PER TONE ECHO CANCELLATION FOR DMT-BASED SYSTEMS

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ABSTRACT

A new echo canceller for discrete multitone (DMT) systems is presented where each used tone has its own per tone echo canceller (PT-EC) in addition to a per tone equalizer (PT-EQ) [1, 2]. This enables us to optimize the Signal-to-Noise Ratio (SNR) for each tone separately by solving a Minimum Mean Square Error (MMSE) problem for each tone. Simulation results confirm improved performance over time domain echo cancellation.

1. INTRODUCTION

DMT-modulation [3] has become an important transmission method, for instance for asymmetric digital subscriber line (ADSL). Usually, a DMT-receiver has a T-taps time domain equalizer (TEQ) before the prefix removal, such that the combined effect of channel and TEQ is sufficiently short [4, 5, 6], i.e. shorter than, or equal to the prefix length plus one.

In [1, 2] a new receiver structure, based on *'per tone equalization'* (PT-EQ), has been derived. Each tone then has its own (complex) T-taps equalizer. With a comparable complexity during data transmission, it becomes possible to optimize the SNR for each tone separately while at the same time the sensitivity to the decision delay is decreased compared to the TEQ-approach.

In [7] joint shortening of far end and echo impulse responses by a TEQ was introduced. By first shortening the echo impulse response, one can use shorter echo cancellers. However the optimization criterion based on channel shortening has no direct relation to the resulting SNR of the system. An echo canceller which is implemented partly in the time domain and partly in the frequency domain was developed in [8]. The advantage of this structure is an efficient echo emulation and a filter updating scheme consisting of LMS algorithms on 1 (complex) tap per tone.

In this paper, *'per tone echo cancellation'* (PT-EC) is introduced. The time domain echo canceller (TEC) is moved to the frequency domain to result in a (complex) echo canceller for each tone separately. By minimizing an MMSE criterion, the SNR is optimized in a per tone fashion.

The paper is organized as follows. Section 2 describes the data model. PT-EC is introduced in section 3. Section 4 explains joint initialization of the PT-EQ and the PT-EC by means of an MMSE cost function per tone. Simulation results are presented in section 5. Finally conclusions are drawn in section 6.

2. DATA MODEL

The following notation is adopted in the description of the DMT-system, analogous to [1, 2]. N is the symbol size of the far end signal expressed in samples, k is the time index of a symbol, $X_i^{(k)}$ is a complex subsymbol for tone i ($i = 1 \dots N$) of the far end signal transmitted at symbol period k. Note that $X_i^{(k)} = X_{N-(i-2)}^{*(k)} \quad i = 2 \dots \frac{N}{2}$. Further, ν denotes the length of the cyclic prefix of the far end signal, $s = N + \nu$ the length of a far end symbol including prefix, $\bar{\mathbf{h}} = [h_L \dots h_0 \dots h_{-K}]$ the far end channel impulse response in reverse order, n_l additive channel noise and y_l the received signal with l being the sample index.

To describe the data model, we consider three successive symbols[1] $X_{1:N}^{(c)}$ transmitted at time $c = k-1, k, k+1$ respectively. The kth symbol is the symbol of interest, the previous and the next symbol are used to include interferences with neighboring symbols in our model.

If echo is present, one can model the echo in a similar way as the far end signal. $U_i^{(k)}$ is a complex subsymbol for tone i ($i = 1 \dots N$) of the echo signal transmitted at symbol period k. $\bar{\mathbf{h}}_E$ is the echo channel impulse response in reverse order. For the sake of compact notation, we assume

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[1] $X_{1:N}^{(c)}$ denotes vector $[X_1^{(c)} \dots X_N^{(c)}]^{\mathrm{T}}$.

