COMPUTATIONALLY EFFICIENT MULTIUSER DETECTION FOR APERIODICALLY SPREADING CDMA SYSTEMS

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ABSTRACT

Multiuser CDMA systems with aperiodic spreading codes have received considerable attention. One major difficulty in multiuser detection is the user's time-varying signature. In a multipath communication environment, this signature is not a priori known, making direct design of the detector more intractable. In this paper, multiuser detection is performed by a two-step approach: estimate the unknown channel coefficients and construct the detector based on the MMSE criterion. We model the interference as colored Gaussian noise with unknown correlations and estimate the channel parameters based on the correlation matching idea. The spreading codes are assumed random with known up to the fourth order statistics, resulting in computationally efficient method. In the case of unknown statistics however, the method is still applicable by estimating those statistics from given codes. To guarantee identifiability, both the channel's output and the output of the matched filter for the desired user are considered in one cost function. Comparisons with previously proposed method and other existing methods are made by simulations.

1. INTRODUCTION

In multiuser detection, two different approaches exist: one is a two-step procedure which constructs the detector based on the estimated channel parameters [1]; the other one is to directly design multiuser detectors [4], obviating the channel estimation step. Although in commercial CDMA systems, a user's information may be spread by a periodic spreading sequence such as Walsh-Hadamard sequence, the code sequence assigned to each user finally exhibits aperiodic nature due to multiplication by a scrambling long code sequence. For such systems with aperiodic spreading, direct design of multiuser receivers is very difficult due to time-varying signatures of desired users.

For this reason, most approaches explicitly or implicitly involve channel estimation. Based on the finite alphabet property of the input, an iterative method to estimate the FIR channels and the transmitted symbols is presented in [6]. In [7], subspace concept is adopted to identify the multipath channel. Blind uplink channel estimation method using correlation matching techniques is proposed in [8] based only on the correlation of the channel's output conditioned on the long spreading codes. The computationally efficient algorithm has been reported for downlink channel estimation in [11] and uplink channel estimation in [10] when the exact knowledge of the number of users is known. Without knowing interfering users within the cell and from neighboring cells, the detector to detect a desired user has to be derived.

In this paper, we adopt a two-step approach to detect the user of interest by employing the correlation matching technique [2]. We model the interference as colored Gaussian noise with unknown correlation which is treated as a nuisance parameter. We also assume the spreading codes are random. First we construct and minimize a correlation matching cost function to estimate the multipath channel experienced by the desired user. Both the channel's output and the output of the matched filter for the desired user are employed in the cost function to guarantee the identifiability of the channel parameters. Secondly, the minimum mean-square-error (MMSE) detector is constructed based on the channel estimate. Two different versions of the detector are derived depending on whether the channel's output or the matched filter's output is employed.

Since we estimate only channel parameters for the desired user, the method requires much lower computational cost especially when the statistics of the random spreading codes are known. In that case some code related matrices can be pre-computed to save computations. In the absence of code statistics however, the method is still applicable by estimating those statistics from given spreading codes. Comparisons among the proposed detector, the previously proposed estimator and other existing methods are made in our simulations.

2. SYSTEM MODEL

In a wireless CDMA system, the communication link is built between the base station and the desired mobile station. Without loss of generality, we may assume mobile user 1 is the desired user. Other users either in the same cell or from the neighboring cells are treated as interferers. User 1 has the information bit stream $w_1(n)$ to transmit through an unknown multipath channel $g_1(m)$ of order q. The transmit power is assumed $\sigma_{w_1}^2 = E\{|w_1(n)|^2\}$. During the n-th bit period, this user is assigned a random spreading code $c_{1,n}(k)$ for the k-th chip ($k = 0, \cdots, P-1$) with spreading factor P. Assume the receiver is synchronized to the desired user. We collect $P+q$ chip samples in a data vector $\boldsymbol{y}(n) = [y(nP), \cdots, y(nP+P+q-1)]^T$. Similarly channel coefficients for user 1 are arranged in $\boldsymbol{g}_1 = [g_1(0), \cdots, g_1(q)]^T$. Then the received discrete-time signal can be written as (see [8])

