T_{x_1}. Also, let the time support of the object signal part be confined within the interval less than T_{x_1}.

Let $G(t, \omega; \phi)$ be a time-frequency filter associated with $g(t)$, and $y(t)$ the filtered output given by the time-domain convolution: i.e., $y(t) = z(t) \otimes g(t)$. Also, its time-frequency distribution can be expressed in terms of $C_z(t, \omega; \phi)$ and $C_h(t, \omega; \phi)$ [7]:

$$\begin{aligned} C_y(t, \omega; \phi) &= C_z(t, \omega; \phi) \otimes_t C_g(t, \omega; \phi) \\ &= \int C_z(\alpha, \omega; \phi) C_g(t - \alpha, \omega; \phi) d\alpha \end{aligned} \quad (3)$$

Now, consider the decision statistics [3] defined by

$$\Lambda(g;\phi|H_i) = \int\int C_y(t,\omega;\phi|H_i)dtd\omega \quad (4)$$

As the performance index for the detection, the following SNR is defined [1] as follows:

$$\text{SNR}(g;\phi) = \frac{|\text{E}\{\Lambda(g;\phi|H_1) - \text{E}\{\Lambda(g;\phi|H_0)\}|}{[\text{var}\{\Lambda(g;\phi|H_0)\}]^{1/2}} \quad (5)$$

Then, by maximizing (5), we can derive the optimal time-frequency filter $C_g(t,\omega)$. Within a short time interval ($< T_{x_1}$), it can be assumed that

$$\text{var}\{\Lambda(g;\phi|H_0)\} = \gamma_0^2 \quad (6)$$

From (4)and (5), the numerator part of (5) can be expressed by

$$\begin{aligned} & |\text{E}\{\Lambda(g;\phi|H_1) - \text{E}\{\Lambda(g;\phi|H_0)\}| \\ = \; & |\text{E}\{\int\int[C_y(t,\omega|H_1) - C_y(t,\omega|H_0))]dtd\omega\}| \\ = \; & |\int\int[\int \text{E}\{C_{x_0+x_1}(\alpha,\omega;\phi) - C_{x_0}(\alpha,\omega;\phi)\} \\ & \cdot C_g(t-\alpha,\omega;\phi)d\alpha]dtd\omega| \end{aligned} \quad (7)$$

For a given time-frequency distribution function $C(t,\omega)$, the time-frequency distribution can be decomposed into the sum of the outer products as follows [8]:

$$C(t,\omega;\phi) = \sum_{n=1}^{\infty}\frac{1}{\sigma_n}u_n(t)v_n^*(\omega) \quad (8)$$

where σ_n denotes the singular value, and $u_n(t)v_n^*(\omega)$ corresponds to the outer product term obtained by the following equations:

$$u_n(t) = \sigma_n\int C(t,\omega)v_n(\omega)d\omega \quad (9)$$

$$v_n(\omega) = \sigma_n\int C^*(\omega,t)u_n(t)dt \quad (10)$$

Then, we get

$$\text{E}\{C_{x_0+x_1}(\alpha,\omega;\phi) - C_{x_0}(\alpha,\omega;\phi)\} = \sum_{i=1}^{\infty}\frac{1}{\sigma_{1i}}u_i(t)v_i^*(\omega) \quad (11)$$

Similarly, we can rewrite the time-frequency distribution of the time-varying filter, $C_g(t,\omega)$ as

$$C_g(t,\omega) = \sum_{j=1}^{\infty}\frac{1}{\sigma_{2j}}u_j(t)v_j^*(\omega) \quad (12)$$

Then, the optimal solution of $g_{opt}(t,\omega;\phi)$ from (7) can be found :

$$\begin{aligned} g_{opt}(t,\omega;\phi) &= \arg_g\max\{|\int\int[\int(\sum_{i=1}^{\infty}\frac{1}{\sigma_{1i}}u_i(t)v_i^*(\omega))\cdot \\ & (\sum_{j=1}^{\infty}\frac{1}{\sigma_{2j}}u_j^*(t-\alpha)v_j(\omega))d\alpha]dtd\omega|\} \\ &= \arg_g\max\{|\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}(\int\int u_i(t)u_j^*(t-\alpha)d\alpha dt) \\ & \frac{1}{\sigma_{1i}}\cdot\frac{1}{\sigma_{2j}}\cdot(\int v_i(\omega)v_j^*(\omega)d\omega)^*|\} \end{aligned} \quad (13)$$

From the unitary properties of $u_n(t)$ and $v_n(\omega)$, we can see that (13) can be maximized only when $i = j$ and $\alpha = 0$. Therefore, the optimal detection problem can be solved by finding a proper set of σ_i, $u_i(t)$, and $v_i(\omega)$ that captures the unique feature (in time-frequency domain) of $x_1(t)$.

