

# SEMI-BLIND SPACE-TIME CHIP EQUALIZER RECEIVERS FOR WCDMA FORWARD LINK WITH CODE-MULTIPLEXED PILOT

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## ABSTRACT

In the forward link of WCDMA systems, the multipath propagation channel destroys the orthogonality of the spreading codes and therefore causes multi-user interference (MUI). In this paper, we propose new training-based and semi-blind space-time chip equalizer receivers for the forward link of WCDMA systems with a continuous code-multiplexed pilot. Both Least-Squares (LS) algorithms for block processing and Recursive Least-Squares (RLS) algorithms for adaptive processing are derived. The proposed receivers can track fast fading multipath channels and outperform the RAKE receiver with perfect channel knowledge.

## 1. INTRODUCTION

In the forward link of WCDMA systems, the different users are multiplexed synchronously to the transmission channel by using orthogonal spreading codes. Multipath propagation destroys however the orthogonality between the different user signals and causes multi-user interference (MUI). Channel equalization can restore the orthogonality and therefore suppress the MUI [1]. Non-adaptive ZF and MMSE chip equalizer receivers based on blind channel identification have been introduced in [2]. A direct blind chip equalizer can be derived by making use of the fact that, in the absence of noise, the received signal after equalization should lie in the subspace spanned by the user codes. This was done for symbol by symbol processing in [3] and for block processing in [4]. Reduced-rank adaptive chip equalizers based on the Multi-Stage Nested Wiener Filter (MSNWF) have been discussed in [5].

In this paper, we propose new training-based and semi-blind space-time chip equalizer receivers for the forward link of WCDMA systems with a continuous code-multiplexed pilot. Whereas the training-based receiver only assumes knowledge of the pilot symbols, the pilot code and the desired user code, the semi-blind receiver additionally assumes knowledge of the other active user codes. For both receivers we derive a Least Squares (LS) algorithm for block processing and a Recursive Least Squares (RLS) algorithm for adaptive processing.

The rest of the paper is organised as follows. Section 2 lays out the WCDMA forward link data model. Section 3 develops the semi-blind chip equalizer receiver while section 4 presents the training-based chip equalizer receiver. Section 5 discusses the simulation results while section 6 summarizes the conclusions.

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## 2. WCDMA FORWARD LINK DATA MODEL

### 2.1. Multi-channel framework

Let us consider the forward link of a single-cell WCDMA system with  $K$  active user terminals. The base-station transmits a synchronous code division multiplex, employing user specific orthogonal Walsh-Hadamard spreading codes and a base-station specific aperiodic scrambling code. The transmitted multi-user chip sequence consists of  $K$  user signals and a continuous pilot signal:

$$x[n] = \sum_{k=1}^K s_k[i] c_k[n] + s_p[i] c_p[n] \quad (1)$$

with  $i = \lfloor \frac{n}{N} \rfloor$ . Each user's data symbol sequence  $s_k[i]$  (pilot symbol sequence  $s_p[i]$ ) is spread by a factor  $N$  with the user code sequence  $c_k[n]$  (pilot code sequence  $c_p[n]$ ). The  $k$ -th user aperiodic code sequence  $c_k[n]$  (pilot code sequence  $c_p[n]$ ) is the concatenation of the corresponding Walsh-Hadamard spreading code and the base-station specific aperiodic scrambling code.

Assume that the user terminal is equipped with  $M$  receive antennas and let  $h_m(t)$  denote the continuous-time channel from the base-station to the  $m$ -th receive antenna, including the transmit and receive filters. By sampling the different received antenna signals at the chiprate  $\frac{N}{T}$ , we obtain the following received vector sequence :

$$\mathbf{y}[n] = [ y_1[n] \quad y_2[n] \quad \dots \quad y_M[n] ]^T$$

which can be written as :

$$\mathbf{y}[n] = \sum_{n'=0}^L \mathbf{h}[n'] x[n-n'] + \mathbf{e}[n] \quad (2)$$

where  $\mathbf{e}[n]$  is similarly defined as  $\mathbf{y}[n]$  and  $\mathbf{h}[n]$  is the discrete-time  $M \times 1$  vector channel from the base-station to the  $M$  receive antennas, given by :

$$\mathbf{h}[n] = [ h_1[n] \quad h_2[n] \quad \dots \quad h_M[n] ]^T$$

In this equation  $h_m[n] = h_m(n \frac{N}{T})$  is the discrete-time channel impulse response from the base-station to the  $m$ -th receive antenna. Note that we model  $\mathbf{h}[n]$  as an  $M \times 1$  FIR vector filter of order  $L$ .

