

ADAPTIVE PARAUNITARY FILTER BANKS FOR CONTRAST-BASED MULTICHANNEL BLIND DECONVOLUTION

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ABSTRACT

In this paper, we present novel algorithms for multichannel blind deconvolution under output whitening constraints. The algorithms are inspired by recently-developed procedures for gradient adaptive paraunitary filter banks. Several algorithms are developed, including one algorithm that successfully deconvolves mixtures of arbitrary non-zero kurtosis source signals. We provide detailed local stability analyses of the proposed methods to verify their capabilities. Simulations show that the methods successfully deconvolve spatio-temporal mixtures of statistically-independent source signals.

1. INTRODUCTION

In multichannel blind deconvolution, an m -dimensional vector sequence $\mathbf{s}(k)$ containing statistically-independent samples $s_i(k)$, $1 \leq i \leq m$ is mixed by an $(n \times m)$, $m \leq n$ unknown multichannel linear system with impulse response \mathbf{A}_i , $-\infty < i < \infty$, to produce the measured sequence

$$\mathbf{x}(k) = \sum_{i=-\infty}^{\infty} \mathbf{A}_i \mathbf{s}(k-i). \quad (1)$$

The goal is to process $\mathbf{x}(k)$ by an $(m \times n)$ multichannel adaptive linear system to produce estimates of the source sequences $\{s_i(k)\}$ in the adaptive system's outputs without precise knowledge of $\{s_i(k)\}$ or $\{\mathbf{A}_i\}$. Multichannel blind deconvolution is particularly useful in wireless communications employing smart antennas [1].

Although single-channel blind deconvolution is a well-studied topic [2], there exist relatively few successful multichannel blind deconvolution algorithms. Two simple multichannel blind deconvolution algorithms are described in [3] and [4], respectively. The former algorithm is an extension of the constant modulus algorithm (CMA) equalizer, and the latter algorithm is an extension of the natural gradient blind signal separation (BSS) algorithm in [5]. Both of these algorithms rely on knowledge of the probability density functions (p.d.f.'s) of each $s_i(k)$, and they fail to extract these signals if the chosen density models do not accurately match the p.d.f.'s of each $s_i(k)$. More recently, contrast-based criteria for BSS have been extended to the multichannel blind deconvolution task [6, 7]. These criteria identify separated and deconvolved sources regardless of their p.d.f.'s.

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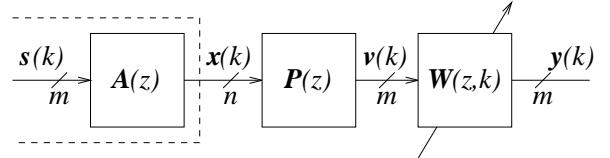


Fig. 1: Contrast-based multichannel blind deconvolution.

In this paper, we derive novel multichannel blind deconvolution algorithms based on the contrast functions in [6, 7]. Our methods are inspired by recent work on gradient adaptive paraunitary filter banks [8]. Unlike the approaches in [3, 4], ours do not require precise knowledge about the underlying source p.d.f.'s; rather, they only require knowledge of the number of positive-kurtosis and negative-kurtosis sources within the mixture. The algorithms are simple, requiring between four and seven multiply/adds per adaptive system coefficient at each time instant. We provide detailed local stability analyses of the proposed methods to verify their extraction capabilities. Simulations of the proposed method show their abilities to extract spatio-temporal mixtures of statistically-independent source signals.

2. STRUCTURES AND CRITERIA

Our proposed methods share the common system structure shown in Fig. 1. The $(m \times n)$ prewhitening filter, denoted by $\mathbf{P}(z)$, calculates the m -dimensional prewhitened signal

$$\mathbf{v}(k) = \sum_{i=0}^M \mathbf{P}_i \mathbf{x}(k-i). \quad (2)$$

where \mathbf{P}_i , $0 \leq i \leq M$ is this system's impulse response. The coefficients of this system are designed so that $\mathbf{v}(k)$ is approximately spatially- and temporally-uncorrelated, *i.e.*

$$E\{\mathbf{v}(k)\mathbf{v}^T(k-j)\} \approx \mathbf{I}\delta(j), \quad -M < j < M. \quad (3)$$

Any one of a number of procedures can be used to calculate \mathbf{P}_i , such as the multichannel Levinson algorithm or other adaptive approaches [9]. The $(m \times m)$ separation filter $\mathbf{W}(z, k)$ calculates the source signal estimates as

$$\mathbf{y}(k) = \sum_{l=0}^L \mathbf{W}_l(k) \mathbf{v}(k-l), \quad (4)$$

where $\mathbf{W}_l(k)$, $0 \leq l \leq L$ are the adaptive coefficients of the separation system. In this paper, we develop adaptive algorithms for adjusting $\{\mathbf{W}_l(k)\}$ to obtain separated and deconvolved source estimates in $\mathbf{y}(k)$.

