DIVERSITY SIGNAL RECEPTION VIA SOFT DECISION COMBINING

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Abstract

The problem of diversity signal reception using quantized data is considered. This problem arises in situations where multiple receivers are employed over a geographically broad area. We propose a method for the design of the quantizers that are used by the receiver to make soft decisions. This quantizer minimizes the mean-squared error between the quantizer output and the log-likelihood ratio associated with the receiver observation. The soft decisions are then combined at a central location to yield the final decision. Our quantizers are then applied to the coherent detection of a BPSK signal over Rayleigh fading channels. Simulation results show that using coarse quantizers yields close to optimum performance.

1. Introduction

Techniques for exploiting multiple receivers for signal detection have been known for many years and have been successful in many applications, in particular for diversity signal reception in telecommunications. These techniques are essentially analog oriented and have been implemented on digital platforms via the use of high rate quantizers, i.e. A/D converters, so that the quantization error will have little impact on the overall signal detection performance. In recent years, there has been increasing interest in using low rate quantizers. A potential benefit is reduction in data volume that may help speed up the signal detection process. This can be critical in high data rate applications. For a more important reason, low data rate quantizers are desirable in applications in which multiple receivers are geographically separated and connected to a data network that has limited bandwidth. For example, in a cellular wireless network, multiple base stations may communicate with a common mobile user during handover. In such applications, data obtained at different receivers must be efficiently quantized before they are transmitted over the data network to a combining device. The quantizers employed at the receivers should be designed to best preserve the discriminating power of the original data while using minimum number of bits.

Hard decision combining has been employed in several communication applications to compensate fading channels [1][2][3][4][5]. It has been shown that using hard decision combining can reduce bit-error-rate (BER). In [6][7], attempts have been made to use multi-bit soft decisions to compensate Rayleigh fading channels and to combat near-far effect in cellular wireless communications. In these works, the soft decisions were obtained via a numerical method. A quantizer design methodology is still lacking.

In this paper, we propose a quantizer based on distributed detection theory. It is a well-known result that the optimum quantizers employed at the receivers are likelihood ratio quantizers [8]. We take the point of view that the discriminating power associated with receiver observations is carried by likelihood ratios. From this point of view, we adopt the criterion that the receiver quantizers should best preserve the fidelity of likelihood ratios. Based on this criterion, we develop a quantizer that minimizes the mean squared error of the log likelihood ratio at the quantizer output. This quantizer is referred to minimum mean-squared error log-likelihood ratio quantizer (MMSE-LLRQ).

In Section 2, we will obtain a set of equations that the parameters of this quantizer must satisfy. We will then propose a Lloyd type of algorithm that yields MMSE-LLRQs. In Section 3, MMSE-LLRQs are employed for BPSK signal reception through Rayleigh fading channels. Some concluding remarks are made in Section 4.

2. MMSE Log-Likelihood Ratio Quantizer

Let us consider a network of N receivers. The network is used to observe and decide on an unknown hypothesis which may be either H_0 or H_1. Let $\mathbf{x}_n$ denote the observation obtained by the nth sensor. Given the unknown hypothesis H_i, $\mathbf{x}_n$ follows probability density function $p_i(\mathbf{x}_n)$, $i = 0,1$. The receiver observations are corrupted by statistically independent noises. Hence, we obtain $p_i(\mathbf{x}_1,\cdots,\mathbf{x}_N) = p_i(\mathbf{x}_1)\cdots p_i(\mathbf{x}_N)$.

Let Q_n denote the quantizer that is employed at the nth sensor. Let τ_n denote the log-likelihood ratio $log\frac{p_1(\mathbf{x}_n)}{p_0(\mathbf{x}_n)}$. τ_n is the input to Q_n. Let D_n denote the number of quantization levels. The D_n output levels are l_{n,u_n}, namely $l_{n,1}$, … and l_{n,D_n}. Let u_n denote index of the output of Q_n. We recall that the optimal quantizer Q_n is a likelihood ratio quantizer that has the following structure

$$u_n = d \text{ if } t_{n,d-1} \le \tau_n < t_{n,d}\ ,\ 1 \le d \le D_n\ ,$$

where $-\infty = t_{n,0} < \cdots < t_{n,d} < \cdots < t_{n,D_n} = +\infty$ are the thresholds. Let R_n denote the rate of quantizer Q_n, where $R_n = \log_2 D_n$. These quantizer parameters are computed off line and stored and stored at the decision-combining device.

The receivers send the indices u_n, $n = 1,\cdots,N$, to the decision-combining device. Based upon received indices, the decision-combining device retrieves the corresponding quantizer output levels and decides the identity of the unknown hypothesis using the following decision rule

$$l_{1,u_1} + \cdots + l_{N,u_N} \underset{0}{\overset{1}{\gtrless}} log\frac{\pi_0}{\pi_1} = \lambda \tag{1}$$

where π_0 and π_1 are prior probabilities of H_0 and H_1 respectively.

