ACTIVE NOISE CONTROL USING ROBUST FEEDBACK TECHNIQUES

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ABSTRACT

With the resurgence of active noise control in the past two decades, the design and implementation of optimal noise cancellers has been a topic of considerable research. Recently, much of the investigation is focused on fixed controllers that rely on the robustness of closed loop systems to deal with poorly parameterized or changing plants. Many acoustic systems employ *internal model controller* structures [1-5] designed via H_∞ techniques to attain optimality. Absent from the literature, however, is a qualitative study of classical robust controllers applied to the active noise problem. This paper looks at the practical issues and possible pitfalls of applying H_∞ optimal feedback controllers to one-dimensional acoustic ducts. In particular, it is observed that the relatively high degree of non-minimum-phase zeros, typical of acoustic plants, limits the ultimate performance of the optimal H_∞ solution.

1. INTRODUCTION

The use of active noise control is becoming increasingly important to both the industrialist and environmentalist. Applications reported to date [6,7] are generally applied to one-dimensional structures such as air-conditioning systems, or in small enclosures such as aircraft cockpits where the acoustic dynamics can be accurately modelled. Traditionally, active noise controllers have an adaptive feedforward structure. The advantages of this strategy are that satisfactory performance can be achieved with FIR controllers for time-varying acoustic plants and, assuming algorithm convergence, will remain inherently stable. It is assumed that the time variance of the plant is slow in comparison to the algorithm convergence time. Furthermore, this type of canceller can deal with both tonal and broadband disturbances. The disadvantages of this type of solution are the requirements of at least two sensors and the extensive computational expense.

In the case of narrowband noise, an alternative approach employs a non-adaptive feedback only controller, which requires just a single feedback sensor. In the late 1970's Zames [8] introduced a new form of feedback control, referred to as H_∞ control, which allows for the systematic design of controllers for optimal performance in the presence of plant uncertainty. In addition to this robust performance feature, a robust stability constraint may be also included. The remainder of this paper outlines the strategy required and the possible hazards of applying this type of robust control to active noise cancellation.

The feedback only active noise control problem of figure 1 can be represented in block diagram form in figure 2. The requirement of the system is to design $C(z)$ to maintain robust stability and provide satisfactory disturbance rejection for an uncertain secondary path, $G(z)$. In many classical control applications, robust set point tracking is also a requirement, but is not obligatory in this instance, as the controller is utilized purely in a disturbance rejection mode.

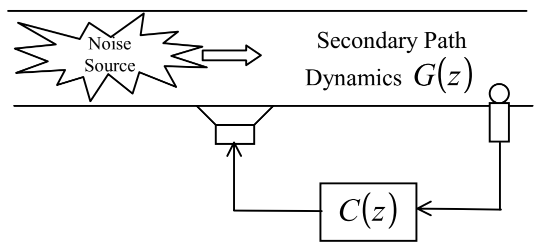

Figure 1. Feedback active noise controller

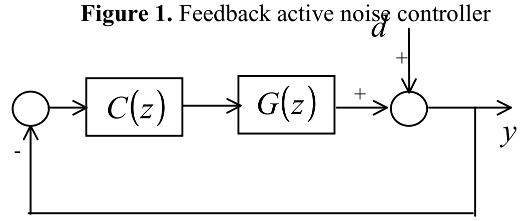

Figure 2. Classical feedback controller

Central to H_∞ control is the *sensitivity function*, which is given by $S(z) = 1/(1 + C(z)G(z))$. Equally important is the *complementary sensitivity function* $T(z) = C(z)G(z)/(1 + C(z)G(z))$. The plant, $G(z)$, is assumed to be parameterized by a *multiplicative disk-like uncertainty* (1)

$$G(z) = \{G_0(z)(1 + W_2(z)\Delta) \ ; \quad \|\Delta\|_\infty < 1\} \qquad (1)$$

G_0 is referred to as the *nominal plant*, and W_2 represents the uncertainty bound about the nominal plant (where the dependence on z has been dropped for convenience of notation). For satisfactory disturbance rejection with the nominal plant, it can be shown [9] that C must be designed such that the following condition is met.

$$\|W_1 S\|_\infty < 1 \qquad (2)$$

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In this case, W_1 represents a bound on the sensitivity function, i.e. on the disturbance rejection capabilities of the closed loop system. It has been shown [9] that robust performance is achievable *iff*

$$\left\| |W_1 S| + |W_2 T| \right\|_\infty < 1 \qquad (3)$$

Thus, if a controller, C, is designed that is stable for the nominal plant, and (3) holds, then the system has good robust stability and performance properties.