### 2.2. Data model for block processing

Let us now introduce the  $(Q+1)M \times BN$  output matrix  $\mathbf{Y}_a$  (with Hankel structure), shown in equation 3, where  $B$  is the block

$$\mathbf{Y}_a = \begin{bmatrix} \mathbf{y}[a] & \mathbf{y}[a+1] & \dots & \mathbf{y}[a+BN-1] \\ \vdots & \vdots & & \vdots \\ \mathbf{y}[a+Q] & \mathbf{y}[a+Q+1] & \dots & \mathbf{y}[a+Q+BN-1] \end{bmatrix} \quad (3)$$

length,  $a$  is the processing delay and  $Q+1$  is the temporal smoothing factor. This output matrix can be written as :

$$\mathbf{Y}_a = \mathcal{H}\mathbf{X}_a + \mathbf{E}_a \quad (4)$$

where the noise matrix  $\mathbf{E}_a$  is similarly defined as  $\mathbf{Y}_a$  and  $\mathcal{H}$  is the  $(Q+1) \times r$  ( $r = L+Q+1$ ) channel matrix (with Toeplitz structure). The  $r \times BN$  input matrix  $\mathbf{X}_a$  (with Hankel structure) is given by :

$$\mathbf{X}_a = [ \mathbf{x}_{a-L}^T \dots \mathbf{x}_{a+Q}^T ]^T$$

with the transmitted multi-user chip sequence vector at delay  $a$  :

$$\mathbf{x}_a = [ x[a] \ x[a+1] \ \dots \ x[a+BN-1] ]$$

### 3. SEMI-BLIND CHIP EQUALIZER RECEIVER

In this section, we discuss a new semi-blind chip equalizer receiver, that exploits all code information on one hand and pilot symbol information on the other hand. This receiver is based on the fully blind chip equalizer receiver for the reverse link of WCDMA systems, presented in the second part of [6]. One algorithm for block processing and one for adaptive processing is derived.

#### 3.1. Block processing

The block processing algorithm for the semi-blind chip equalizer detects  $B$  data symbols at once. We can write the transmitted multi-user chip sequence at delay  $a = 0$  as follows :

$$\mathbf{x}_0 = \mathbf{s}_d \mathbf{C}_d + \mathbf{s}_p \mathbf{C}_p \quad (5)$$

where  $\mathbf{s}_d$  is the  $1 \times KB$  total transmitted data symbol vector :

$$\mathbf{s}_d = [ \mathbf{s}_1 \ \dots \ \mathbf{s}_K ]$$

and  $\mathbf{s}_k$  is the  $k$ -th user's  $1 \times B$  transmitted data symbol vector :

$$\mathbf{s}_k = [ s_k[0] \ s_k[1] \ \dots \ s_k[B-1] ]$$

The transmitted pilot symbol vector  $\mathbf{s}_p$  is similarly defined as  $\mathbf{s}_k$ . The  $KB \times BN$  user code sequence matrix  $\mathbf{C}_d$  stacks the code sequence matrices of the individual users :

$$\mathbf{C}_d = [ \mathbf{C}_1^T \ \dots \ \mathbf{C}_K^T ]^T$$

where  $\mathbf{C}_k$  is the  $k$ -th user's  $B \times BN$  code sequence matrix :

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{c}_k[0] \\ \ddots \\ \mathbf{c}_k[B-1] \end{bmatrix}$$

and  $\mathbf{c}_k[i]$  is the  $k$ -th user's code sequence vector used to spread the data symbol  $s_k[i]$  :

$$\mathbf{c}_k[i] = [ c_k[iN] \ \dots \ c_k[iN+N-1] ]$$

The  $B \times BN$  pilot code sequence matrix  $\mathbf{C}_p$  and the pilot code sequence vector  $\mathbf{c}_p[i]$  are similarly defined as  $\mathbf{C}_k$  respectively  $\mathbf{c}_k[i]$ .