Table 1: Contrast-Based Multichannel Blind Deconvolution Algorithm for Arbitrary Mixtures

Initialize: $\{w_{ijl}(0)\}$ paraunitary
for $k \geq 0$ do
$u_{0j}(k) = 0, z_{0j}(k) = 0, 1 \leq j \leq m$
for $i = 1$ to m do
$y_i(k) = \sum_{j=1}^m \sum_{l=0}^L w_{ijl}(k) v_j(k-l)$
$\epsilon_i(k) = y_i^3(k)$
for $j = 1$ to m do
$u_{ij}(k) = u_{(i-1)j}(k) + \sum_{q=0}^L w_{ij(L-q)}(k) \epsilon_i(k-q)$
end
if $i \leq \bar{m}$ do
$\psi_i(k) = \sum_{j=1}^{\bar{m}} \sum_{l=0}^L w_{ijl}(k) u_{ij}(k-l)$
end
end
for $i = 1$ to \bar{m} do
for $j = 1$ to \bar{m} do
$z_{ij}(k) = z_{(i-1)j}(k) + \sum_{q=0}^L w_{ij(L-q)}(k) \psi_i(k-q)$
end
$\zeta_i(k) = \sum_{j=1}^{\bar{m}} \sum_{l=0}^L w_{ijl}(k) z_{ij}(k-l)$
end
for $i = 1$ to m do
$\bar{\epsilon}_i(k) = \beta_i \epsilon_i(k), \bar{y}_i(k) = \beta_i y_i(k)$
for $j = 1$ to \bar{m} do
$w_{ijl}(k+1) = w_{ijl}(k) + \bar{\epsilon}_i(k-L) v_j(k-L-l) - \bar{y}_i(k-L) u_{ij}(k-l)$
end
for $j = (\bar{m}+1)$ to m do
$w_{ijl}(k+1) = w_{ijl}(k) + \bar{\zeta}_i(k) v_j(k-2L-l) - \bar{y}_i(k-2L) u_{ij}(k-L-l)$
end
end

In [6, 7] a contrast function for the multichannel blind deconvolution task is proposed. A *contrast function* is a cost function that depends on the source signal estimates whose extrema over the separation system parameters extract all of the source signals [10]. This formulation assumes that (i) the sources are spatially- and temporally-uncorrelated, such that $E\{\mathbf{s}(k)\mathbf{s}^T(k-l)\} \approx \mathbf{D}\delta(l), -\infty < l < \infty$ for an arbitrary nonsingular diagonal scaling matrix \mathbf{D} with diagonal entries d_{ii} , and (ii) each source is spatially- fourth-order uncorrelated, such that $E\{s_i(k)s_j(k)s_l(k)s_p(k)\} = \kappa[s_i(k)]\delta_{ijlp} + d_{ii}d_{jj}\delta_{ij}\delta_{lp} + d_{ii}d_{jj}[\delta_{ijlp} + \delta_{ip}\delta_{jl}]$.

Under these assumptions, the following procedure solves the multichannel blind deconvolution task:

$$\text{maximize } \mathcal{J}(\{\mathbf{W}_i\}) = \sum_{i=1}^m |\kappa[y_i(k)]| \quad (5)$$

$$\text{such that } E\{\mathbf{y}(k)\mathbf{y}^T(k-l)\} \approx \mathbf{I}\delta(l), -\infty < l < \infty. \quad (6)$$

where $\mathcal{J}(\{\mathbf{W}_i\})$ is the contrast function and $\kappa[y_i(k)] = E\{|y_i(k)|^4\} - 3E^2\{|y_i(k)|^2\}$ is the *kurtosis* of $y_i(k)$. As-

sume for the current discussion that all $\{s_i(k)\}$ have the same kurtosis sign, such that $\kappa[s_i(k)] > 0$ or $\kappa[s_i(k)] < 0$ for all $1 \leq i \leq m$. Then, an equivalent formulation to (5)–(6) is

$$\text{maximize } \widehat{\mathcal{J}}(\{\mathbf{W}_i\}) = \frac{\beta}{4} \sum_{i=1}^m E\{|y_i(k)|^4\} \quad (7)$$

$$\text{such that } \sum_{l=0}^L \mathbf{W}_i(k) \mathbf{W}_{i+l}^T(k) \approx \mathbf{I}\delta(i), \quad (8)$$

where β satisfies $\beta\kappa[s_i(k)] > 0$ for all $1 \leq i \leq m$.