$$\boldsymbol{y}(n) = \boldsymbol{C}_1(n)\boldsymbol{g}_1 w_1(n) + \boldsymbol{v}(n) \quad (1)$$

where code matrix $\boldsymbol{C}_1(n)$ has the following form

$$\boldsymbol{C}_1(n) = \begin{bmatrix} c_{1,n}(0) & & \boldsymbol{0} \\ \vdots & \ddots & c_{1,n}(0) \\ c_{1,n}(P-1) & & \vdots \\ \boldsymbol{0} & \ddots & c_{1,n}(P-1) \end{bmatrix} \quad (2)$$

and $\boldsymbol{v}(n)$ accounts for all interference plus background noise.

In (1) the interference structure is unknown. Moreover, the signature of the desired user is time varying (TV) because of the random spreading codes. For the fore-mentioned reason, we will approach detection of user 1 in two steps: first estimate the channel vector $\boldsymbol{g}_1$ with low complexity based on $\boldsymbol{y}(n)$ and the statistical knowledge of user's random spreading codes, then construct the MMSE detector based on $\boldsymbol{y}(n)$ and the channel estimate. Before we derive our solutions, following assumptions are first explicitly made:

(1) $c_{1,n}(k)$ is i.i.d. random in n and k, with zero-mean, variance $\sigma_c^2 = E\{|c_{1,n}(k)|^2\}$ and fourth order moment $m_{4c} = E\{|c_{1,n}(k)|^4\}$; if $c_{1,n}(k)$ is complex, its real and imaginary parts are also i.i.d.

(2) $w_1(n)$, $c_{1,n}(k)$ and $v(n)$ are mutually independent.

Under these assumptions, we will first estimate $\boldsymbol{g}_1$ from autocorrelations of $\boldsymbol{y}(n)$ and the output of the matched filter based on correlation matching techniques.

From (1), the autocorrelation $\boldsymbol{R} \triangleq E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\}$ can be easily obtained

$$\boldsymbol{R} = E\{\boldsymbol{C}_1(n)\boldsymbol{G}_1\boldsymbol{C}_1^H(n)\} + \boldsymbol{R}_{int} \quad (3)$$

where superscript ^{H} represents conjugate transpose, $\boldsymbol{G}_1 \triangleq \sigma_{w_1}^2 \boldsymbol{g}_1\boldsymbol{g}_1^H$, and $\boldsymbol{R}_{int}$ is the unknown autocorrelation matrix of $\boldsymbol{v}(n)$ (interference plus noise).

In order to identify $\boldsymbol{g}_1$, we also exploit the output of the matched filter at time n which correlates $\boldsymbol{y}(n)$ with the code matrix $\boldsymbol{C}_1^H(n)$ as $\boldsymbol{y}_1(n) \triangleq \boldsymbol{C}_1^H(n)\boldsymbol{y}(n)$

$$\boldsymbol{y}_1(n) = \boldsymbol{C}_1^H(n)\boldsymbol{C}_1(n)\boldsymbol{g}_1 w_1(n) + \boldsymbol{C}_1^H(n)\boldsymbol{v}(n) \quad (4)$$

from which the autocorrelation of $\boldsymbol{y}_1(n)$ can be computed

$$\begin{aligned} \boldsymbol{R}_1 &= E\{\boldsymbol{C}_1^H(n)\boldsymbol{C}_1(n)\boldsymbol{G}_1\boldsymbol{C}_1^H(n)\boldsymbol{C}_1(n)\} \\ &+ E\{\boldsymbol{C}_1^H(n)\boldsymbol{R}_{int}\boldsymbol{C}_1(n)\} \end{aligned} \quad (5)$$

According to [3], to build an MMSE detector, the channel coefficients and the autocorrelation of the channel's output are required. The latter can be easily estimated from the data. Thus multiuser detection can be performed if the channel parameters are estimated. Noticing that channel information is embedded in (3) and (5) which are linearly parameterized by $\boldsymbol{G}_1$ and $\boldsymbol{R}_{int}$. Two equations are sufficient to solve these unknowns. If $\boldsymbol{G}_1$ is estimated, then the estimate for $\boldsymbol{g}_1$ is obtained by performing SVD on this rank one matrix within a complex scalar ambiguity. However, the performance of the MMSE detector is insensitive to such ambiguity if the phase of the channel is corrected or estimated based on some pilot symbols.