Using *a priori* information of $x_1(t)$, one can find an approximate solution: i.e., the sampled time-frequency distribution of a specific signal component with B_{x1} and T_{x1} can be located in the time-frequency plane. In particular, the interference terms due to the noise component can exist within such time-frequency bandwidth. When the sampled time-frequency distribution is decomposed using SVD, the dominant component takes a larger singular value σ_i. Furthermore, the energy fraction ε_i, is defined as the energy fraction of the decomposed signal $\frac{1}{\sigma_i}u_i(t)v_i^*(\omega)$ with respect to the total signal energy. In addition, the relatively low energy fraction ε_i indicates that the corresponding outer product term includes the interference effects between a object signal and a noise signal [8]. Thus, such cross-term effects can be effectively suppressed by choosing only the indices with dominant singular values and large energy fraction values. Thus, the time-frequency optimal detection filtering, $y_{TF}(t)$, can be achieved as follows:

$$y_{TF}(t) = \int\int C_z(\tau,\omega;\phi)\cdot C_g(\tau-t,\omega;\phi)d\omega d\tau \quad (14)$$

If the kernel satisfies $\phi(\theta,\tau) \neq 0, \forall\theta, \forall\tau$, the time-domain expression of the $C_g(t,\omega;\phi)$ can be obtained using the SVD and the inversion formula of the Cohen's class [6]. Also, the time-domain filtered output, $y_{time}(t)$, can be expressed by

$$y_{time}(t) = \int z(\tau)g(t-\tau)d\tau \quad (15)$$

Note that the optimal time-frequency filtering in (14) is expressed in the time-domain as in (15). That is, from the input-output relation of the proposed time-frequency filter, we can see that the classical 1-D matched-filter is a special case of the proposed time-frequency filter. Now, let's investigate in the next section how the conventional time-domain filtering and the proposed time-frequency filtering provide different detection results, where the theoretical results derived in this section will be applied to a real musical sound signal.

3. APPLICATION EXAMPLE

In this section, we demonstrate how the optimal time-frequency filter can be utilized for real signal detection. In the time series, a snare drum (object signal, $x_1(t)$) is mixed with voice and sound signals which are from other instruments (noise, $x_0(t)$). Using the optimal filter design schemes discussed in previous section, we want to detect the event time of the snare drum sound.

The musical signal and its corresponding time-frequency distribution, RID, of the musical signal are provided in Fig. 1, where the frequency bandwidth of the snare drum is estimated to be B =150 Hz, and its time duration is to be T =25 ms.

With the *a priori* time-frequency information on the snare drum sound component, the singular values (σ_i) and corresponding energy index (ε_i) for the sampled time-frequency distribution of the object non-stationary signal is provided in Fig. 2. Close observation of the singular values and energy fraction indices of the RID

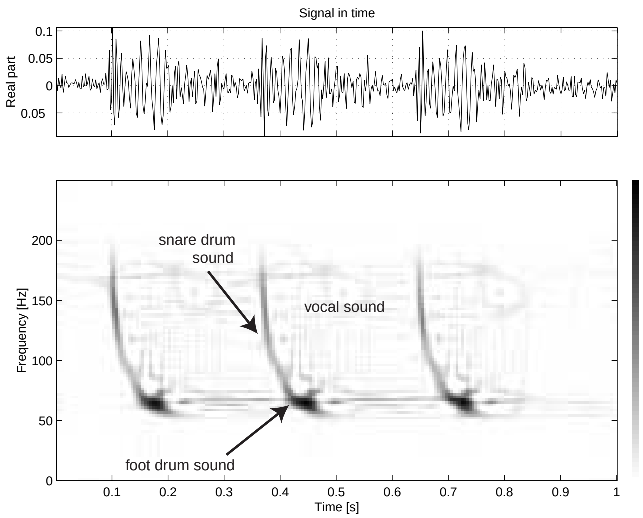

Fig. 1. A musical signal (top) and its binomial reduced interference distribution (bottom)