a symmetric rate set-up, i.e. the same symbol and prefix size for echo and far end signal. Extensions of the data model to asymmetric rate set-ups are straightforward. The received signal then becomes

$$\overbrace{\begin{bmatrix} y_{k\cdot s+\nu-T+2+\delta_1} \\ \vdots \\ y_{(k+1)\cdot s+\delta_1} \end{bmatrix}}^{\mathbf{y}} = \mathbf{H}\cdot\hat{\mathbf{x}} + \mathbf{H}_E\cdot\hat{\mathbf{u}} + \mathbf{n} =$$

$$\left[\begin{array}{c|ccc|c} \mathbf{O}_{(1)} & \boxed{\bar{\mathbf{h}}} & 0 & \cdots & \mathbf{O}_{(2)} \\ & & \ddots & \ddots & \\ & 0 & \cdots & \boxed{\bar{\mathbf{h}}} & \end{array}\right] \cdot\hat{\mathbf{P}}\cdot\hat{\mathcal{I}}_N\cdot \overbrace{\begin{bmatrix} X_{1:N}^{(k-1)} \\ X_{1:N}^{(k)} \\ X_{1:N}^{(k+1)} \end{bmatrix}}^{\hat{\mathbf{x}}} +$$

$$\left[\begin{array}{c|ccc|c} \mathbf{O}_{(3)} & \boxed{\bar{\mathbf{h}}_E} & 0 & \cdots & \mathbf{O}_{(4)} \\ & & \ddots & \ddots & \\ & 0 & \cdots & \boxed{\bar{\mathbf{h}}_E} & \end{array}\right] \cdot\hat{\mathbf{P}}\cdot\hat{\mathcal{I}}_N\cdot \overbrace{\begin{bmatrix} U_{1:N}^{(k-1)} \\ U_{1:N}^{(k)} \\ U_{1:N}^{(k+1)} \end{bmatrix}}^{\hat{\mathbf{u}}} +$$

$$\overbrace{\begin{bmatrix} n_{k\cdot s+\nu-T+2+\delta_1} \\ \vdots \\ n_{(k+1)\cdot s+\delta_1} \end{bmatrix}}^{\mathbf{n}} \tag{1}$$

Here, T is the length of the equalization filters. $\mathbf{O}_{(1)}$ and $\mathbf{O}_{(2)}$ are zero matrices of size $(N+T-1)\times(N+\nu-T+1-L+\nu+\delta_1)$ and $(N+T-1)\times(N+\nu-K-\delta_1)$ respectively. Matrix $\hat{\mathbf{P}} = \text{diag}(\mathbf{P},\mathbf{P},\mathbf{P})$ is a block diagonal matrix where $\mathbf{P} = \left[\begin{array}{c|c} \mathbf{O} & \mathbf{I}_\nu \\ \hline \multicolumn{2}{c}{\mathbf{I}_N} \end{array}\right]$ adds the cyclic prefix. Matrix $\hat{\mathcal{I}}_N = \text{diag}(\mathcal{I}_N,\mathcal{I}_N,\mathcal{I}_N)$ is a block diagonal matrix where $\mathcal{I}_N$ is an $N\times N$ IDFT matrix which modulates the input symbols. The zero reference delay of the far end signal corresponds to the head $[h_{-K}\dots h_{-1}]$ and the tail $[h_{\nu+1}\dots h_L]$ that maximize the energy in $[h_0\dots h_\nu]$. Finally, δ_1 is the (relative) decision delay of the far end signal. Analogous definitions hold for the corresponding echo parameters $\mathbf{O}_{(3)}$, $\mathbf{O}_{(4)}$, $\bar{\mathbf{h}}_E$, L_E, K_E and δ_2.

In case of a 2-fold oversampled receiver, one has a vector $\mathbf{y}$ consisting of even and odd samples. This vector can be split into a vector of even samples $\mathbf{y}_e$ and a vector of odd samples $\mathbf{y}_o$, both of which may be specified by means of a formula of the form of (1). The corresponding impulse responses are $\mathbf{h}_e$, $\mathbf{h}_{E,e}$ and $\mathbf{h}_o$, $\mathbf{h}_{E,o}$ and the noise vectors are $\mathbf{n}_e$ and $\mathbf{n}_o$.