3. CORRELATION BASED CHANNEL ESTIMATION

In our estimation problem, there are two unknown matrices: $\boldsymbol{G}_1$ and $\boldsymbol{R}_{int}$, and we also have equal number of equations (3) and (5). Therefore the problem is well determined. If the correlations are matched with their estimates from the data, resulting errors can be minimized to solve $\boldsymbol{G}_1$. (3) and (5) can be written in vector forms with vec operation [5] in order for a closed form solution. Define

$$\boldsymbol{r} \triangleq vec(\boldsymbol{R}), \;\; \boldsymbol{r}_1 \triangleq vec(\boldsymbol{R}_1), \;\; \boldsymbol{d}_1 \triangleq vec(\boldsymbol{G}_1)$$

Since $\boldsymbol{R}_{int}$ is a nuisance parameter, it can be eliminated from (5) based on (3). Substituting (3) into (5) and performing vec operation on both sides of the equation, it can be easily verified that our unknown vector $\boldsymbol{d}_1$ satisfies the following equation

$$\boldsymbol{S}\boldsymbol{d}_1 = \boldsymbol{z} \quad (6)$$

where

$$\boldsymbol{S} \triangleq E\{\boldsymbol{Q}_1^H(n)\boldsymbol{Q}_1(n)\} - \boldsymbol{A}^H\boldsymbol{A}, \quad \boldsymbol{A} \triangleq E\{\boldsymbol{Q}_1(n)\} \quad (7)$$

$$\boldsymbol{Q}_1(n) \triangleq \boldsymbol{C}_1^*(n) \otimes \boldsymbol{C}_1(n), \quad \boldsymbol{z} \triangleq \boldsymbol{r}_1 - \boldsymbol{A}^H\boldsymbol{r} \quad (8)$$

the superscript * denotes complex conjugate, $\otimes$ represents the Kronecker product.

In (6), $\boldsymbol{S}$ and $\boldsymbol{A}$ are constant depending on system parameters such as σ_c^2, m_{4c}, P, q. They can be pre-computed for a given system. Due to lack of space, their expressions will be reported elsewhere. In the absence of such high order information however, they can be estimated from given codes. In the sequel, we will assume $\boldsymbol{S}$ and $\boldsymbol{A}$ are available. Since $\boldsymbol{Q}_1^H(n)$ and $\boldsymbol{Q}_1(n)$ are highly correlated, $\boldsymbol{S}$ is not zero and verified by simulations to be non-singular. Therefore unique solution is guaranteed. It can also be observed that if short codes are employed, $\boldsymbol{S}$ becomes zero which gives us a trivial equation. Then $\boldsymbol{d}_1$ can not be determined from the correlations under the current input/output model. However, it has been shown in [9] that in a multiuser system with white Gaussian noise, the solution is still achievable.

Based on (6), a batch algorithm to estimate $\boldsymbol{d}_1$ can be easily derived. Assume data vectors corresponding to N bit periods are received. Then as a common practice, the estimates for $\boldsymbol{r}$ and $\boldsymbol{r}_1$ can be obtained by their sample averages. Therefore the estimate for $\boldsymbol{z}$, denoted by $\widehat{\boldsymbol{z}}_N$, can be obtained from (8). According to (6), $\boldsymbol{d}_1$ is then estimated in the least square sense

$$\widehat{\boldsymbol{d}}_1 = \boldsymbol{S}^{-1}\widehat{\boldsymbol{z}}_N \tag{9}$$

The complexity to perform (9) is significantly reduced if $\boldsymbol{S}$ and $\boldsymbol{A}$ are pre-computed. In the case of unknown code statistics, $\boldsymbol{S}$ and $\boldsymbol{A}$ should be estimated from given spreading codes.