We define the mean squared error associated with Q_n as

$$\varepsilon_n = \pi_0 E_0\left(l_{n,u_n} - \tau_n\right)^2 + \pi_1 E_1\left(l_{n,u_n} - \tau_n\right)^2 . \tag{2}$$

where E_i stands for average when the unknown hypothesis is H_i, $i = 0,1$.

Now we formulate the problem of designing quantizer Q_n for a fixed number of levels D_n as

$$\left\{t_{n,0}, \cdots, t_{n,D_n}, l_{n,1}, \cdots, l_{n,D_n}\right\}_{optimal} = \arg \min_{\substack{t_{n,0},\cdots,t_{n,D_n} \\ l_{n,1},\cdots,l_{n,D_n}}} \varepsilon_n , \tag{3}$$

where the left side is the set of optimal parameters. The resulting quantizer is referred to as the minimum mean-squared error log-likelihood ratio quantizer (MMSE-LLRQ). This quantizer is expected to preserve the discriminating power of receiver observations, by approximating the log-likelihood ratio under the MMSE criterion.

Since Q_n works in the log-likelihood ratio space as shown in (2), we will derive its structure in the log-likelihood ratio space and then transform back into the observation space. The mean-squared-error ε_n in (2) is expressed as

$$\varepsilon_n = \sum_{d=1}^{D_n} \int_{t_{n,d-1}}^{t_{n,d}} \left[\pi_0 p_0(\tau_n) + \pi_1 p_1(\tau_n)\right]\left(l_{n,d} - \tau_n\right)^2 d\tau_n . \tag{4}$$

Taking partial derivative of ε_n with respect to each quantizer parameter and set it to zero, we obtain a set of necessary conditions that the optimum quantizer Q_n must satisfy. This result is stated in the following theorem.

The optimal quantizer that minimizes the ε_n must satisfy the following necessary conditions

Theorem 1: The optimal quantizer Q_n that minimizes the ε_n must satisfy the following necessary conditions

$$t_{n,d} = \frac{l_{n,d} + l_{n,d+1}}{2}, \; d = 1, \cdots, D_n - 1 , \tag{5}$$

$$l_{n,d} = \frac{\int_{t_{n,d-1}}^{t_{n,d}} \tau\left(\pi_0 p_0(\tau_n) + \pi_1 p_1(\tau_n)\right) d\tau_n}{\int_{t_{n,d-1}}^{t_{n,d}} \left(\pi_0 p_0(\tau_n) + \pi_1 p_1(\tau_n)\right) d\tau_n}, \; d = 1, \cdots, D_n . \tag{6}$$

This result is similar to the well-known Lloyd's result. The optimal thresholds are the midpoints between consecutive optimal quantized output log-likelihood ratios. The optimal quantized output log-likelihood ratios are the local means of the actual log-likelihood ratios. It is worth pointing out that if the sensor observation $\mathbf{x}_n$ is a vector, then Q_n is a vector quantizer in the sensor observation space, even though it is a scalar quantizer in the log-likelihood ratio space.

Putting (5) and (6) into (2), we obtain the value of the minimum mean-squared error

$$\varepsilon_{n,min} == \left[\pi_0 E_0\left(\tau_n^2\right) + \pi_1 E_1\left(\tau_n^2\right)\right] - \left[\pi_0 E_0\left(l_{n,u_n}^2\right) + \pi_1 E_1\left(l_{n,u_n}^2\right)\right]. \tag{7}$$

A byproduct in the derivation of (7) is

$$\pi_0 E_0\left[l_{n,u_n}\left(l_{n,u_n} - \tau_n\right)\right] + \pi_1 E_1\left[l_{n,u_n}\left(l_{n,u_n} - \tau_n\right)\right] = 0 . \tag{8}$$

Equation (8) states that the quantization error $\left(l_{n,u_n} - \tau_n\right)$ is uncorrelated with the quantizer output l_{n,u_n}.

Next, we propose a Lloyd type of algorithm that cyclically optimizes the parameters of the MMSE-LLRQs. For notation simplicity, we will drop subscript *n* in this section as long as no confusion arises.