The input to the echo canceller (echo reference signal) is modeled by

$$\overbrace{\begin{bmatrix} u_{k\cdot s+\nu-T_E+2+\delta_3} \\ \vdots \\ u_{(k+1)\cdot s+\delta_3} \end{bmatrix}}^{\mathbf{u}} = \left[\begin{array}{c|c|c} \mathbf{O}_{(5)} & \mathbf{I} & \mathbf{O}_{(6)} \end{array}\right]\cdot\hat{\mathbf{P}}\cdot\hat{\mathcal{I}}_N\cdot\hat{\mathbf{u}} = \mathbf{H}_r\cdot\hat{\mathbf{u}} \tag{2}$$

with T_E the length of the echo cancellation filter, $\mathbf{O}_{(5)}$ and $\mathbf{O}_{(6)}$ zero matrices of size $(N+T_E-1)\times(N+\nu-T_E+1+\nu+\delta_3)$ and $(N+T_E-1)\times(N+\nu-\delta_3)$ respectively. There are three delays in the data model: δ_1 for the far end signal, δ_2 for the echo signal and δ_3 for the reference signal. The delays δ_2 and δ_3 are related to each other.

3. COMBINATION OF EQ AND EC IN A DMT-RECEIVER

In [1, 2] it was demonstrated how a TEQ can be moved to the frequency domain to result in a PT-EQ for each tone separately. The advantage of the latter structure is that optimizing the SNR for each tone separately results in an MMSE problem per tone. The same idea can also be applied to a TEC. By moving the TEC to the frequency domain, one obtains an EC for each tone separately which can be initialized by solving an MMSE problem. For each tone i, the operation of the transceiver with TEQ and TEC is based on the following operation

$$Z_i^{(k)} = D_i\cdot\text{row}_i\,\overbrace{(\mathcal{F}_N)\cdot(\mathbf{Y}\cdot\mathbf{w}-\mathbf{U}\cdot\mathbf{w}_E)}^{\text{1 FFT}} \tag{3}$$

where D_i is the (complex) 1-taps FEQ for tone i, $\mathcal{F}_N$ is an $N\times N$ DFT matrix, $\mathbf{w} = [w_0\,w_1\dots w_{T-1}]^{\text{T}}$ is the (real) T-taps TEQ, and $\mathbf{Y}$ an $N\times T$ Toeplitz matrix of received samples. Similarly, $\mathbf{w}_E$ is the (real) T_E-taps TEC and $\mathbf{U}$ an $N\times T_E$ Toeplitz matrix of samples of the echo reference signal. Note that $\mathbf{Y}$ and $\mathbf{U}$ contain the same samples as vector $\mathbf{y}$ in formula (1) and $\mathbf{u}$ in formula (2) respectively. Formula (3) may be rewritten as follows

$$\begin{aligned} Z_i^{(k)} &= \text{row}_i\,(\mathcal{F}_N)\cdot(\mathbf{Y}\cdot\mathbf{w}\cdot D_i-\mathbf{U}\cdot\mathbf{w}_E\cdot D_i) \\ &= \text{row}_i\,\underbrace{(\mathcal{F}_N\cdot\mathbf{Y})}_{T\text{ FFT's}}\cdot\mathbf{w}_i-\text{row}_i\,\underbrace{(\mathcal{F}_N\cdot\mathbf{U})}_{T_E\text{ FFT's}}\cdot\mathbf{w}_{E,i} \end{aligned} \tag{4}$$

By putting D_i to the right one obtains a complex T-taps PT-EQ $\mathbf{w}_i$ and a complex T_E-taps PT-EC $\mathbf{w}_{E,i}$ for each tone separately. As demonstrated in [2], T respectively T_E successive FFT's can be calculated efficiently by means of a sliding FFT. A per tone filter (PTF) needs only one 'full' FFT and the other FFT's can be derived as a complex linear combination of this FFT and $T-1$ respectively T_E-1 (real) difference terms. As a result, it has been shown in [2] that a T-taps time domain filter and T-taps per tone filtering have the same complexity during data transmission.