The vector  $\mathbf{x}_0$  is a row of every input matrix from the set  $\{\mathbf{X}_a\}_{a=-Q}^L$  and is therefore 'contained' in every output matrix from the set  $\{\mathbf{Y}_a\}_{a=-Q}^L$ . The semi-blind block processing problem addressed here is to compute the desired user's data symbol sequence  $\mathbf{s}_1$  (we assume the first user to be the user of interest) from  $\mathbf{Y}_a$ , with  $-Q \leq a \leq L$ , based on the knowledge of the user code sequence matrix  $\mathbf{C}_d$ , the pilot code sequence matrix  $\mathbf{C}_p$  and the pilot symbol vector  $\mathbf{s}_p$ . In order to solve this problem we make the following rather standard assumptions :

**Assumption 1** *The channel matrix  $\mathcal{H}$  has full column rank  $r$ .*

**Assumption 2** *The input matrix  $\mathbf{X}_a$  has full row rank  $r$ .*

The first assumption requires that :

$$(Q+1)(M-1) \geq L$$

Therefore the number of antennas should be at least  $M = 2$ . The second assumption, on the other hand, requires that :

$$BN \geq r$$

Let us first, for the sake of clarity, assume there is no additive noise present in  $\mathbf{Y}_a$  ( $-Q \leq a \leq L$ ). Because of assumptions 1 and 2, the rows of  $\mathbf{Y}_a$  span the row space of  $\mathbf{X}_a$ . Hence, there exists a  $1 \times (Q+1)M$  linear chip equalizer  $\mathbf{f}_a$ , for which :

$$\mathbf{f}_a \mathbf{Y}_a - \mathbf{x}_0 = \mathbf{0}$$

and this linear chip equalizer  $\mathbf{f}_a$  is a ZF linear chip equalizer with  $(Q+1)M-r$  degrees of freedom (hence, this linear chip equalizer is only unique when  $(Q+1)M = r$ ). Using Equation 5 we can then write :

$$\mathbf{f}_a \mathbf{Y}_a - \mathbf{s}_d \mathbf{C}_d - \mathbf{s}_p \mathbf{C}_p = \mathbf{0} \quad (6)$$

In order to guarantee the uniqueness of the solution for  $\mathbf{f}_a, \mathbf{s}_d$  up to a complex scaling factor, the matrix  $[ \mathbf{X}_a^T \ -\mathbf{C}_d^T \ -(s_p \mathbf{C}_p)^T ]^T$  should have at most a one-dimensional left null space. This leads to the following identifiability condition :

$$B(N-K) \geq r \quad (7)$$

Therefore the maximum number of users that can be supported is  $K = N-1$ .

Let us now assume that additive noise is present in  $\mathbf{Y}_a$  ( $-Q \leq a \leq L$ ). We then solve the following Least Squares (LS) minimisation problem :

$$\bar{\mathbf{f}}_a, \bar{\mathbf{s}}_d = \arg \min_{\mathbf{f}_a, \mathbf{s}_d} \| \mathbf{f}_a \mathbf{Y}_a - \mathbf{s}_d \mathbf{C}_d - \mathbf{s}_p \mathbf{C}_p \|^2 \quad (8)$$

Since the LS cost function is a quadratic form in  $\mathbf{f}_a, \mathbf{s}_d$ , the minimisation can be done independently for  $\mathbf{f}_a$  and  $\mathbf{s}_d$ . In order to

obtain a direct semi-blind equalizer estimation, we first solve for  $\mathbf{s}_d$ , assuming  $\mathbf{f}_a$  to be known and fixed. The LS solution for  $\mathbf{s}_d$  can be simplified to :

$$\bar{\mathbf{s}}_d = \mathbf{f}_a \mathbf{Y}_a \mathbf{C}_d^H \quad (9)$$

because  $\mathbf{C}_d \mathbf{C}_d^H = \mathbf{I}_{KB}$  and  $\mathbf{C}_p \mathbf{C}_d^H = \mathbf{0}_{B \times KB}$  due to the orthogonality of the user code sequences and the pilot code sequence at each symbol instant. Substituting  $\bar{\mathbf{s}}_d$  into the original LS problem (see Equation 8) leads to a modified LS problem in  $\mathbf{f}_a$  :