Our algorithm uses the following equations to iteratively compute the parameters

$$t_d^{k+1} = \frac{l_d^k + l_{d+1}^k}{2} , \tag{9}$$

$$l_d^{k+1} = \frac{\int_{t_{d-1}^k}^{t_d^k} \tau\left(\pi_0 p_0(\tau) + \pi_1 p_1(\tau)\right) d\tau}{\int_{t_{d-1}^k}^{t_d^k} \left(\pi_0 p_0(\tau) + \pi_1 p_1(\tau)\right) d\tau} , \tag{10}$$

where *k* is the step number. The operation of the algorithm is described as follows

1. Choose the tolerance of stopping criterion σ. Set $k = 1$. Arbitrarily choose distinct t_1^k, … and t_{D-1}^k. Compute l_1^k, … and l_D^k using (10). Compute ε^k using (2).
2. Compute t_1^{k+1}, … and t_{D-1}^{k+1} using (9).
3. Compute l_1^{k+1}, … and l_D^{k+1} using (10).
4. Compute ε^{k+1} using (2). If the difference between ε^{k+1} and ε^k is less than σ, the algorithm terminates. Otherwise, set $k = k + 1$ and then go to step 2.

Our algorithm yields a sequence of quantizers that converge to a local optimum. The convergence of our algorithm is guaranteed. However, in general, the solution may not be unique and may be different for different initial conditions.

3. Numerical Results

In this section, we consider the problem of coherent detection of a BPSK signal over Rayleigh fading channels. We will consider two cases in which the channels information is or is not available. For notation simplicity, we will drop the subscript n.

Let us consider the transmission of a BPSK signal through a fading Rayleigh channel with additive white Gaussian noise (AWGN). The output x of the receiver is

$$x = br + z ,$$

where r is the amplitude of the received the signal that follows Rayleigh distribution, z is the additive white Gaussian noise with double side spectral density $\frac{N_0}{2}$ and b is the transmitted information bit, where $b \in \{-1, +1\}$.

Let E_b denote the average energy of the received signal. With x normalized by $\sqrt{\frac{N_0}{2}}$, r follows $p(r)=\frac{r}{\bar{\gamma}}\exp\left(-\frac{r^2}{2\bar{\gamma}}\right)$, where $\bar{\gamma}=\frac{E_b}{N_0}$ is the average signal to noise ratio (SNR). Given b and r, x follows $p_b(x|r)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x-br)^2}{2}\right)$.

Case 1: channel information available

In this case, the value of r is accurately estimated during the signaling interval. The log-likelihood ratio τ associated with this signaling interval is $\tau=2rx$. The cumulative distribution function of τ given the transmitted bit b is

$$F_b(\tau)=\text{Prob}(2rx\le\tau|b)=\int_0^{+\infty}\frac{r}{\bar{\gamma}_b}e^{-\frac{r^2}{2\bar{\gamma}_b}}dr\int_{-\infty}^{\frac{\tau}{2r}}\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-br)^2}{2}}dx\,. \quad (11)$$

Taking derivative of $F_b(\tau)$ with respect to τ, we obtained the density function of the corresponding probability density function of τ as

$$p_b(\tau)=\begin{cases}\frac{\beta^2-1}{4\beta}e^{-\frac{\tau}{2}(\beta-b)} & \tau\ge 0\\ \frac{\beta^2-1}{4\beta}e^{\frac{\tau}{2}(\beta+b)} & \tau<0\end{cases}. \quad (12)$$

where $\beta=\sqrt{\frac{1+\bar{\gamma}_b}{\bar{\gamma}_b}}$.

In turn, the cumulative distribution function of τ is

$$F_b(\tau)=\begin{cases}\frac{1}{2}\left(1-\frac{b}{\beta}\right)+\frac{1}{2}\left(1+\frac{b}{\beta}\right)\left(1-e^{-\frac{\tau}{2}(\beta-b)}\right) & \tau\ge 0\\ \frac{1}{2}\left(1-\frac{b}{\beta}\right)e^{\frac{\tau}{2}(\beta+b)} & \tau<0\end{cases}. \quad (13)$$

Equations (12) and (13) are used in our MMSE-LLRQ design algorithm to obtain the optimum quantizers.

Now, we consider a system of two receivers. The average signal-to-noise ratio γ is assumed to be the same for all receivers. Identical MMSE-LLRQs are employed at all the receivers.

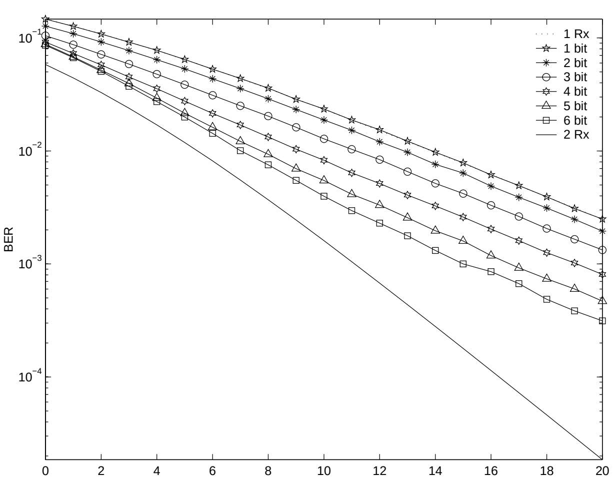

Figure 1: BER vs. γ for Case 1.

