ROBUST ITERATIVE SYNCHRONIZATION ALGORITHM USING CALABRO WOLF PERFECT ARRAYS

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ABSTRACT

Next generation wireless communication systems are characterized by high bit-rate services. UMTS systems, already subject to standardization process, require a strict synchronization also for the uplink (mobile-to-base) channel. The operation is not trivial due to the fact that during the acquisition process, non synchronous users can impact on the ongoing transmissions reducing the overall systems efficiency. The following proposal describes a different approach to the synchronization problem which is based on a dedicated channel, shared by more users with a OFDM-CDMA transmission technique. The paper focuses on the adaptative algorithm used by the base station to track the users into the synchronization dedicated channel. The results obtained by computer simulations explore the dependence of the performances from the many parameters involved.

1. INTRODUCTION

Most of IMT-2000 [1, 2] proposals rely on a synchronous uplink to increase the efficiency of the communication systems. The timing of the signals from different users must be the same within a few microseconds at the receiving base station. The procedure of synchronization employs a closed loop mechanism where the base station communicates time advance commands to the remote users until a perfect synch is obtained. The commands, inserted in the downlink frames, will delay or anticipate the users timing base of an amount equal to the timing error measured by the base station. This can be done with a single command if the delay is completely measured by the BS, or with intermediate steps allowing a reduced processing power at the BS at the expense of a longer acquisition delay. Until a synchronization is obtained, the unruled users are harmful to the transmissions on the same channel. Two solutions proposed: to employ a dedicated uplink syncrhonization channel for each new user or to allow unruled signals to be present in the traffic channels [11]. In the former case, a considerable amount of bandwidth is employed for sync, while in the latter an unavoidable degradation of the performances has to be expected. We propose a synchronization scheme that, based on OFDM-CDMA [4, 3] technology described so far, will allow several users to share the same dedicated sync channel. Thanks to some characteristics of the signal employed, a fast, two-stage acquisition process has been developed and is described in the algorithm section.

2. A MULTIPLE ACCESS SCHEME FOR SYNCHRONIZATION USING CALABRO-WOLF ARRAYS

The proposed multiple access scheme is the result of the combination of classical DS-CDMA and the OFDM technique. In the proposed scheme, each user is given a unique **synchronization key** and is allowed to access a common synchronization channel along with other users.

The **synchronization key** is constituted by a Q x P Calabro-Wolf perfect array [8, 5]. It is useful to recall how a Calabro-Wolf array is defined:

$$A(x, y) = e^{(j\frac{2\pi}{N}xy)} \qquad (1)$$

with $0 \le x < N - 1$ and $0 \le y < N - 1$. So the array is $N \times N$ where N is also the alphabet size.

The synchronization signal for the generic i-th user, can be expressed by:

$$C_i(t) = \sqrt{\frac{2}{T_c}} \sum_{k=0}^{P-1} \sum_{l=0}^{Q-1} A_{l,k}^{(i)} g(t - lT_c) e^{j2\pi kt/T_c} \qquad (2)$$

$$0 \le t < T_f$$

where T_f is the frame duration, $T_c = T_f/Q$ is the chip interval and $A_{l,k}^{(i)}$ is the synchronization key for the i-th user. The pulse $g(t)$ limits the bandwidth occupancy of the signal. In the proposal we considered a rectangular pulse ($g(t) = rect(\frac{t - T_c/2}{T_c})$).

The elements associated to the same row of $A_{l,k}^{(i)}$ refer to the same chip interval while those associated with the same column are transmitted on the same sub-carrier. Equation (3) is the baseband complex representation of the i-th user synchronization signal.

A new synchronization signal, for a new user, is generated by the array $B_{u,v}$ which is obtained by the cyclic rotation of the columns of $A_{l,k}$.

Formally $B_{u,v} = A_{l,k+n}$ where n is an integer and the indexes are taken respectively modulo P and Q, with P x Q being the size of the original array.

The synchronization signal is periodically repeated every T_f seconds. The base station (BS) will perform a correlation operation in order to produce the needed metric for the timing search.

The shifted correlation for a non synchronized user affected by timing error of τ is $\frac{1}{2}\int_0^{T_f} C_i(t)C_i^*(t+\tau)dt$.