The final modem set-up has one FFT operation for the received signal and one FFT operation for the reference echo signal together with equalization/echo filters mainly acting upon difference terms. The FFT's have as their inputs the elements in the first column of $\mathbf{Y}$ and $\mathbf{U}$ respectively. Remark that, if the modem works frame-synchronously, the only FFT needed in the PT-EC, reproduces the transmitted echo symbol $U_{1:N}^{(k)} = \mathcal{F}_N\cdot\mathbf{U}(:,1)$. Hence, in that case, the

FFT in the echo branch becomes superfluous. Furthermore, in frame-synchronous mode, the first $\nu + 1$ columns of $\mathbf{U}$ are equal up to a rotation, so the first ν difference terms are zero. In **Fig. 1** a PTF block is defined, including an N-point FFT operation and T-taps per tone filters $\mathbf{v}_i$ for all used tones. The notation $\mathbf{v}$ instead of $\mathbf{w}$ is adopted for the PTF's because we work here with the efficient implementation of these filters, i.e. their inputs are one FFT-output and $T-1$ difference terms. A complete DMT-receiver with PT-EQ and PT-EC[2] is then shown in **Fig. 2**.

We also study the performance of two-fold oversampled PT-EQ combined with PT-EC. If the received signal is sampled at twice the original sample rate, then the TEQ can be rewritten by means of its polyphase components which results in an even (e) and odd (o) TEQ at the sample rate. When transforming such a receiver structure to a per tone structure, one obtains a receiver with an even and an odd PT-EQ.

4. JOINT INITIALIZATION OF PT-EQ AND PT-EC

In the sequel, we immediately write all the formulas for the oversampled PT-EQ, which is more general. In case of sampling at the original sample rate, all 'odd' parts in the following formulas can be set to zero.

As in [1, 2], one finds the PTF's by solving an MMSE problem for each tone separately. The MMSE problem for joint initialization of the PT-EQ and the PT-EC, gives rise to an extended version of the MMSE problem for initialization of the PT-EQ only. For each tone i, one minimizes the following cost function

$$J(\mathbf{v}_{i,e}, \mathbf{v}_{i,o}, \mathbf{v}_{E,i}) =$$
$$\mathcal{E}\left\{\left| \begin{bmatrix}\bar{\mathbf{v}}_{i,e}^{\mathrm{T}} & \bar{\mathbf{v}}_{i,o}^{\mathrm{T}}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{F}_{i,T} & \mathbf{O} \\ \mathbf{O} & \mathbf{F}_{i,T}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{y}_e \\ \mathbf{y}_o\end{bmatrix} - \underbrace{\bar{\mathbf{v}}_{E,i}^{\mathrm{T}} \cdot \mathbf{F}_{i,T_E} \cdot \mathbf{u}}_{\text{echo cancellation}} - X_i^{(k)} \right|^2\right\} \tag{5}$$

with $\mathbf{F}_{i,T}$ equal to $\left[\begin{array}{c|c|c}\mathbf{I}_{T-1} & \mathbf{O} & -\mathbf{I}_{T-1} \\ \hline \mathbf{O} & \multicolumn{2}{c}{\mathcal{F}_N(i,:)}\end{array}\right]$, where the first block row extracts the difference terms. Vectors $\mathbf{v}_{i,e}$ and $\mathbf{v}_{i,o}$ are respectively the even and the odd PT-EQ for tone i. $\mathbf{v}_{E,i}$ is the PT-EC for tone i. Formula (5) can be rewritten by including formulas (1) and (2), resulting in formula (6) (see next page) with $\mathbf{e}_i^{(k)} = [0\ldots 0\,1\,0\ldots 0]$ a $1 \times 3N$ vector with the non-zero entry in the $(N+i)$th column. Matrices $\mathbf{R}_{\hat{\mathbf{x}}} = \mathcal{E}\{\hat{\mathbf{x}}\hat{\mathbf{x}}^{\mathrm{H}}\}$ and $\mathbf{R}_{\hat{\mathbf{u}}} = \mathcal{E}\{\hat{\mathbf{u}}\hat{\mathbf{u}}^{\mathrm{H}}\}$ are the autocorrelation matrices of vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{u}}$ respectively. Matrix $\mathbf{R}_{\mathbf{n}} = \mathcal{E}\{\mathbf{n}\mathbf{n}^{\mathrm{H}}\}$ is the autocorrelation of vector $\mathbf{n} = [\mathbf{n}_e \mathbf{n}_o]^{\mathrm{T}}$.