In Figure 1, BER is plotted against γ for different values of R. The BER curve for the one receiver system (1 Rx) is plotted as an upper bound. The BER curve for the two-receiver system (2 Rx) that employs combining of original receiver likelihood ratios is also plotted as the lower bound. It is observed that using multi-bit soft decision combining reduces BER. We notice that there is a significant gap between the lower bound and the BER when quantizers are used. A possible explanation is given below. Most of the errors are attributed to the occurrence of deep fade in both channels. In such situations, the receiver log-likelihood ratios are very small thus are not well approximated by the quantizer outputs. Hence, using quantizers in deep fade situations significant degrade system performance. This accounts for the different between the BER curve when quantizers are employed and the lower bound.

Case 2: channel information unavailable

In this case, it is assumed that the value of the instantaneous value of r is not available, but the average signal-to-noise ratio $\bar{\gamma}$ is known. The log-likelihood ratio τ must be estimated over the probability distribution of r. Given the transmitted bit b, x follows probability density function

$$p_b(x)=E_r\left[\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-br)^2}{2}}\right]=\int_0^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-br)^2}{2}}\frac{r}{\bar{\gamma}_b}e^{-\frac{r^2}{2\bar{\gamma}_b}}dr$$

Using $\beta=\sqrt{\frac{1+\bar{\gamma}_b}{\bar{\gamma}_b}}$ and $b=\pm 1$, we obtain

$$p_b(x)=\frac{1-\frac{1}{\beta^2}}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}+bx\frac{\beta^2-1}{2\beta^3}e^{-\frac{x^2}{2}\left(1-\frac{1}{\beta^2}\right)}erfc\left(-\frac{bx}{\sqrt{2}\beta}\right). \quad (14)$$

where $erfc(z)$ is the complementary error function.

With routine calculus manipulation, we obtain the cumulative probability function of x

$$F_b(x)=\frac{1}{2}erfc\left(-\frac{x}{\sqrt{2}}\right)-\frac{b}{2\beta}e^{-\frac{x^2}{2}\left(1-\frac{1}{\beta^2}\right)}erfc\left(-\frac{bx}{\sqrt{2}\beta}\right). \quad (15)$$

Define $y=x\sqrt{\frac{\beta}{2}}$. Putting y into (3.14) and (3.15), we obtain the log-likelihood ratio

$$\tau=\log\frac{p_1(x)}{p_{-1}(x)}=\log\frac{e^{-y^2}+\sqrt{\pi}\,yerfc(-y)}{e^{-y^2}-\sqrt{\pi}\,yerfc(y)}\,. \quad (16)$$

Equations (14), (15) and (16) are used in our MMSE-LLRQ design algorithm to obtain the optimum quantizers.

Next, we will consider a system of two receivers. The average signal-to-noise ratio γ is again assumed to be the same for all receivers. Identical MMSE-LLRQs are employed at all the receivers.

In Figure 2, BER is plotted against γ for different values of R. Similar to Case 1, the BER curve for two-receiver systems is also plotted as the lower bound. It is observed that using 4 or 5 bits gives close to optimum performance. In this case, we do not observe a considerable gap between the lower bound and the BER curves when high rate quantizers are used.

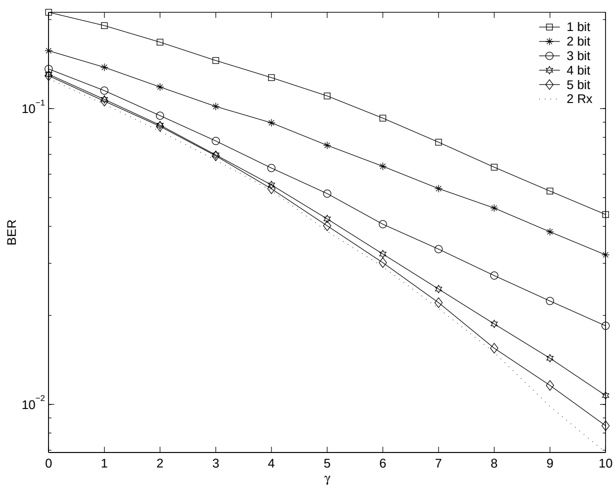

Figure 2: Probability of error vs. γ for Case 2.

4. Summary

In this paper, we considered the design of quantizers that are employed in multi-receiver system for signal reception through fading channels. We used log-likelihood ratios as measures of the discriminating power of receiver observations. We adopted the criterion that the receiver quantizers should minimize the mean-squared error while quantizing log-likelihood ratios. Using this performance criterion, we developed a generalized Lloyd algorithm for designing the quantizers. These quantizers were employed in a two-receiver system that detects a BPSK signal though Rayleigh fading channel. The numerical results showed that using coarse quantizers could yield close to optimum performance.

5. REFERENCES

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