When τ is a multiple of the chip time, $\tau = mT_c$ we obtain:

$$\frac{1}{2}\int_0^{T_f} C_i(t)C_i^*(t+mT_c)\,dt = \Phi_{A^{(i)},A^{(i)}}(m,0) \qquad (3)$$

where Φ is the 2D auto-correlation function of the array A(x,y). If the synchronization key is a perfect array (i.e. has out of phase auto-correlation equal to zero), the shifted auto-correlation function will have *nodes*, the zero valued points, corresponding to delays which are multiples of the chip time.

From this property comes the idea that the BTS could track the users processing a parallel research of these nodes, thus reducing acquisition time and greatly improving accuracy of the phase acquisition. In our proposal we consider square Calabro - Wolf arrays, where $Q = P = N$.

3. ITERATIVE SYNCRHONIZATION ALGORITHM

3.1. Fundamentals

In the proposed timing acquisition algorithms, a set of envelopes are numerically generated, obtained through the OFDM-CDMA technique based on the use of Calabro-Wolf arrays. Different cyclic shifts of the same array correspond to different envelopes, and consequently different users [7, 9, 6]. One of these will be the *desired* user, that is the one we want to synchronize; the other users are also looking for synchronization, but we assume, without loss of generality, that their timing acquisition will occur later than the desired user's one. In this way their spreading signals are treated like interference on the signal from the desired user. Each signal goes through the channel with a random phase unknown to the receiver. It is assumed a zero mean type AWGN noise. If $n(t)$ is the noise, $C_i(t)$ is the desired user, $C_j(t)$ (with $j = 1, ..G < N$ perfect array size) is the interferer users, the signal received to the other link end, will be:

$$r(t) = C_i(t) + \sum_{j=1}^{G} C_j(t) + n(t) \qquad (4)$$

At the receiver, the correlation is performed between r(t) and $C_i(t)$ as follows:

$$R_{r(t),C_i(t)}(\tau) = \frac{1}{2}\int_0^{T_f} r(t)C_i^*(t+\tau)\,dt \qquad (5)$$

The acquisition steps, assuming N chips per frame, are:

- acquisition of N samples of the cross correlation function in the vector $\bar{v}$. Each element in it depends from an **intra-chip offset** τ_0.

$$\bar{v}(\tau_0) \equiv [R_{r(t),C_i(t)}(\tau_0),$$
$$R_{r(t),C_i(t)}(\tau_0 + T_c),,$$
$$R_{r(t),C_i(t)}(\tau_0 + (N-1)T_c)]$$

The value τ_0 is a time offset such that $0 \leq \tau_0 < T_c$. The offset is also discretized from the time resolution of the DSP. Let S_c be the number of samples per chip; the time resolution is indicated by the parameter $T_{res} = T_c/S_c$. So it is possible to characterize the dependence of the vector $\bar{v}$ from τ_0 with the index m:

$$\bar{v}(m) \equiv [R_{r(t),C_i(t)}(mT_{res}),$$
$$R_{r(t),C_i(t)}(mT_{res} + T_c),$$
$$,R_{r(t),C_i(t)}(mT_{res} + (N-1)T_c)]$$

where m ranges in $\{0,......,S_c - 1\}$;

- computing of the *Euclidean distance* between the N-vector $\bar{v}(m)$ and the elements of the orthogonal canonical basis of $\mathbf{R}^N$.

- identification of the element of the canonical basis with the *minimum* distance from the received vector $\bar{v}(m)$. The canonical basis expresses the cross-correlation values of a synchronized user, in ideal channel conditions, when the intra-chip delay is zero.

To fight the effect of the noise added by the channel the metrics are averaged over several frames, resulting in a batch execution of the metrics computation.

The correlation metrics can be represented by two-dimensional array with the index n cycling through the canonical basis and the index m indicating the intra-chip delay. We indicate with $\hat{R}_{r(t),C_i(t)}(mT_{res} + nT_c)$ the averaged metrics, obtained by the analysis over successive frames (indexed by b):

$$\hat{R}_{r(t),C_i(t)}(mT_{res} + nT_c) =$$
$$\frac{\sum_{b=1}^{B} R_{r(t),C_i(t)}^{(b)}(mT_{res} + nT_c)}{B} \qquad (6)$$

If, from previous steps the search got to an intra-chip delay of $\tilde{m}$, during the next steps, the search will focus on the values for m of $\{(\tilde{m} - D_m), ..., \tilde{m}, ..., (\tilde{m} + D_m)\}$ where D_m is the local search window.

