HARMONIC TRANSFORM

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ABSTRACT

The harmonic transform is designed for harmonic signals, which are composed of a base tone and some harmonics (e.g. voiced speech). As a generalization of the Fourier transform, harmonic transform represents signals by the sum of a base tone and some harmonics, which may gives more concise results for harmonic signals than Fourier transform. Some experiments of speech signals are used to demonstrate the advantages of harmonic transform on harmonic signal processing.

1. INTRODUCTION

The Fourier transform (FT) [1, 2] expands the signal in terms of sinusoids of different frequencies. The FT of signal $f(t)$ is defined as

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt. \tag{1}$$

The original signal is recovered by the inverse Fourier transform (IFT),

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega. \tag{2}$$

For the signal consisting of elements assuming fixed frequencies FT is an effective tool to determine the magnitude and the frequency of every element. However, this characteristic is deteriorated quickly for harmonic signals [3, 4], which are signals containing a base tone and some harmonics. One common example of harmonic signals is the voiced speech. To improve the performance of FT for harmonic signals, we have developed the harmonic transform (HT), which represents the signal as the sum of a base tone and some harmonics. In this paper HT is interpreted as the line integral of the signal on time-frequency plane and the parameter of HT is the integral of the center frequency of the signal's base tone. This interpretation is useful for parameter optimization.

This paper is organized as follows. The concept, the definition, and properties of HT are given in section 2, followed by some applications in section 3. The conclusion is the last section.

2. HARMONIC TRANSFORM

Harmonic signals are composed of a base tone and some harmonics. For a harmonic signal if the central frequency of its base tone is $c_0(t)$ in terms of t, the central frequency of its k-th harmonic can be expressed as

$$c_k(t) = kc_0(t), \qquad k = 1, 2, 3, \cdots . \tag{3}$$

For simplicity, in this paper we call the base tone 0-th harmonic written as $c_0(t)$. Figure 1 is a diagrammatic drawing to show the basic concept of HT. Time-frequency representation of a harmonic signal contains one base tone and two harmonics, as shown in Figure 1(b). Because the magnitude of $c_k(t)$ varies greatly in time, the FT of the signal, which can loosely be interpreted as the linear integral of its representation in time-frequency plane along the time axes, may be a plateau, in which we can find no peak corresponding to the base tone and harmonics, as illustrated in Figure 1(c). The basic idea of HT is to integrate the signal in time-frequency plane along the central frequencies of harmonics instead of the time axes. For example, for the base tone, the integral is along $c_0(t)$; for the k-th harmonic, the integral is along $c_k(t)$. So we can expect that there are peaks corresponding to the base tone and harmonics of the signal in the HT, as shown in of Fig. 1 (a).

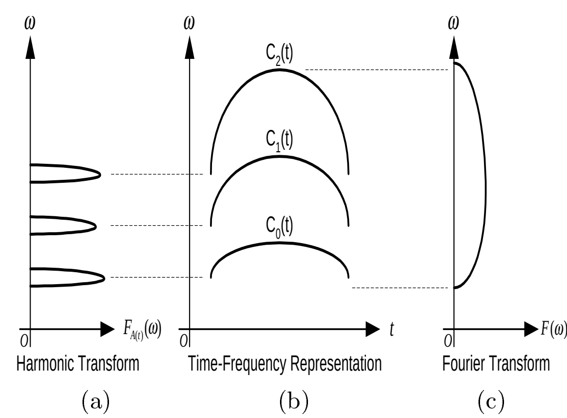

Fig. 1. The Concept of Harmonic Transform

The HT of $f(t)$ is defined by

$$F_{A(t)}(\omega) = \int_{-\infty}^{+\infty} f(t)A'(t)e^{-j\omega A(t)}dt, \tag{4}$$

where $A(t)$ called phase function is a differentiable and invertible function and $A'(t)$ is the first-order derivative of $A(t)$. The inverse harmonic transform (IHT) is

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_{A(t)}(\omega)e^{j\omega A(t)}d\omega. \tag{5}$$

We note that HT becomes FT when $A(t) = t$.