$$\bar{\mathbf{f}}_a = \arg \min_{\mathbf{f}_a} \|\mathbf{f}_a \mathbf{Y}_a (\mathbf{I}_{BN} - \mathbf{C}_d^H \mathbf{C}_d) - \mathbf{s}_p \mathbf{C}_p\|^2 \quad (10)$$

which can be interpreted as follows. The equalized signal  $\mathbf{f}_a \mathbf{Y}_a$  is projected on the orthogonal complement of the space spanned by the user code sequences. Furthermore, the projected equalized signal  $\mathbf{f}_a \mathbf{Y}_a (\mathbf{I}_{BN} - \mathbf{C}_d^H \mathbf{C}_d)$  should be as close as possible to the transmitted pilot chip sequence  $\mathbf{s}_p \mathbf{C}_p$  in a Least Squares sense. Eventually, the LS solution for  $\mathbf{f}_a$  can be written as :

$$\bar{\mathbf{f}}_a = \mathbf{s}_p \mathbf{C}_p \mathbf{Y}_a^H \left\{ \mathbf{Y}_a (\mathbf{I}_{BN} - \mathbf{C}_d^H \mathbf{C}_d) \mathbf{Y}_a^H \right\}^{-1} \quad (11)$$

### 3.2. Adaptive processing

In this subsection, we derive a Square-Root Information (SRI) RLS type of adaptive algorithm for the semi-blind chip equalizer. By having a closer look at Equation 10, we notice that each new incoming  $(Q+1)M \times N$  output matrix block  $\mathbf{Y}_a[i]$  is first projected on the orthogonal complement of the space spanned by the user code sequences, by using the projection matrix  $\tilde{\mathbf{C}}_d[i]$  :

$$\tilde{\mathbf{C}}_d[i] = \mathbf{I}_N - \sum_{k=1}^K \mathbf{c}_k[i]^H \mathbf{c}_k[i]$$

The new projected output matrix block  $\mathbf{Y}_a[i] \tilde{\mathbf{C}}_d[i]$  is then used together with the new transmitted pilot chip sequence vector  $\mathbf{s}_p[i] \mathbf{c}_p[i]$  in a QRD-updating step, shown in Equation 12. In this equation,  $\lambda$  is the forget factor, that should be chosen in correspondence with the coherence time of the time-varying channel. The QRD-updating step tracks a lower-triangular factor  $\mathbf{R}_a[i]$  and a corresponding right-hand side  $\mathbf{z}_a[i]$ . The new value for the semi-blind chip equalizer  $\mathbf{f}_a[i]$  then follows from the backward substitution step :

$$\mathbf{f}_a[i] \cdot \mathbf{R}_a[i] = \mathbf{z}_a[i] \quad (13)$$

## 4. TRAINING-BASED CHIP EQUALIZER RECEIVER

In this section, we discuss a training-based chip equalizer receiver [7], that exploits (besides the desired user's code information) only pilot code information on one hand and pilot symbol information on the other hand. Again, one algorithm for block processing and one for adaptive processing is derived.

### 4.1. Block processing

The block processing algorithm for the training-based chip equalizer, that detects  $B$  data symbols at once, can again be formulated as a LS minimisation problem :

$$\bar{\mathbf{g}}_a = \arg \min_{\mathbf{g}_a} \|\mathbf{g}_a \mathbf{Y}_a \mathbf{C}_p^H - \mathbf{s}_p\|^2 \quad (14)$$

which can be interpreted as follows. The equalized signal  $\mathbf{g}_a \mathbf{Y}_a$  is despread with the pilot code sequence matrix  $\mathbf{C}_p$ . The equalized signal after despread  $\mathbf{g}_a \mathbf{Y}_a \mathbf{C}_p^H$  should then be as close as possible to the transmitted pilot symbol vector  $\mathbf{s}_p$  in a Least Squares sense. The LS solution for  $\mathbf{g}_a$  can be written as :

$$\bar{\mathbf{g}}_a = \mathbf{s}_p \mathbf{C}_p \mathbf{Y}_a^H \left\{ \mathbf{Y}_a (\mathbf{C}_p^H \mathbf{C}_p) \mathbf{Y}_a^H \right\}^{-1} \quad (15)$$