It is not hard to derive a corresponding adaptive algorithm to estimate the channel $\boldsymbol{g}_1$. Furthermore, the identifiability for $\boldsymbol{g}_1$ can be established based on the rank evaluation of $\boldsymbol{S}$. However, it will not be presented due to limited space.

4. BLIND MULTIUSER DETECTION

The MMSE detector can be built once channel is estimated. Based on the data either directly from the channel's output (eq. (1)), or from the matched filter's output (eq. (4)), two different versions of the detector will be discussed successively.

4.1. Based on the channel's output

If we focus on the first case, then the detector must be time-varying (TV) since the signature of the desired symbol is also TV due to the random spreading codes. The detector $\boldsymbol{f}_1(n)$ is designed according to the MMSE criterion [3]

$$\min E\{\|w_1(n) - \boldsymbol{f}_1^H(n)\boldsymbol{y}(n)\|^2\}$$

From (1) and matrix inversion lemma [3], it can be easily shown that for given spreading codes

$$\begin{aligned} \boldsymbol{f}_1(n) &= \sigma_{w_1}^2 \left[\boldsymbol{R}_{int} + \sigma_{w_1}^2 \boldsymbol{h}_1(n)\boldsymbol{h}_1^H(n)\right]^{-1} \boldsymbol{h}_1(n) \\ &= \frac{\sigma_{w_1}^2 \boldsymbol{R}_{int}^{-1}\boldsymbol{h}_1(n)}{1 + \sigma_{w_1}^2 \boldsymbol{h}_1^H(n)\boldsymbol{R}_{int}^{-1}\boldsymbol{h}_1(n)} \end{aligned} \tag{10}$$

where $\boldsymbol{h}_1(n) \triangleq \boldsymbol{C}_1(n)\boldsymbol{g}_1$ for short, and $\boldsymbol{R}_{int}$ is the autocorrelation of interference plus noise vector $\boldsymbol{v}(n)$. It can be estimated according to (3) once $\boldsymbol{d}_1$ thus $\boldsymbol{G}_1$ are obtained.

Notice that this detector is time-varying as $\boldsymbol{C}_1(n)$ is time-varying. It has more parameters to obtain if P is large. Next we derive a time-invariant (TI) detector with less computations based on the matched filter's output.

4.2. Based on the matched filter's output

The matched filter's output and its autocorrelation are given in (4), (5) respectively. The MMSE detector to detect the desired user can be constructed as [3]

$$\begin{aligned} \boldsymbol{f}_1 &= \sigma_{w_1}^2 \boldsymbol{R}_1^{-1} E\{\boldsymbol{C}_1^H(n)\boldsymbol{C}_1(n)\}\boldsymbol{g}_1 \\ &= \sigma_{w_1}^2 \sigma_c^2 P \boldsymbol{R}_1^{-1}\boldsymbol{g}_1 \end{aligned} \tag{11}$$

where $\boldsymbol{R}_1$ is the autocorrelation of $\boldsymbol{y}_1(n)$, and the second equality comes from the statistics of spreading codes. In (11), $\boldsymbol{R}_1$ is similarly estimated by its sample average. Its inverse can be computed directly or updated based on RLS method [3]. $\boldsymbol{g}_1$ is obtained from the proposed method. Thus the TI detector $\boldsymbol{f}_1$ can be easily obtained.