The cited values of intra-chip delay will be averaged by the batch mechanism in (6) and the resulting $2D_m + 1$ vectors $\hat{\bar{v}}(m)$ compared with the space basis. The same process applies to the basis index n, around a value of $\tilde{n}$ for an interval of $2D_n + 1$ values.

A the end of this stage we obtain a local metrics matrix centered on $(\tilde{n}, \tilde{m})$ values. The matrix will contain $(2D_n + 1)$ x $(2D_m + 1)$ values.

Two possible situations can occur at this point:

1. the center value is the local minimum in the correlation matrix,

2. the center value is not.

In the first case, due to the properties of the synchronization signals, the global minimum is searched in a grid of values centered in the local minimum. The deterministic location of the local and global minima are useful for an efficient and time saving search.

In the second occurrence, i.e. when the local minimum is not centered, the algorithm moves toward the measured minimum and a new correlation matrix is computed.
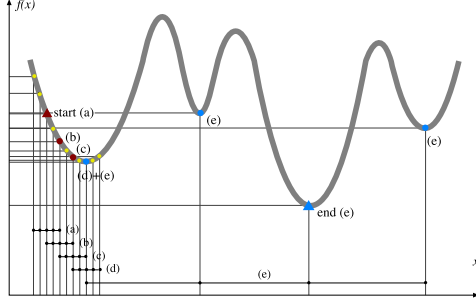
Fig. 1. Mono-dimensional local search
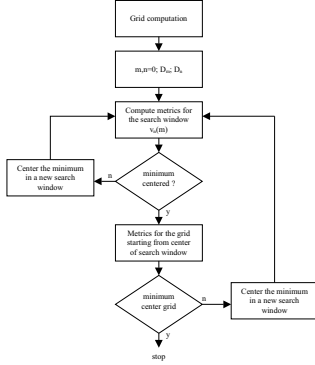
Fig. 2. Local and Grid Search Algorithm

A mono-dimensional representation of the local plus grid search is in figure 1. As an example of minima displacement we consider a Calabro-Wolf array $A(x, y)$ sized $N = 8$, with $S_c = 100$. The desired user is assigned the key $A(x, y + 3)$, i.e. obtained from a shift of value 3 across the columns. The search windows' sizes are $D_n = D_m = 3$.

The grid found from the cross-correlation profiles is (expressed in T_{res} steps):

$$[-360, -330, -260, -180, -160, 0, 160, 180, 260, 330, 360]$$

The overall process of timing search is represented in the flow diagram of figure 2.

It is to note that the grid is also dependent on the local search window. If D_n and D_m are large, the local search approximates the exhaustive search, and the number of global off-window minima is reduced.

4. SIMULATIONS AND PERFORMANCE EVALUATION

4.1. The test-bed

To obtain an evaluation of the benefits in using the OFDM-CDMA technique for synchronization a series of computer simulations have been produced. The system is implemented with $\mathrm{Matlab}^{\copyright}$, and evaluated through the Monte-Carlo method. The following performance indexes have been considered:
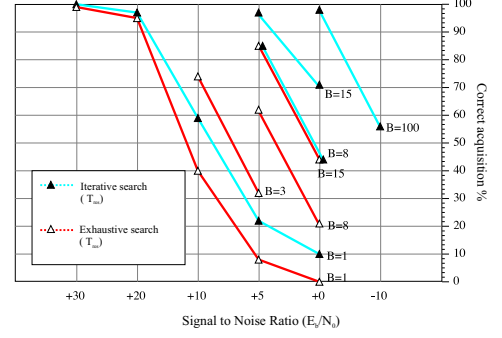
Fig. 3. Correct acquisitions vs. SNR and B; 4 users, $N = 8$, $S_c = 100$

- **percentage of correct acquisition**: i.e. a timing acquisition within the resolution of the considered algorithm (T_{res}).

- **percentage of quasi-correct acquisition**: i.e. a timing acquisition within three times the resolution of the considered algorithm ($3T_{res}$).

- **mean acquisition time**: averaged number of frames to get the timing for the desired user.

- **mean complexity**: mean number of real multiplications to reach the timing estimate. Since the number of multiplications devoted to the the distances is negligible with respect to those for the cross-correlations, only the last ones have been considered.