2. THE MODIFIED SOLUTION

The performance criterion stated in (3) is difficult to solve and is replaced in practice by the following sufficient (but not necessary) condition

$$\left\| |W_1 S|^2 + |W_2 T|^2 \right\|_\infty < \frac{1}{2} \qquad (4)$$

Even though this performance objective will probably produce a solution that is non-optimal for the inequality specified in (3), it is far more widely used in practice because it is relatively easier to solve. A polynomial solution is presented in [10], but a state-space approach is generally preferred [11]. The latter technique tends to be more suited to plants parameterized by longer filter orders and offers greater numerical robustness [9].

3. H_∞ CONTROL IN THE PRESENCE OF NON-MINIMUM-PHASE DYNAMICS

It has been shown [9,10,12] that robust control is seriously compromised in the presence of plants containing right half plane poles and right half plane zeros. Acoustic plants are generally stable and thus possess no right half plane poles. Right half plane zeros are common, however, mainly due to the inherent propagation delay of sound. A Padê approximation indicates the dynamic equivalence of right half plane zeros to a delay. Indeed, it may be shown that a plant with right half plane zeros exhibits the following adverse properties: -

Consider first the *waterbed effect,* which states that the improvement of the sensitivity at a certain frequency deteriorates it at another frequency. This phenomenon applies only to non-minimum-phase plants. Secondly, it has been shown that the weight, W_1, must be less than unity at the position of right half plane zeros in the plant or controller. This causes the sensitivity to be reduced at this frequency, but following from the waterbed effect, it will be increased at some other frequency. It is immediately obvious that this makes the shaping of the sensitivity function for acoustic systems inherently more difficult.

4. SIMULATED RESULTS FOR THE PROTOTYPE SYSTEM

An LMS algorithm was employed to adapt an FIR model to the secondary path dynamics. Figure 3 depicts a set of results for this system where the only parameter varied being the order of the model. Figure 3(a) indicates the error variance of the converged system versus the filter order. Nominating a value of 0.2 as the maximum allowable error variance, it follows that an FIR filter of greater than 120 taps is required to model the duct to a reasonable degree of accuracy. Figure 3(b) illustrates a plot of the number of non-minimum-phase zeros versus the filter order. Once again it may be noticed that the number of non-minimum-phase zeros converged (albeit noisily) at a filter order of approximately 120. Consequently, the controller must provide robust performance in the presence of almost 40 non-minimum-phase zeros. The central finding of this work was the experimental evidence that confirmed the deterioration of optimal disturbance rejection as the number of non-minimum-phase zeros increased.

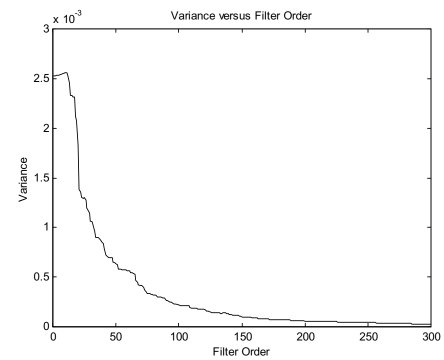

Figure 3(a). Variance versus filter order for the adaptive parameterization of the secondary path

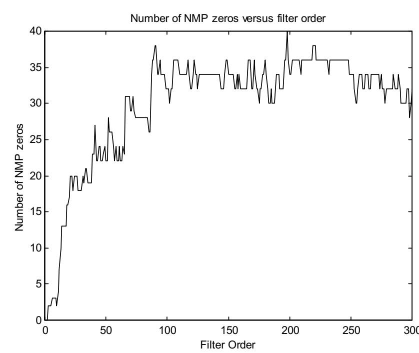

Figure 3(b). Number of non-minimum-phase zeros versus the filter order of the secondary path model