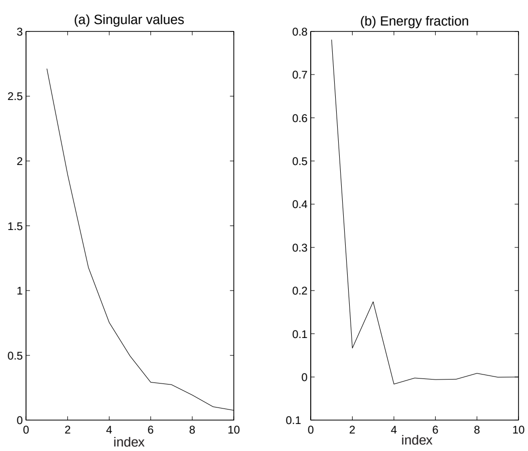

Fig. 2. Principal singular values (a) and the corresponding energy fraction (b) for snare drum sound components

distribution indicate that (i) the first index (i.e., here, index 1) is the most appropriate choice for the basis(or outer product term) because the singular and energy fraction values are large enough, (ii) while the singular value is relatively high in the case of the second index (i.e., index 2), the second energy fraction has a small value due to the interference between the object and noise signals, (iii) the third index(i.e., index 3) has a higher energy fraction than index 2, but it corresponds to a small singular value, and (iv) in the case of the fourth and higher indices, both their singular values and energy fractions are negligible. In particular, the optimal basis $\frac{1}{\sigma_1}u_1(t)v_1^*(\omega)$ and its time-domain filtering via (15) are provided in Fig. 5 and 3, respectively, where the plotted bases are normalized in time-domain so that its correlation is bounded between -1 and 1.

After convolving the measured composite signal with the time-domain detection filter in Fig. 3, the filtered output in the time-

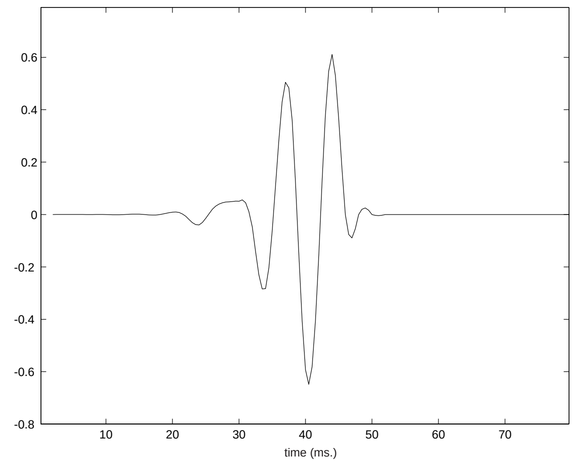

Fig. 3. The detection filter in time-domain

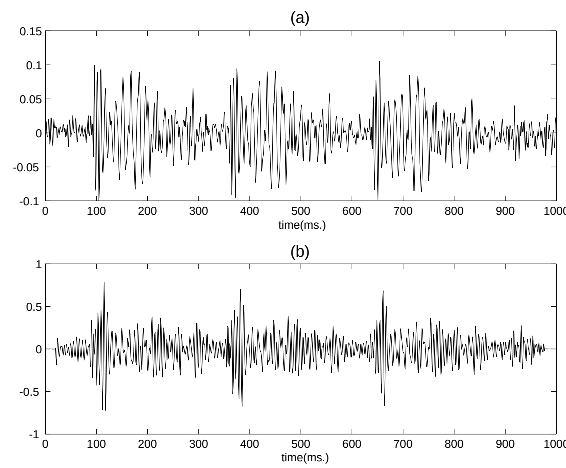

Fig. 4. Detection of the snare drum sound event using time-domain matched filtering: (a) a composite sound waveform and (b) the detected snare drum sound event

domain (i.e., via (15)), the detection of the event of the snare drum part, is displayed in Fig. 4. At the event time of the snare drum sound, the filtered output is close to 1, while the values at the other instant times are less than 0.5. From Fig.4 (b), the snare drum sound events are detected at 110ms., 380 ms., and 660 ms, respectively. However, the filtered output of the snare drum sound shows a high oscillation, and, also, the time-domain SNR is not particularly high. If the amplitude of the noise part increases, the detection may not provide an acceptable detection result.