This set of equations has a particular structure. The first block row is constructed with the far end impulse responses and the second block row with the echo impulse responses. The third block row consists of noise data. The first and second block column corresponds to the even and the odd PT-EQ respectively. The third block column corresponds to the PT-EC, so it consists of the echo reference signal.

Cost function (6) incorporates optimal *per tone joint shortening* for the general case of a T-taps equalizer and a T_E-taps echo canceller. The per tone joint shortening improves upon the original time domain joint shortening [5, 7, 9] because it maximizes the SNR for each tone separately.

[2]More efficient PT-EC implementations and corresponding complexity savings can be derived for asymmetric rate set-ups. The derivation will be reported elsewhere.

5. SIMULATION RESULTS

Table 1 presents simulation results for a 3 km downstream channel (26 AWG) with white noise of -140dBm/Hz. The downstream IFFT and FFT have size $N = 512$. The upstream IFFT and FFT have size 128. An FDM set-up for the used tones is adopted: upstream on tones 8 till 30 and downstream on tones 39 till 256. Due to relaxed transmit and receive filter specifications, downstream symbols experience echo from upstream symbols. Downstream and upstream PSD on the used tones are -40dBm/Hz and -38dBm/Hz respectively.

The bitrate is calculated by:

$$rate = (\sum_{i=used\ tone} b_i) \cdot \frac{F_s}{N+\nu} \tag{7}$$

with $F_s = 2.208$MHz the sample rate. The zero reference delay of far end and echo signal corresponds to the $\nu + 1$ successive samples with highest energy in the far end and echo impulse response respectively (with $\nu = 32$). The zero reference delay of the echo canceller corresponds to the T_E successive samples with highest energy in the echo impulse response. Bitrates are calculated for relative delays $\delta_1 = 0$ and $\delta_2 = \delta_3 = 20$.

Different scenarios are considered: without echo cancellation, TEC and PT-EC. In each scenario PT-EQ is performed. We assume a 32-taps PT-EQ in case of a non-oversampled receiver and two 16-taps PT-EQ in case of a two-fold oversampled receiver. Hence, the total number of equalizer taps is equal in both cases.

Without EC, the bitrate is around 3.3Mbits/s. The echo channel length is roughly 60 taps sampled at F_s. With a 60-taps TEC, the bitrate increases to 4.28Mbits/s. Oversampling increases the bitrate even more to 4.94Mbits/s. PT-EC combined with two-fold oversampled PT-EQ gives the highest bit rate, namely 4.96Mbits/s. When decreasing the number of EC taps to $T_E = 32$ (to save complexity), the bitrate in case of TEC decreases. The PT-EC however, keeps the same bit rates.

6. CONCLUSIONS

Per tone echo cancellation is proposed as an alternative to time domain echo cancellation. The resulting receiver struc-