Initially, the H_∞ controller was designed to give *nominal* disturbance rejection for a frequency region about 230Hz, using an FIR plant model with 32 filter taps. However, it can be observed from figure 3(a) that this value of filter length does not meet the specified variance requirements. On the other hand, it was found that, due to numerical limitations, a candidate controller is difficult to find for a plant model whose filter length is greater than 32. Hence, this makes a low order and consequently highly uncertain plant model obligatory. Figure 4 depicts a plot of $|W_1 S|$ versus frequency for the nominal plant. It can be seen that $\|W_1 S\|_\infty = 0.73$ satisfies the constraint outlined in (2), thus achieving nominal performance. The sensitivity plot depicted in figure 5 shows that at least 20dB cancellation is achievable for a 6Hz band about 230Hz. Moreover, it may be noted that further sensitivity improvement occurs at other frequency bands. However, at the remaining frequencies, disturbance amplification occurs. The richness of nulls in the sensitivity function can be attributed to the many non-minimum-phase zeros. Combining this observation with the waterbed effect accounts for the excessive amplification of the disturbance at the other frequencies. This exemplifies the difficulty in shaping the sensitivity function.

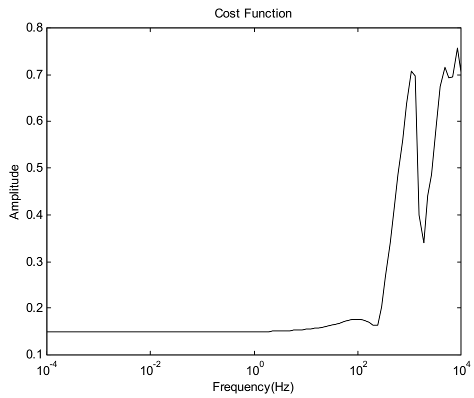

Figure 4. Plot of $|W_1 S|$ versus frequency

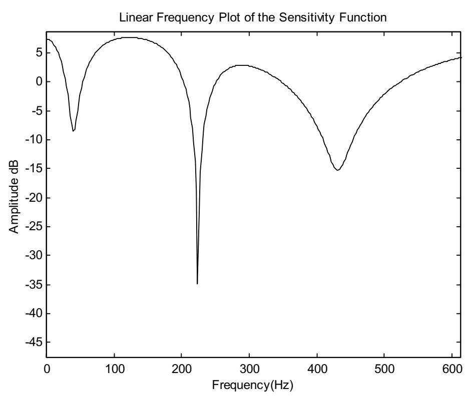

Figure 5. Sensitivity function, i.e. disturbance rejection capabilities of the closed loop system with a single constraint

In acoustic applications, controllers designed purely for nominal plants are generally not used. This is due to the fact that the candidate plant models generally have a degree of uncertainty associated with them. Moreover, the plant dynamics tend to be time-variant. Thus, to achieve robust performance an extra loopshaping constraint, W_2, is required. Figure 6 illustrates bode plots of $1/W_1$ and W_2, which are designed to give maximum disturbance rejection at a frequency band about 230Hz, in the presence of high frequency uncertainty. An additional advantage of this robust approach is that an accurate nominal plant model is no longer obligatory, provided the actual plant is within the uncertainty bound of the nominal plant. Therefore, a nominal FIR plant model of a considerably small number of taps can be used to design a suitable controller for acceptable disturbance rejection. This alleviates the complexity of shaping the sensitivity function. These calculations were performed using the MATLAB *dhinf* command [13].