Before the mathematical proofs, let us see the relation between $A(t)$ and the central frequency $c_k(t)$ of harmonic signals. For the signal given in Fig. 1 (b), if we omit the amplitude variations of its base tone and harmonics, it can be expressed as

$$f_h(t) = \sum_{k=0}^{2} a_k e^{j(k+1)\alpha(t)}, \tag{6}$$

where a_k is the amplitude of the k-th harmonic and $\alpha(t)$ is the phase function of the base tone. The relation between $\alpha(t)$ and $c_0(t)$ is

$$c_0(t) = \alpha'(t). \tag{7}$$

When $A(t) = \alpha(t)$, the HT of $f_h(t)$ is

$$\begin{aligned} F_{h\alpha(t)}(\omega) &= \int_{-\infty}^{+\infty}\sum_{k=0}^{2} a_k e^{j(k+1)\alpha(t)}\alpha'(t)e^{-j\omega\alpha(t)}dt \\ &= \sum_{k=0}^{2} a_k \int_{-\infty}^{+\infty} e^{j(k+1-\omega)\alpha(t)}d\alpha(t) \\ &= \sum_{k=0}^{2} 2\pi a_k \delta(\omega - k - 1), \end{aligned} \tag{8}$$

which has three ideal peaks located at $\omega = 1, 2, 3$. This example shows that the most narrow spectrum is achieved when $A(t)$ is the phase function of the base tone. This is the reason why $A(t)$ is called phase function.

HT gives clear spectra for harmonic signals if a suitable $A(t)$ is employed. The users can design a suitable $A(t)$ according to their applications. As (7) shown, we can integrate $c_0(t)$, the central frequency of the base tone, to obtain $A(t)$. Since in many situation, such as voiced speech, the base tone is stronger than other harmonics, we can find $c_0(t)$ via other methods such as FT. Another way to find $A(t)$ is via optimizing parameters of the model of $A(t)$. In this way the users should first setup a model for $A(t)$ based on the properties of specific signals. Then the parameters are assigned according to certain criteria. Both methods are used in experiments given in this paper.

The proof of the existence of HT and IHT is as follows. Let $B(z)$ be the inverse function of $A(t)$, we have $z = A(t)$ and $t = B(z)$. Substituting $t = B(z)$ and $z = A(t)$ in (4) gives

$$\begin{aligned} F_{A(t)}(\omega) &= \int_{-\infty}^{+\infty} f(t)e^{-j\omega A(t)}dA(t) \\ &= \int_{-\infty}^{+\infty} f(B(z))e^{-j\omega z}dz. \end{aligned} \tag{9}$$

Substituting $t = B(z)$ and $z = A(t)$ in (5), we obtain

$$\begin{aligned} f(B(z)) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_{A(t)}(\omega)e^{j\omega A(t)}d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_{A(t)}(\omega)e^{j\omega z}d\omega. \end{aligned} \tag{10}$$

Comparing (9) and (10) with (1) and (2), we can see that (9) and (10) are the FT and the IFT of $f(B(z))$, respectively. The existence of the FT and the IFT of $f(B(z))$ guarantees the existence of the HT and the IHT defined by (4) and (5). ■

The properties of HT are very important for its applications. Some useful properties are listed in Table 1. We note that most of these properties are extensions of corresponding properties of FT. However, some properties of FT such as scaling property can not be extended, for the scaling property of HT is determined by the relation between $A(t)$ and $A(\frac{t}{a})$(a is a constant). The linear property can be obtained with the definition of HT given in (4) and (5). The properties of FT and the relation between FT and HT described by (9) and (10) can be used to deduce the properties of HT.

Table 1. The Properties of Harmonic Transform

	Time domain	Frequency domain
1	$x(t), y(t)$	$X_{A(t)}(\omega), Y_{A(t)}(\omega)$
2‡	$ax(t) + by(t)$	$aX_{A(t)}(\omega) + bY_{A(t)}(\omega)$
3†	$x(-t)$	$X_{A(t)}(-\omega)$
4	$x^*(t)$	$X^*_{A(t)}(\omega)$
5	$x(t) * y(t)$	$X_{A(t)}(\omega)Y_{A(t)}(\omega)$
6	$x(t)y(t)$	$X_{A(t)}(\omega) * Y_{A(t)}(\omega)$
7	$\frac{X_{A(t)}(t)}{A'(t)}$	$x(-\omega)$
8	$\delta(t - t_0)$	$A'(t_0)e^{-j\omega A(t_0)}$
9	$e^{j\omega_0 A(t)}$	$\delta(\omega - \omega_0)$
10	$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2 A'(t)dt$	$\int_{-\infty}^{+\infty} \lvert X_{A(t)}(\omega)\rvert^2 d\omega$
†	$A(t)$ is odd.	‡ a and b are constants.

The existence condition of HT requires that the product of the signal $f(t)$ and $A'(t)$ should be absolutely integrable over the whole time domain, that is,

$$\int_{-\infty}^{+\infty} |f(t)|A'(t)dt < M, \tag{11}$$

where M is a positive constant. This property is proved via the existence condition of FT,

$$\int_{-\infty}^{+\infty} |f(t)|dt < M.$$

Applying this existence condition to (9) gives the existence condition of HT,

$$\int_{-\infty}^{+\infty} |f(B(z))| dz = \int_{-\infty}^{+\infty} |f(t)| A'(t) dt < M. \qquad (12)$$

In conclusion, HT can provide a better spectrum than FT for harmonic signals with suitable phase functions. The phase functions of HT, which are the key to HT's performance, should be the integral of the base tone's central frequency. How to find the phase function $A(t)$ is left to the users.