3. ALGORITHM DERIVATION

Multichannel linear systems whose impulse responses obey (8) are called *paraunitary filter banks*. In [8], the following differential update is proposed to adapt the coefficients of a paraunitary filter bank to maximize $\widehat{\mathcal{J}}(\{\mathbf{W}_i\})$:

$$\frac{d\mathbf{W}_i}{dt} = \mathbf{W}_i * \mathbf{W}_{-l}^T * \mathbf{G}_l - \mathbf{W}_i * \mathbf{G}_{-l}^T * \mathbf{W}_l, \quad (9)$$

where $\mathbf{G}_l = \partial\widehat{\mathcal{J}}(\{\mathbf{W}_i\})/\partial\mathbf{W}_l$ and “*” denotes discrete-time convolution of matrix sequences. Eqn. (9) maintains (8) for a doubly-infinite multichannel IIR system. Modifications are required, however, to make the resulting system both causal and numerically-stable [11].

Applying (9) to (7)–(8), discretizing the resulting updates, and assuming slow adaptation of the system’s coefficients results in the first proposed stochastic gradient algorithm for the multichannel blind deconvolution task:

$$\mathbf{W}_i(k+1) = \mathbf{W}_i(k) + \mathbf{D}_\beta [\psi(k)\mathbf{v}^T(k-L-l) - \mathbf{y}(k-L)\mathbf{u}^T(k-l)] \quad (10)$$

$$\mathbf{u}(k) = \sum_{q=0}^L \mathbf{W}_{L-q}^T(k) \mathbf{f}(\mathbf{y}(k-q)) \quad (11)$$

$$\psi(k) = \sum_{l=0}^L \mathbf{W}_l(k) \mathbf{u}(k-l), \quad (12)$$

where \mathbf{D}_β is a diagonal matrix of step sizes $\{\beta_i\}$. This algorithm is simple, requiring $5m^2(L+1) + 3m$ multiply/adds per adaptive filter coefficient. Unfortunately, this algorithm fails to maintain the paraunitary constraint in (8) over time due to numerical effects. Similar difficulties have appeared in algorithms for minor subspace analysis and contrast-based BSS [12, 13], and they can often be addressed by modifying the updates to allow numerically-stable behavior. To this end, we propose the following modified algorithms for all $\beta_i > 0$ and $\beta_i < 0$, respectively:

$$\mathbf{W}_i(k+1) = \mathbf{W}_i(k) + \mathbf{D}_\beta [\mathbf{f}(\mathbf{y}(k-L))\mathbf{v}^T(k-L-l) - \mathbf{y}(k-L)\mathbf{u}^T(k-l)] \quad (13)$$

$$\text{and } \mathbf{W}_i(k+1) = \mathbf{W}_i(k) + \mathbf{D}_\beta [\zeta(k)\mathbf{v}^T(k-2L-l) - \mathbf{y}(k-2L)\mathbf{u}^T(k-L-l)] \quad (14)$$

$$\mathbf{z}(k) = \sum_{q=0}^L \mathbf{W}_{L-q}^T(k) \psi(k-q) \quad (15)$$

$$\zeta(k) = \sum_{l=0}^L \mathbf{W}_l(k) \mathbf{z}(k-l). \quad (16)$$

Table 2: Local stability analysis results for the multichannel blind deconvolution algorithms.