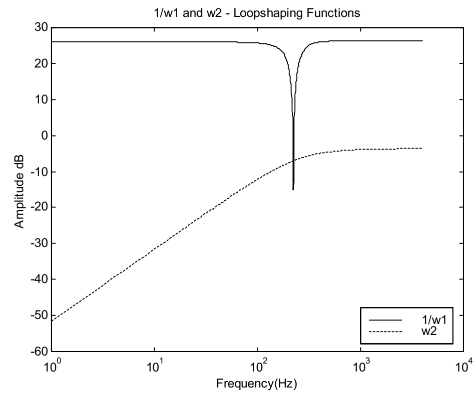

Figure 6. Plot of $1/W_1$ and W_2 loopshaping constraints

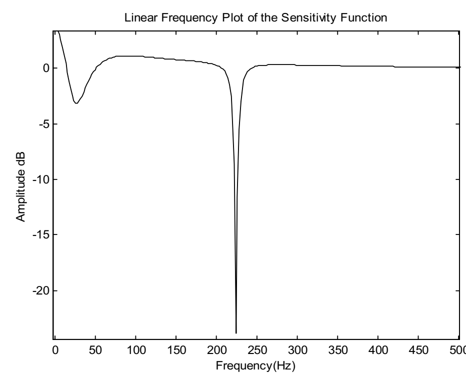

Figure 7. Sensitivity function, i.e. disturbance rejection capabilities of the nominal closed loop system with both constraints

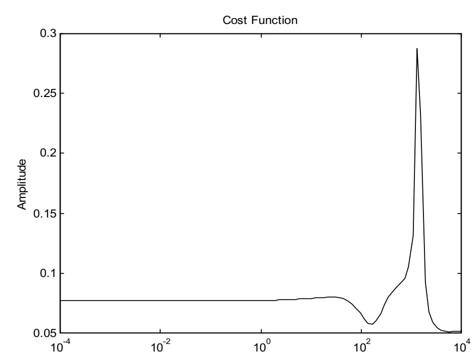

Figure 8. Plot of $|W_1 S| + |W_2 T|$ versus frequency for the nominal plant

Figure 7 depicts the disturbance rejection properties of the optimally designed closed loop system. A comparison of figure 5 with figure 7 indicates that the introduction of the robustness constraint causes a slight degradation of the cancellation capabilities within the 230Hz region. It may be observed that the rejection bandwidth is now reduced from 6Hz to 2Hz, with the sensitivity at the other frequencies not varying significantly from 0dB. Figure 8 illustrates a plot of the cost function $|W_1 S| + |W_2 T|$ versus frequency. It is obvious from this plot that the constraint outlined in (3) is met, with $\| |W_1 S| + |W_2 T| \|_\infty = 0.29$.

5. RESULTS FOR THE PROTOTYPE SYSTEM

With the design of the robust controller completed, closed loop control was implemented on the acoustic duct for an input disturbance frequency of 230Hz. It may be seen from figure 9 that a 23dB reduction of this tone was achieved. It was also empirically verified that noise attenuation of greater than 20dB now occurred over the same 2Hz band.

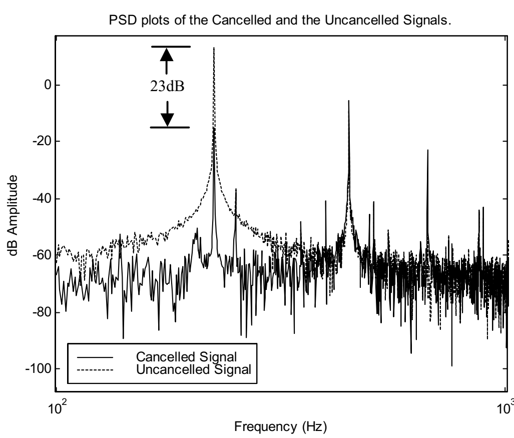

Figure 9. Cancellation of 230Hz tonal disturbance

6. CONCLUSIONS

A H_∞ robust controller was successfully applied to a one-dimensional acoustic plant in the presence of model uncertainty. The performance achieved, when compared to the more conventional adaptive solutions [6,7] proved inferior. Besides the absence of the feedforward feature, the relatively poor performance of the robust controller was attributed to the high level of non-minimum-phase zeros typical of acoustic plants.

For a SISO system, the robust strategy, in comparison to the adaptive approach, requires a more involved initial design phase, but with less processing requirements thereafter. It also requires one sensor less and is accordingly more suited to short ducts where modest cancellation is acceptable. Adaptive control is better suited to longer ducts where a higher level of cancellation is obligatory.

Due to the inherent complexity of acoustic dynamics, high order complex models would be required to ensure satisfactory performance of conventional active noise controllers. The use of H_∞ robust control offers a systematic approach to designing optimal controllers for lower orders and consequently less complex nominal plant models.

7. REFERENCES

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