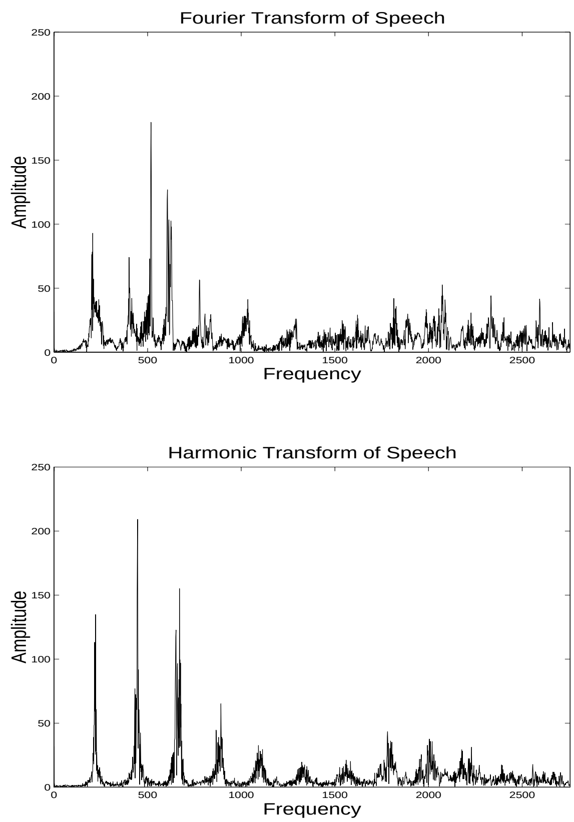

Fig. 2. The FT and the HT of PA

3. APPLICATIONS

Speech signals are non-stationary with harmonics that vary in time and frequency. It can be categorized into voiced speech and unvoiced speech. Voiced speech, mainly consisting of a base tone and some harmonics, is one of the most common harmonic signals. The speech used for the experiments is a short sentence of a male voice, "Mailman, your mail".

3.1. Harmonic Transforms of Voiced Speech

The HTs of the sentence is calculated to demonstrate the advantages of HT. Since there is a short break in the sentence, we divide the sentence in two parts, PA and PB, and calculate their HTs separately. The duration of every part is about one second. The phase function $A(t)$ is the integral of the central frequency of the speech's base tone, which was extracted from the result of the sentence's short-time harmonic transform mentioned in the next application. Figure 2 gives the FT and the HT of PA. Figure 3 shows the FT and the HT of PB. There are clear peaks in the spectra provided by HT, which means that the bandwidths of the HTs are much narrow than the bandwidths of FTs. This comparison reveals the potentiality of HT on speech enhancement and coding.

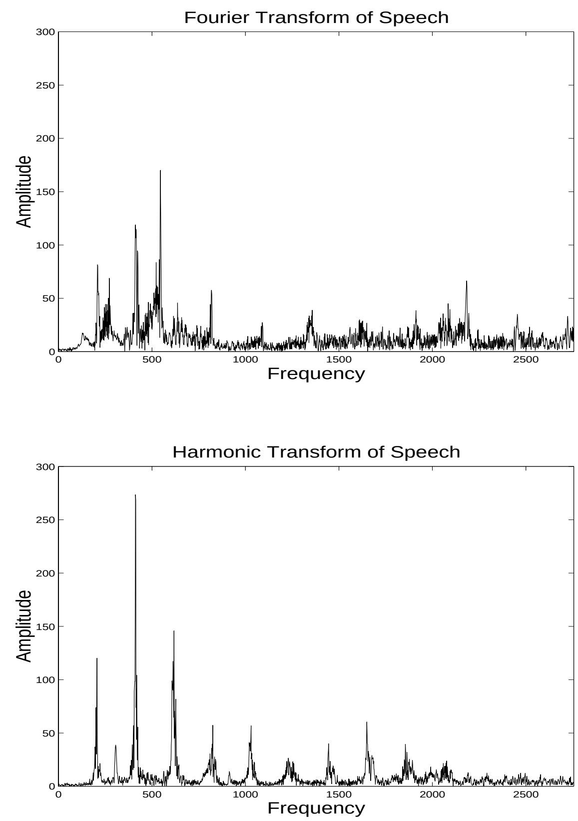

Fig. 3. The FT and the HT of PB

3.2. Short-Time Harmonic Transform

HT is applied to improve the performance of the short-time Fourier transform (STFT) [5] for harmonic signals. Time-frequency distributions [5] describe the signal in time-frequency plane and provide spectra of signals at any given time. STFT is a widely used technique to calculate the time-frequency representation. The STFT of $f(t)$ is

$$\mathrm{STFT}(\omega, t) = \int_{-\infty}^{+\infty} f(\tau) w(\tau - t) e^{-j\omega\tau} d\tau, \qquad (13)$$

where $w(t)$ is the window function. Replacing the FT used in STFT with HT gives the short-time harmonic transform