In the present paper, due to space limitation, only the *correct acquisition* performance index are presented.

4.2. Performance Evaluation

This series of simulations has been conducted with SNR ranging from $+30$ dB to -10 dB. The constant parameters are:

- Synchronization key size: $N = 8$;

- Number of users: 4.

- Local search window: $D_m = D_n = 3$;

- Minima distances grid: $D_G = 13$. It represents the location of the minima for the selected key. It is computed a priori and is known for each synchronization key.

$$([\pm 270, \pm 255, \pm 170, \pm 155, \pm 85, \pm 15, 0]);$$

- Time resolution : $S_c = 100$ samples/chip.

The trend in figure 4 and figure 3 shows the dependence of the performance from the quality of AWGN channel, with a fixed batch of iterations.

To combat the degradation induced by noise, the **batch** parameter is relevant. If B, length of the batch for the averaging window in frames, increases, the number of correct estimates increases as well. This can be appreciated in the figures 5 and 6.

Comparing the two proposed approaches, the exhaustive and the iterative ones, we note that the iterative algorithm is more accurate. In particular, as shown in figure 3, for $E_b/N_0 = 5$ and for
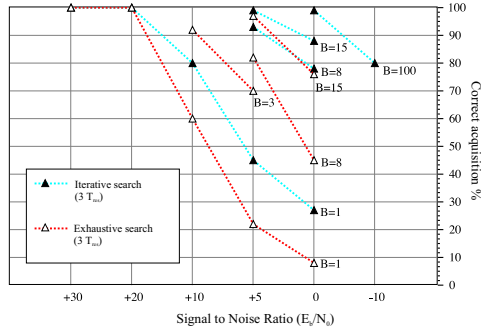
Fig. 4. Correct acquisitions vs. E_b/N_0, for different values of the batch dimension B; $N = 8$, 4 users, $S_c = 100$
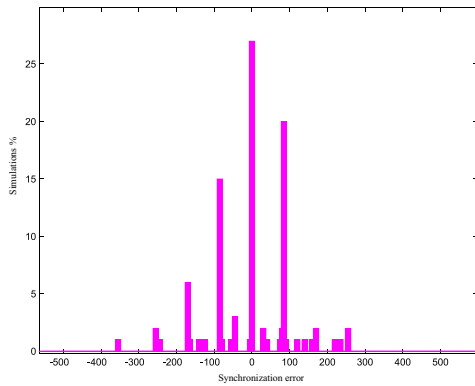
Fig. 5. Iterative Search:correct acquisition histogram; $N = 8$, SNR=0 dB, **batch of 1** ($B = 1$), 4 users ,$S_c = 100$, $D_n = D_m = 3$, $D_G = 13$
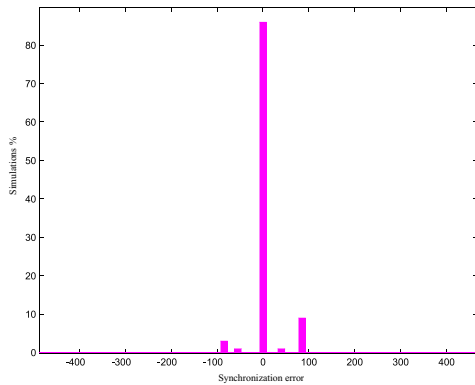
Fig. 6. Iterative Search:correct acquisition histogram; $N = 8$, SNR=0 dB, **batch of 15** ($B = 15$), 4 users ,$S_c = 100$, $D_n = D_m = 3$, $D_G = 13$

$E_b/N_0 = 0$ dB, the iterative algorithm obtains, with $B = 8$, the same performances that the exhaustive one obtains with $B = 15$. The exhaustive algorithm reach faster the estimate, with a lower number of operations while the iterative one distributes a slightly higher computational complexity over a longer time.

5. CONCLUDING REMARKS

The paper investigates a novel approach to the synchronization problem for wireless, multi-user communication systems. The opportunity of using an OFDM-CDMA technique for a common synchronization channel is analyzed, and two algorithms are proposed for the implementation of the technique. Detailed computer simulations have been provided to analyze the performances of the proposed synchronization method with respect to accuracy, acquisition time and computational complexity. The considered communication environment is a multi-user AWGN channel; successive works will analyze the method over multipath fading environments.

6. REFERENCES

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