Algorithm	$\mathbf{H}_{ijl}, i < j$			$\mathbf{H}_{ill}, l \neq 0$			h_{iil}
Eqn. (10)	-			$-\beta_i \kappa_i \quad -\beta_i \kappa_j \quad \beta_j \kappa_j$			0
Eqn. (13)	-			$\beta_i(\kappa_i + \kappa_j + 3) \quad 3\beta_i \quad 3\beta_j$			$-2\beta_i(\kappa_i + 3)$
Eqn. (14)	-			$\beta_i(\kappa_i - \kappa_j - 3) \quad -\beta_i(2\kappa_j + 3) \quad \beta_j(\kappa_j - \kappa_i - 3)$			$2\beta_i(\kappa_i + 3)$
Eqn. (17)	-			$\beta_i \kappa_i \quad 0 \quad \beta_j(\kappa_j + \kappa_i + 3)$			$-2\beta_i(\kappa_i + 3)$
Eqn. (18)	-			$\beta_i \kappa_i \quad 0 \quad -\beta_j(2\kappa_i + 3) \quad \beta_j(\kappa_j - \kappa_i - 3)$			$2\beta_i(\kappa_i + 3)$

These algorithms require $4m^2(L + 1) + 3m$ and $7m^2(L + 1) + 3m$ multiply/adds per iteration, respectively.

While useful for separating mixtures of positive- or negative-kurtosis sources, respectively, the updates in (13) and (14) cannot separate mixtures containing both positive- and negative-kurtosis sources. We now propose modified algorithms that are the spatio-temporal extensions of those in [14]. These algorithms can be approximately described using the convolution operator “*” over the time index l as

$$\mathbf{W}_l(k+1) = \mathbf{W}_l(k) + \mathbf{D}_\beta[\mathbf{f}(\mathbf{y}(k-L))\mathbf{v}^T(k-L-l) - \text{tri}[\mathbf{y}(k-L)\mathbf{f}^T(\mathbf{y}(k-L-l)) * \mathbf{W}_l(k)] \quad (17)$$

for all $\beta_i > 0$ and

$$\mathbf{W}_l(k+1) = \mathbf{W}_l(k) + \mathbf{D}_\beta[\bar{\zeta}(k)\mathbf{v}^T(k-2L-l) - \text{tri}[\mathbf{y}(k-2L)\mathbf{f}^T(\mathbf{y}(k-2L-l)) * \mathbf{W}_l(k)] \quad (18)$$

$$\bar{\zeta}(k-l) = \text{tri}[\mathbf{W}_l(k) * \mathbf{W}_{-l}^T(k)] * \text{tri}[\mathbf{W}_l(k) * \mathbf{W}_{-l}^T(k)] * \mathbf{f}(\mathbf{y}(k-2L-l)) \quad (19)$$

for all $\beta_i < 0$, respectively, in which

$$\text{tri}[\mathbf{F}] = \begin{cases} f_{ij} & \text{if } i \geq j \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

for an $(m \times m)$ matrix \mathbf{F} with entries $\{f_{ij}\}$. Assuming slow adaptation, these algorithms can be combined and simplified to produce the algorithm listed in Table 1, where choosing $\bar{m} = 0$ and $\bar{m} = m$ yields updates identical to (17) and (18), respectively, as $|\beta_i| \rightarrow 0$. These algorithms have the same computational complexities as (13) and (14). In addition, when \bar{m} corresponds to the number of negative-kurtosis sources in $\mathbf{s}(k)$, the combined algorithm in Table 1 can potentially separate arbitrary source mixtures, as will be shown.

4. STABILITY ANALYSES

We now provide stability analyses of (10), (13), (14), (17), and (18). These analyses determine the constraints on the step size parameters $\{\beta_i\}$ to guarantee stability of the algorithms about separating and deconvolving solutions. The procedure used is the ordinary differential equation (ODE) method, in which the dynamics of the linearized averaged ODE of the coefficient updates is elucidated. For a similar analysis of orthonormal-update BSS algorithms, see [13]. Our analyses use a simplified notation whereby time indices are suppressed, such that $\mathbf{y}(k+p) = \mathbf{y}_p$. Our analyses also ignore truncation of the prewhitening and deconvolution filters. Without loss of generality, we describe the evolutionary behaviors of the algorithms near a separating solution

$$\mathbf{C}_l = \mathbf{W}_l * \mathbf{P}_l * \mathbf{A}_l = \mathbf{I}\delta(l) + \Delta_l, \quad (21)$$

where Δ_l are $(m \times m)$ matrices with $\sum_{l=-\infty}^{\infty} \|\Delta_l\|_F^2 \ll m$.