(STHT), written as

$$\mathrm{STHT}_{A(t)}(\omega, t) = \int_{-\infty}^{+\infty} f(\tau) w(\tau - t) \times A'(\tau) e^{-j\omega A(\tau)} d\tau, \tag{14}$$

where $f(t)$ is the signal and $w(t)$ is the window function. The STHT and the STFT of the sentence are given in Fig. 4 for comparisons. Although at some times (e.g. t=0.2) the STHT and the STFT have the similar performance because the base tone's frequencies are stable, it is clear that when there are variations on the base tone's frequencies (e.g. t=1.2) the spectra of the STHT are more concentrated than the spectra of the STFT, which means the bandwidth of the STHT is more narrow than the STFT's. For a further comparison, the areas $t \in [1.1, 1.3]$ are enlarged in Fig. 5, which confirms the conclusion obtained from Fig. 4. In conclusion, STHT with suitable phase functions has a better performance for harmonic signals than STFT.

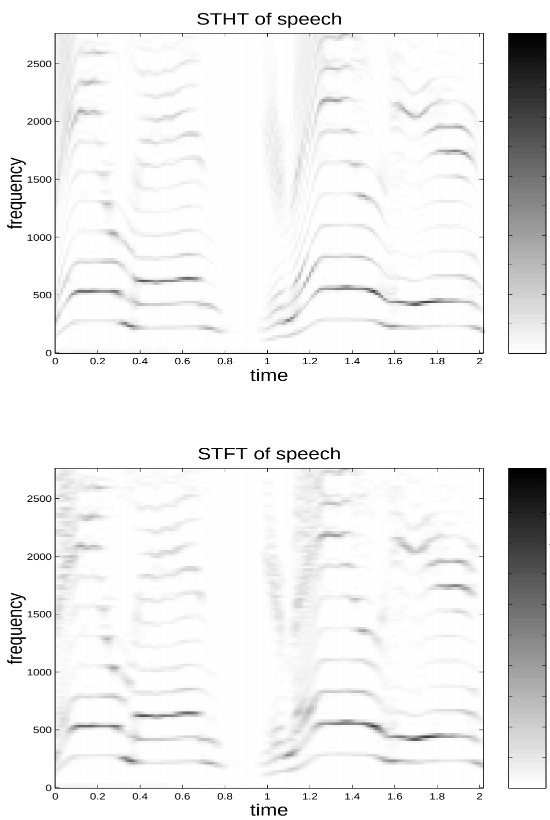

Fig. 4. The STHT and the STFT of the Sentence

4. CONCLUSIONS

The proposed HT, which represents signals as the sum of a base tone and some harmonics, is a generalization of FT and provides better performances than FT for harmonic signals. Besides the definition of HT, some useful properties are also

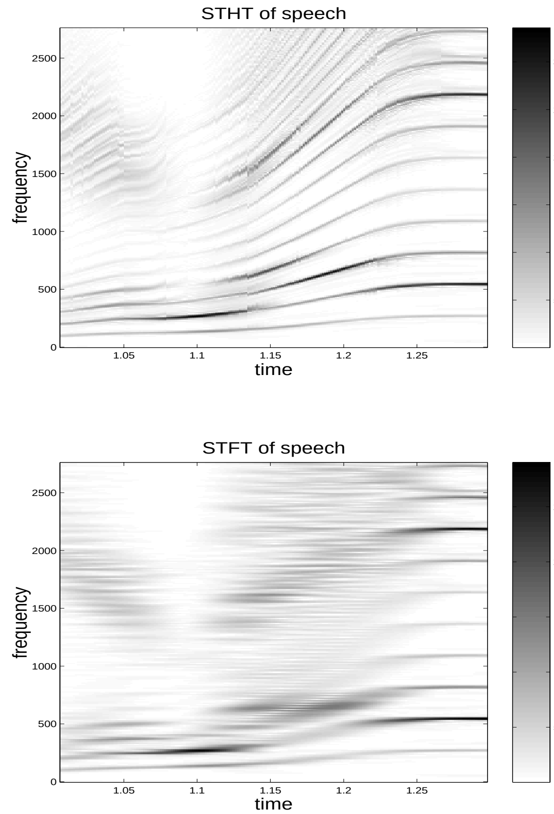

Fig. 5. The Enlarged Areas of the STHT and the STFT

discussed in this paper. Some applications on voiced speech are given to certify that HT is a power tool for harmonic signal processing.

5. REFERENCES

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