In order to guarantee the uniqueness of the solution for  $\mathbf{g}_a$  up to a complex scaling factor, the matrix  $\begin{bmatrix} (\mathbf{X}_a \mathbf{C}_p^H)^T & -\mathbf{s}_p^T \end{bmatrix}^T$  should have at most a one-dimensional left null space. This leads to the following identifiability condition :

$$B \geq r \quad (16)$$

Note that when  $K = N - 1$ ,  $\mathbf{I}_{BN} - \mathbf{C}_d^H \mathbf{C}_d = \mathbf{C}_p^H \mathbf{C}_p$ . This means that for a fully loaded system ( $K = N - 1$ ) the semi-blind method is exactly the same as the training-based method. This is also indicated by the identifiability conditions 7 and 16.

### 4.2. Adaptive processing

In this subsection, we derive an SRI-RLS type of adaptive algorithm for the training-based chip equalizer. By having a closer look at Equation 14, we notice that each new incoming  $(Q+1)M \times N$  output matrix block  $\mathbf{Y}_a[i]$  is first despread with the pilot code sequence vector  $\mathbf{c}_p[i]$ . The new despread output matrix block  $\mathbf{Y}_a[i] \mathbf{c}_p[i]^H$  is then used together with the new pilot symbol  $\mathbf{s}_p[i]$  in a QRD-updating step, shown in Equation 17. In this equation  $\lambda$  is again the forget factor. The QRD-updating step tracks a lower-triangular factor  $\hat{\mathbf{R}}_a[i]$  and a corresponding right-hand side  $\hat{\mathbf{z}}_a[i]$ . The new value for the training-based chip equalizer  $\mathbf{g}_a[i]$  then follows from the backward substitution step :

$$\mathbf{g}_a[i] \cdot \hat{\mathbf{R}}_a[i] = \hat{\mathbf{z}}_a[i] \quad (18)$$

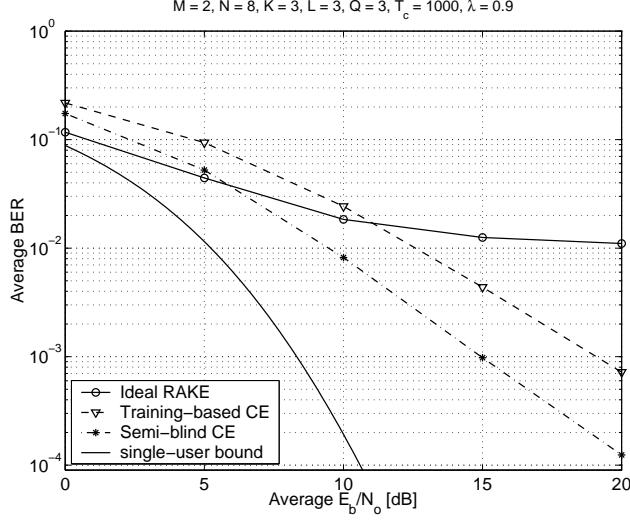
## 5. SIMULATION RESULTS

The simulations are performed for the forward link of a WCDMA system with  $K$  active, equal power users, QPSK data modulation, real orthogonal Walsh-Hadamard spreading codes of length  $N = 8$  along with a random overlay code for scrambling, whose period measures 10 symbols. The user terminal is equipped with the minimum number of receive antennas  $M = 2$  for chiprate sampling. The time-varying vector channel with order  $L = 3$  has  $M(L+1) = 8$  Rayleigh distributed channel taps of equal average power and a normalized coherence time of  $T_c = 1000$  symbols. The temporal smoothing factor  $Q = L$  is chosen in correspondence with the channel order  $L$ . The optimal value of the RLS forget factor proved to be  $\lambda = 0.9$ , corresponding to a data memory  $\frac{1}{1-\lambda}$  being  $\frac{1}{0.1}$  of the normalized coherence time  $T_c$ .

