BLIND MULTIUSER DETECTION FOR SPACE-TIME CODED CDMA SYSTEMS

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ABSTRACT

We propose here a linear blind multiuser detector, referred to as the Capon receiver, for space-time coded CDMA (code division multiple access) systems. The proposed Capon receiver is designed to pass desired signals without distortion, meanwhile minimizing the overall interference signals, irrespective of their origins. It is shown that the Capon receiver achieves similar performance in terms of the output SINR (signal-to-interference-and-noise ratio) to that of the optimum, linear, training-assisted MMSE (minimum mean squared error) receiver over a wide range of the input SNR (signal-to-noise ratio). It is also shown that the ratio of the output SINR of the Capon receiver to that of the MMSE receiver converges to a constant very close to one at high SNRs. The Capon receiver also yields a blind channel estimate which converges to the true channel parameters (within a scalar ambiguity) as the SNR increases. Simulation results are provided to support our study.

1. INTRODUCTION

Future wireless mobile networks are envisioned to provide capacities and transmission rates by orders of magnitude higher than state-of-the-art systems [1]. However, due to the inherent difficulties of the wireless transmission medium, characterized by fading, multipath and interference, reliable high-speed transmission over the wireless channel in mobile radio networks is significantly more challenging than in wired networks.

Space-time coding (STC) has been fueling significant interest recently. Making use of multiple transmit antennas, space-time coding provides an effective way to exploit spatial and temporal diversity, and is capable of drastically increase the transmission rate. A number of STC schemes have been proposed (e.g., [2], [3], [4]). Most of previous studies on STC assumed that the receiver has perfect knowledge of the channel state information (CSI). In practice, the receiver has to estimate the CSI by training or blind methods. As multi-channel state information is required for coherent decoding in multiple-antenna systems, channel estimation in the current case is much more difficult than in single-antenna systems [5]. Additionally, when the CSI is unknown (or imperfectly known) to the receiver, STC aggravates the problem of interference [5], [6], since in addition to the standard radio interference encountered in a single-antenna system, the transmitted signals (for the same mobile user) from different antennas are mixed spatially when they arrive at the receiver. They interfere with each other's detection, acting as a sort of *self-interference*.

In this paper, we are interested in blind receivers, which do not require channel estimation and yet are robust to interference, for space-time coded CDMA systems. In particular, we present a blind linear receiver that passes the signals from the desired user undistorted, while minimizing the receiver output power, so that the overall interference, including the multiuser interference (MUI), is suppressed. The receiver is henceforth referred to as the Capon receiver since it resembles a technique used by Capon in 1960s for spectral estimation [7]. The proposed Capon receiver is compared with the training-assisted linear MMSE receiver, the optimum in the class of linear receivers, in terms of the output SINR. It is shown that the output SINR of the Capon receiver is close to that of the MMSE receiver over a wide range of the input SNR. Furthermore, the ratio of the output SINR of the Capon receiver to that of the MMSE receiver converges to a constant very close to one at high SNRs. The Capon receiver also obtains a blind channel estimate. It is shown that the Capon channel estimate converges to the true channel parameters (within a scalar) as the SNR increases.

2. SYSTEM MODEL

We consider the Alamouti's space-time coding scheme [2] involving two transmit antennas and achieving a full-diversity gain based on linear processing. For this coding scheme, during a given symbol interval, b_1 and b_2 drawn from some constellation are transmitted from the first and second antenna, respectively; during the next symbol interval, $-b_2^*$ and b_1^* are transmitted from the two antennas in the same order. Here, $(\cdot)^*$ denotes complex conjugate.

We herein assume a K-user CDMA system that is synchronous (downlink) and experiences flat-fading. Extension to frequency selective fading is being investigated and will be reported elsewhere. Consider the case where the system is employed with $M = 2$ transmit antennas and $N(N \geq 1)$ receive antennas, and a distinct spreading code is assigned to each user signal from each transmit antenna. After down-converting to baseband and chip-matched filtering, the output of the chip-matched filter is sampled at the chip rate. We then collect $J \times 1$ vectors, where J denotes the spreading gain, for processing. We restrict in this paper the spreading codes and data symbols to be real-valued. Extension to the case of complex constellation is straightforward [8]. The received vectors corresponding to the n-th receive antenna over two consecutive symbol periods can be expressed as:

$$\begin{aligned}
\mathbf{x}_n(2t-1) = &\sum_{k=1}^{K} \sqrt{\rho_k}\, [h_{1n}\mathbf{c}_{1k} b_k(2t-1) \\
&+ h_{2n}\mathbf{c}_{2k} b_k(2t)] + \mathbf{w}_n(2t-1), \\
\mathbf{x}_n(2t) = &\sum_{k=1}^{K} \sqrt{\rho_k}\, [-h_{1n}\mathbf{c}_{1k} b_k(2t) \\
&+ h_{2n}\mathbf{c}_{2k} b_k(2t-1)] + \mathbf{w}_n(2t),
\end{aligned}$$