5. NUMERICAL EXAMPLES

We simulate a CDMA system with random spreading codes and multipath effect. The output signal to interference plus noise ratio (SINR) of the detector and the mean square channel estimation error (MSE) are adopted as the performance measure. Assume each of 5 users has 500 bits to transmit taken from the set $\{\pm 1\}$. $P = 16$. $c_{j,n}(k)$ take values randomly from $\{\pm 1 \pm j\}$ with equal probability. Therefore $m_{4c} = 4$, $\sigma_c^2 = 2$. Channel vectors for all users are complex and have 4 coefficients ($q = 3$). Both the real and imaginary parts of each coefficient are randomly selected from the interval $(-1, +1)$. Then the vector is normalized to have unit norm. $15dB$ white Gaussian noise is added to the system. Totally 500 independent realizations are performed to obtain the average results. The matrices $\boldsymbol{S}$ and $\boldsymbol{A}$ can be precomputed (with precomputation) or estimated from the given spreading codes (without precomputation). The output SINRs of the TI detector (11) and the TV detector (10) are shown in Fig. 1(a) and Fig. 1(b) respectively. Solid lines are based on the channel estimate with precomputation, while dashed lines without precomputation. The SINR of the TI detector converges to $5.8dB$ similar to that of the TV detector. However, the SINR of the TI detector converges much faster. It can also be observed that the difference between the method with precomputation and the method without precomputation for each detector disappears after 200 bit periods. This suggests the method with precomputation is preferable when complexity is of particular concern.

We also compare the currently proposed method with that of [8] and the subspace method [7] in a downlink CDMA system with different loads: (1) light load with $P = 32$, 5 users; (2) average load with $P = 16$, 6 users; (3) heavy load with $P = 16$, 14 users. We use a common channel for all users with three coefficients [7]. 50 realizations are performed. The average channel estimation errors are compared in Fig. 2(a), (b), (c) respectively. Solid lines are the results of the currently proposed method (with precomputation), dashed-dotted lines are from the multiuser

(MU) channel estimation method [8], and the dashed lines are based on the subspace method [7]. First it can be seen that the proposed method has performance close to that of the multiuser channel estimation method. These two methods can provide satisfactory channel estimate even when the system is overloaded. However, in [8] much more computations are required. Secondly, the subspace method provides more accurate estimate for a light load system. However, it is very sensitive to the system load, as explained in [8]. In all three cases, the currently proposed method needs less computations compared with both [8] and [7]. In [8], an inverse of a big matrix is mandatory, while in [7], SVD of an accumulated data matrix is required at each time.

6. REFERENCES

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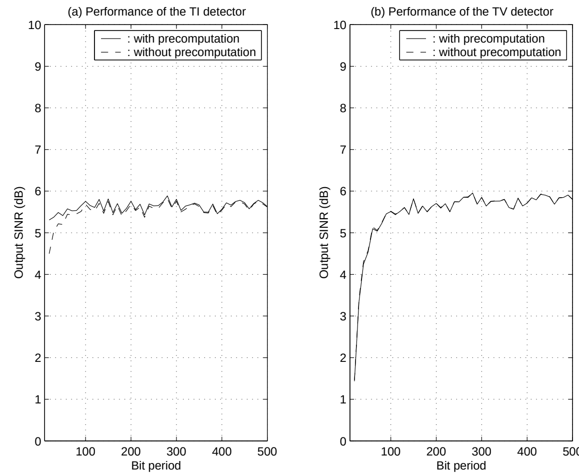

Fig. 1. SINRs of different proposed detectors based on the batch method.

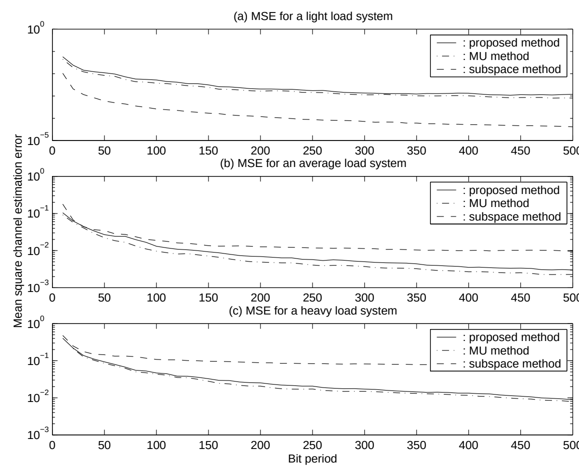

Fig. 2. Comparison of channel estimation errors among different methods.

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