For each algorithm, the first step of the analysis is to find the averaged ODE of the coefficient updates in the combined impulse response \mathbf{C}_l . For (10), the ODE is

$$\frac{d\mathbf{C}_l}{dt} = \mathbf{D}_\beta \sum_{p,q} \mathbf{C}_p E\{\mathbf{C}_{p+q}^T \mathbf{f}(\mathbf{y}_{-q}) \mathbf{s}_{-l}^T - \mathbf{s}_{-p} \mathbf{f}^T(\mathbf{y}_{-q}) \mathbf{C}_{l-q}\} \quad (22)$$

The averaged ODE is then linearized about a separating solution satisfying (21), yielding an averaged ODE for Δ_l . The statistical properties of the sources are then used to simplify the relations. In every case, we can express these updates in terms of $\{\Delta_{ijl}\}$, the (i, j) th entries of all Δ_l , as

$$\frac{d}{dt} \begin{bmatrix} \Delta_{ijl} \\ \Delta_{ji(-l)} \end{bmatrix} = \mathbf{H}_{ijl} \begin{bmatrix} \Delta_{ijl} \\ \Delta_{ji(-l)} \end{bmatrix}, \quad i < j, \forall l \quad (23)$$

$$\frac{d}{dt} \begin{bmatrix} \Delta_{ill} \\ \Delta_{ii(-l)} \end{bmatrix} = \mathbf{H}_{ill} \begin{bmatrix} \Delta_{ill} \\ \Delta_{ii(-l)} \end{bmatrix}, \quad \forall l \neq 0 \quad (24)$$

$$\frac{d}{dt} \Delta_{iil} = h_{iil} \Delta_{iil}, \quad (25)$$

where the \mathbf{H}_{ijl} , \mathbf{H}_{ill} , and h_{iil} depend on β_i , β_j , and the statistics of $s_i(k)$ and $s_j(k)$. Shown in Table 2 are the expressions for these quantities for the five proposed algorithms; detailed derivations for each case are omitted for brevity. Because (23)–(25) are linear ODEs, local stability is guaranteed if \mathbf{H}_{ijl} , \mathbf{H}_{ill} , and h_{iil} have negative eigenvalues, producing constraints on the values of $\{\beta_i\}$ and $\{\kappa_i\}$.

From these results, a number of conclusions can be drawn:

- Because $h_{iil} = 0$ for the analysis of (10), this algorithm fails to maintain a paraunitary filter impulse response in the multichannel blind deconvolution task.
- Eqns. (13) and (17) are locally stable for positive-kurtosis source mixtures ($\kappa_i > 0$) if $\beta_i > 0$ for all i . Similarly, Eqns. (14) and (18) are locally stable for negative-kurtosis source mixtures ($\kappa_i < 0$) if $\beta_i < 0$ for all i .
- Due to the lower-triangular nature of \mathbf{H}_{ijl} for (17) and (18), these algorithms can be combined, such that the algorithm in Table 1 is locally stable if $\kappa_i < 0$ for $1 \leq i \leq \bar{m}$, $\kappa_j > 0$ for $\bar{m} < j \leq m$, and $\text{sgn}[\beta_i] = \text{sgn}[\kappa_i]$ for $1 \leq i \leq m$. This algorithm can successfully deconvolve arbitrary source mixtures, so long as the numbers of negative- and positive-kurtosis sources are known.

5. SIMULATIONS

We now explore the behavior of the proposed multichannel blind deconvolution algorithms via simulations. In these simulations, we have generated $\mathbf{v}(k)$ directly from $\mathbf{s}(k)$ using a paraunitary filter of the form

$$v_i(k) = \alpha_i v_i(k-1) + \sum_{j=1}^m q_{ij} [\alpha_i s_i(k) - s_i(k-1)], \quad (26)$$

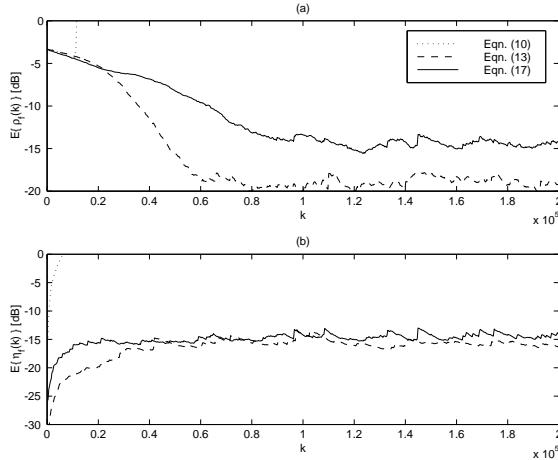


Fig. 2: Evolutions of (a) $E\{\rho_f(k)\}$ and (b) $E\{\eta_f(k)\}$ for the multichannel blind deconvolution algorithms in the first simulation example.