$$
\begin{aligned}
J(\mathbf{v}_{i,e},\mathbf{v}_{i,o},\mathbf{v}_{E,i}) &= \mathcal{E}\left\{ \left| \begin{bmatrix}\bar{\mathbf{v}}_{i,e}^{\mathrm{T}} & \bar{\mathbf{v}}_{i,o}^{\mathrm{T}}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{F}_{i,T} & \mathbf{O}\\ \mathbf{O} & \mathbf{F}_{i,T}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{H}_e\hat{\mathbf{x}}+\mathbf{H}_{E,e}\hat{\mathbf{u}}+\mathbf{n}_e\\ \mathbf{H}_o\hat{\mathbf{x}}+\mathbf{H}_{E,o}\hat{\mathbf{u}}+\mathbf{n}_o\end{bmatrix} - \bar{\mathbf{v}}_{E,i}^{\mathrm{T}}\mathbf{F}_{i,T_E}(\mathbf{H}_r\hat{\mathbf{u}}) - \mathbf{e}_i^{(k)}\hat{\mathbf{x}} \right|^2 \right\} \\
&= \left\| \left[\begin{array}{cc|c} \mathbf{R}_{\hat{\mathbf{x}}}^{1/2}\mathbf{H}_e^{\mathrm{H}}\mathbf{F}_{i,T}^{\mathrm{H}} & \mathbf{R}_{\hat{\mathbf{x}}}^{1/2}\mathbf{H}_o^{\mathrm{H}}\mathbf{F}_{i,T}^{\mathrm{H}} & \mathbf{O}\\ \mathbf{R}_{\hat{\mathbf{u}}}^{1/2}\mathbf{H}_{E,e}^{\mathrm{H}}\mathbf{F}_{i,T}^{\mathrm{H}} & \mathbf{R}_{\hat{\mathbf{u}}}^{1/2}\mathbf{H}_{E,o}^{\mathrm{H}}\mathbf{F}_{i,T}^{\mathrm{H}} & -\mathbf{R}_{\hat{\mathbf{u}}}^{1/2}\mathbf{H}_r^{\mathrm{H}}\mathbf{F}_{i,T_E}^{\mathrm{H}}\\ \multicolumn{2}{c|}{\mathbf{R}_{\mathbf{n}}^{1/2}\begin{bmatrix}\mathbf{F}_{i,T}^{\mathrm{H}} & \mathbf{O}\\ \mathbf{O} & \mathbf{F}_{i,T}^{\mathrm{H}}\end{bmatrix}} & \mathbf{O} \end{array}\right] \cdot \begin{bmatrix}\bar{\mathbf{v}}_{i,e}^{*}\\ \bar{\mathbf{v}}_{i,o}^{*}\\ \bar{\mathbf{v}}_{E,i}^{*}\end{bmatrix} - \begin{bmatrix}\mathbf{R}_{\hat{\mathbf{x}}}^{1/2}\mathbf{e}_i^{(k)\mathrm{H}}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} \right\|_2^2
\end{aligned}
\qquad (6)
$$

		$T_E = 32$		$T_E = 60$	
T	no EC	TEC	PT-EC	TEC	PT-EC
32	3.24	3.26	4.31	4.28	4.31
2*16	3.39	4.34	4.96	4.94	4.96

Table 1. Simulation results for a 3km downstream channel. The first column gives the number of taps of the PT-EQ. The following columns present the capacity in Mbit/s.

ture enables us to optimize the SNR for each tone separately by solving an MMSE problem. This MMSE problem is the basis for adaptive algorithms which truly optimize capacity, unlike the TEC-based approaches.

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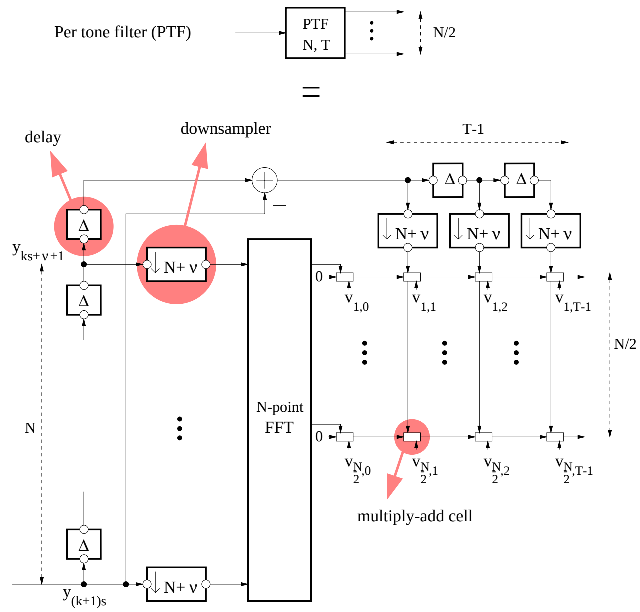

Fig. 1. An (N, T)-PTF block includes one N-point FFT operation and T-taps per tone filters $\mathbf{v}_i$ for all used tones.

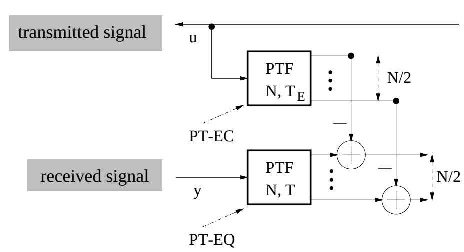

Fig. 2. Receiver with T-taps PT-EQ and T_E-taps PT-EC.