Next consider the *time-frequency* domain filtering scheme. The time-frequency distribution with the selected singular value and its unitary vectors, $\frac{1}{\sigma_1}u_1(t)v_1^*(\omega)$, is provided in Fig. 5. The time-frequency representation indicates that the snare drum sound is transient with a small time duration, while the frequency bandwidth is approximately 150 Hz. Note that the time-frequency plot in Fig.5 corresponds to the optimal time-frequency domain filter $C_g(t, \omega; \phi)$. With the time-frequency domain filter, the 2-D time-frequency domain correlation can be obtained by applying (14), and the time-frequency domain filtering result is provided in Fig.6,

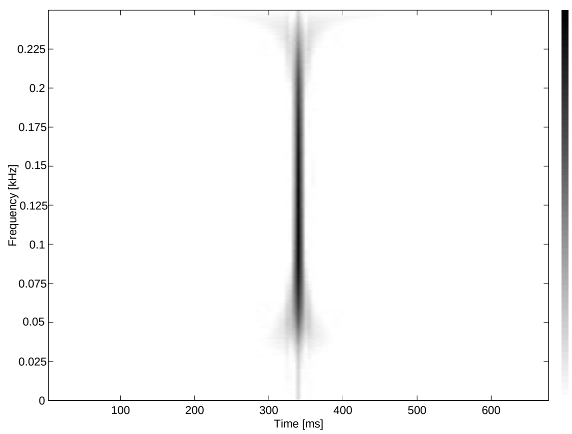

Fig. 5. The optimal detection filter in time-frequency domain

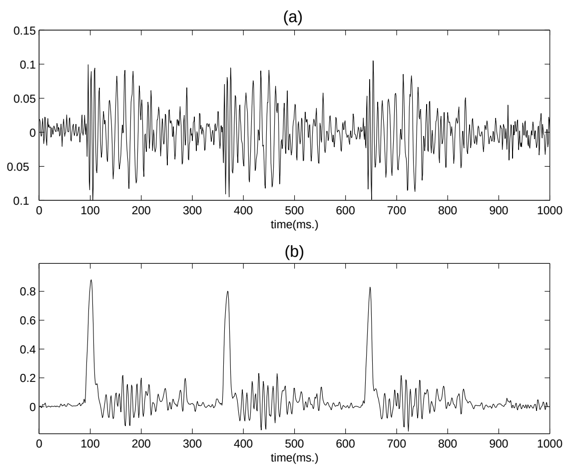

Fig. 6. Detection of the snare drum sound event using a time-frequency domain optimal filtering: (a) a composite sound waveform and (b) the detected snare drum sound event

where the filter output shows the detection of the snare drum sound event with high SNR. For each snare drum sound event, the filtered output value is over 0.8, while the filtered outputs of the noise parts are under 0.2, which implies that even if the amplitude of the noise increases, the time-frequency domain filtering leads to the robust detection results than the just time-domain filtering approach including the conventional matched filtering. Also, the time indices of the three major peaks in Fig. 6 correspond to the exact event times in Fig. 1.

When the time-domain filtering output in Fig.4 is compared with the time-frequency domain filtering output in Fig. 6, it is clear that the time-frequency domain filtering approach provides more desirable and reliable results for the detection of the snare drum sound events. This is due to the fact that the time-varying feature of the signal component can be utilized for the detection process by applying the time-frequency domain representation of the basis to the object signal, which yields the better detection result with higher SNR than the conventional time-domain filtering does.

4. CONCLUSION

In this paper, a practical and effective method is proposed to detect a transient signal component of interest from a composite signal waveform. The approach is based on an extension of the conventional 1-D (i.e., time-domain) matched filter approach to the 2-D (here, time-frequency domain) optimal filtering. In particular, the proposed approach, assuming *a priori* available information of the specific signal part of interest, is very effective in localizing a transient signal part or a non-stationary signal part from a measured composite signal waveform. Also, the proposed detection scheme provides a filtered output with higher SNR, when compared with the result obtained by the classical matched filter approach. Note that the detection filter design in the time-frequency domain needs to be further refined and generalized by exploiting the truncation criterion for the outer product expansion of the time-frequency distribution of a given composite signal.

5. REFERENCES

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