where $m = 4$, $\{\alpha_1 \alpha_2 \alpha_3 \alpha_4\} = \{-0.4 \ 0.5 \ 0.2 \ 0.3\}$, and q_{ij} are elements from a different random orthonormal matrix for each simulation run. Thus, no prewhitening is necessary. For the combined system in (21), the performance factors

$$\rho_f(k) = \frac{N+1}{m(N+1)-1} \left[m - \frac{1}{2} \sum_{i=1}^m \left(\frac{\max_{1 \leq j \leq m, 0 \leq l \leq N} |c_{ijl}(k)|^2}{\sum_{j=1}^m \sum_{l=0}^N |c_{ijl}(k)|^2} \right. \right. \\ \left. \left. + \frac{\max_{1 \leq j \leq m, 0 \leq l \leq N} |c_{jil}(k)|^2}{\sum_{j=1}^m \sum_{l=0}^N |c_{jil}(k)|^2} \right) \right] \quad \text{and} \quad (27)$$

$$\eta_f(k) = \frac{\sum_{l=-L}^L \|\sum_{i=-L}^L \mathbf{W}_i(k) \mathbf{W}_{i+l}^T(k) - \mathbf{I}\delta(l)\|_F^2}{\sum_{l=-L}^L \|\sum_{i=-L}^L \mathbf{W}_i(k) \mathbf{W}_{i+l}^T(k)\|_F^2} \quad (28)$$

were computed for each algorithm using ten simulation runs, where $L = 32$, $N = 3L$, and $w_{ijl}(0) = \delta_{ij}\delta(l - L/2)$.

Figs. 2(a) and (b) show the average values of $\rho_f(k)$ and $\eta_f(k)$ for Eqns. (10), (13), and (17) for common step sizes of 0.000012, 0.000012, and 0.00002, respectively, in which each $s_i(k)$ is approximately Laplacian-distributed ($\kappa_i = 3$). As can be seen, (10) quickly diverges due to its inability to maintain a paraunitary impulse response. Both (13) and (17) successfully separate and deconvolve the source signals, and (13) outperforms (17) due to its use of additional constraints within the updates.

Figs. 3(a) and (b) show the performance of the algorithm in Table 1 in separating and deconvolving mixtures of two Laplacian ($\kappa_i > 0$) and two binary ($\kappa_i < 0$) source signals. In this case, we have chosen $\beta_i = -0.00002$ for $i \in \{1, 2\}$ and $\beta_i = 0.00002$ for $i \in \{3, 4\}$. As can be seen, this algorithm separates and deconvolves the signal mixture without precise knowledge of and without estimating the characteristics of the sources.

6. CONCLUSIONS

In this paper, we propose novel adaptive paraunitary fil-

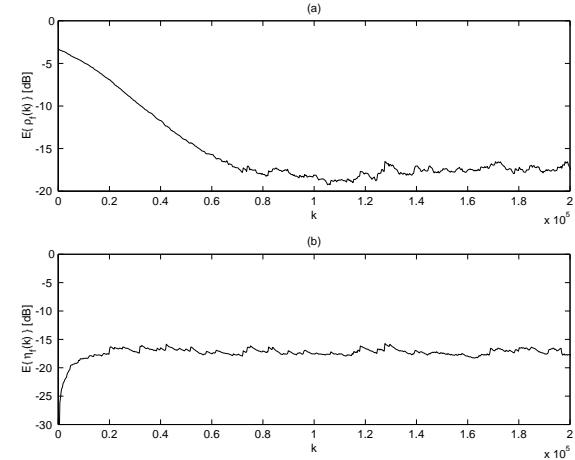


Fig. 3: Evolutions of (a) $E\{\rho_f(k)\}$ and (b) $E\{\eta_f(k)\}$ for the algorithm in Table 1 in the second simulation example.

ter bank algorithms for contrast-based multichannel blind deconvolution. One of these algorithms deconvolves spatio-temporal mixtures of arbitrary non-zero-kurtosis sources without precise knowledge of the sources' statistics. Several algorithm forms are proposed and their local stability properties analyzed. These results, along with simulations, verify the capabilities of the self-stabilized methods. Characterizations of the steady-state MSEs of the proposed algorithms are underway.

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