Figure 1 and 2 compare the performance of the adaptive processing algorithms for half respectively full system load. Both figures show the average BER versus the average SNR per bit for the conventional Maximum Ratio Combining (MRC) RAKE receiver with perfect channel knowledge at each symbol instant, the training-based and the semi-blind chip equalizer (CE) receiver. Also shown in the figures is the theoretical BER-curve of QPSK with  $M(L+1)$ -th order diversity in Rayleigh fading channels (single-user bound).

$$\left[ \begin{array}{c|c|c} \mathbf{R}_a[i] & \mathbf{0}_{(Q+1)M \times 1} & \mathbf{0}_{(Q+1)M \times (N-1)} \\ \hline \mathbf{z}_a[i] & * & \mathbf{0}_{1 \times (N-1)} \end{array} \right] \leftarrow \left[ \begin{array}{c|c} \frac{\lambda \mathbf{R}_a[i-1]}{\lambda \mathbf{z}_a[i-1]} & \mathbf{Y}_a[i] \tilde{\mathbf{C}}_d[i] \\ \hline s_p[i] \mathbf{c}_p[i] & s_p[i] \end{array} \right] \cdot \mathbf{T}_a[i] \quad (12)$$

$$\left[ \begin{array}{c|c} \hat{\mathbf{R}}_a[i] & \mathbf{0}_{(Q+1)M \times 1} \\ \hline \hat{\mathbf{z}}_a[i] & * \end{array} \right] \leftarrow \left[ \begin{array}{c|c} \frac{\lambda \hat{\mathbf{R}}_a[i-1]}{\lambda \hat{\mathbf{z}}_a[i-1]} & \mathbf{Y}_a[i] \mathbf{c}_p[i]^H \\ \hline s_p[i] & s_p[i] \end{array} \right] \cdot \hat{\mathbf{T}}_a[i] \quad (17)$$



**Fig. 1.** Adaptive processing : half system load.

For half system load ( $K = 3$ ), shown in Figure 1, both the training-based and the semi-blind CE receiver outperform the ideal RAKE receiver at high SNR per bit. The ideal RAKE receiver clearly exhibits an error floor due to the MUI. The semi-blind approach realizes a 3 dB gain at a BER of  $10^{-2}$  compared to the training-based approach.

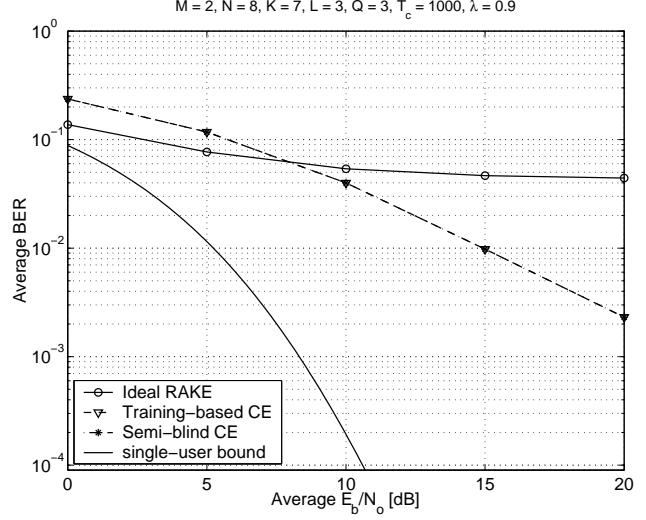
For full system load ( $K = 7$ ), shown in Figure 2, the training-based and the semi-blind approach have exactly the same performance (as discussed earlier) but still outperform the ideal RAKE receiver.

## 6. CONCLUSION

We have developed new training-based and semi-blind space-time chip equalizer receivers for the forward link of WCDMA systems employing a continuous code-multiplexed pilot. The proposed receivers can track fast fading multipath channels and outperform the conventional RAKE receiver with perfect channel knowledge. For full system load, the training-based and the semi-blind approach have exactly the same performance. However, for low to medium system load, the semi-blind approach outperforms the training-based approach.

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**Fig. 2.** Adaptive processing : full system load.

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