where

- ρ_k received signal power per transmit per receive antenna for user k;
- h_{mn} channel coefficient from m-th transmit antenna to n-th receive antenna; modeled as complex (possibly non-Gaussian) random variables with zero mean and unit variance;
- $\mathbf{c}_{mk}$ $J \times 1$ spreading code for signal of user k from transmit antenna m;
- $b_k(t)$ t-th information symbol for user k;
- $\mathbf{w}_n(t)$ $J \times 1$ noise vector sampled during t-th symbol period at receive antenna n; modeled as stationary (possibly colored and non-Gaussian) with zero mean and variance σ_w^2.

Let $\mathbf{y}_n(t) \triangleq [\, \mathbf{x}_n^T(2t-1),\ \mathbf{x}_n^T(2t)\,]^T, \mathbf{v}_n(t) \triangleq [\, \mathbf{w}_n^T(2t-1),\ \mathbf{w}_n^T(2t)\,]^T$, where $(\cdot)^T$ denotes transpose. Then we have

$$\begin{aligned}
\mathbf{y}_n(t) = &\sqrt{\rho_1}\, [\mathbf{g}_{n1} b_1(2t-1) + \bar{\mathbf{g}}_{n1} b_1(2t)] \\
&+ \sum_{k=2}^{K} \sqrt{\rho_k}\, [\mathbf{g}_{nk} b_k(2t-1) + \bar{\mathbf{g}}_{nk} b_k(2t)] + \mathbf{v}_n(t),
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{g}_{nk} &\triangleq \begin{bmatrix} h_{1n}\mathbf{c}_{1k}^T, & h_{2n}\mathbf{c}_{2k}^T \end{bmatrix}^T = \mathbf{D}_k \mathbf{h}_n, \\
\bar{\mathbf{g}}_{nk} &\triangleq \begin{bmatrix} h_{2n}\mathbf{c}_{2k}^T, & -h_{1n}\mathbf{c}_{1k}^T \end{bmatrix}^T = \bar{\mathbf{D}}_k \mathbf{h}_n, \\
\mathbf{D}_k &\triangleq \begin{bmatrix} \mathbf{c}_{1k} & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_{2k} \end{bmatrix}, \bar{\mathbf{D}}_k \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{c}_{2k} \\ -\mathbf{c}_{1k} & \mathbf{0} \end{bmatrix},
\end{aligned}$$

and $\mathbf{h}_n \triangleq [\, h_{1n},\ h_{2n}\,]^T$. Let

$$\begin{aligned}
\mathbf{h} &\triangleq \begin{bmatrix} \mathbf{h}_1^T, & \cdots, & \mathbf{h}_N^T \end{bmatrix}^T, \\
\mathbf{g}_k &\triangleq \begin{bmatrix} \mathbf{g}_{1k}^T, \cdots, \mathbf{g}_{Nk}^T \end{bmatrix}^T = (\mathbf{I}_N \otimes \mathbf{D}_k)\mathbf{h}, \quad (1) \\
\bar{\mathbf{g}}_k &\triangleq \begin{bmatrix} \bar{\mathbf{g}}_{1k}^T, \cdots, \bar{\mathbf{g}}_{Nk}^T \end{bmatrix}^T = (\mathbf{I}_N \otimes \bar{\mathbf{D}}_k)\mathbf{h}, \quad (2)
\end{aligned}$$

where $\otimes$ denotes the Kronecker product. We collect the outputs of all receive antennas and define

$$\begin{aligned}
\mathbf{y}(t) &\triangleq \begin{bmatrix} \mathbf{y}_1^T(t), & \cdots, & \mathbf{y}_N^T(t) \end{bmatrix}^T \\
&= \sqrt{\rho_1}\, [\, \mathbf{g}_1 b_1(2t-1) + \bar{\mathbf{g}}_1 b_1(2t)\,] \quad (3) \\
&\quad + \sum_{k=2}^{K} \sqrt{\rho_k}\, [\, \mathbf{g}_k b_k(2t-1) + \bar{\mathbf{g}}_k b_k(2t)\,] + \mathbf{v}(t),
\end{aligned}$$

where $\mathbf{v}(t) \triangleq [\, \mathbf{v}_1^T(t), \cdots, \mathbf{v}_N^T(t)\,]^T$.

The problem of interest is to detect $\{b_1(2t-1), b_1(2t)\}_{t=0}^{T-1}$ from the received signal $\{\mathbf{y}(t)\}_{t=0}^{T-1}$.

3. MMSE RECEIVER

The MMSE receiver $\mathcal{F}_M \in \mathbb{C}^{2JN \times 2K}$ is obtained by minimizing the MSE criterion:

$$\mathcal{F}_M = \arg \min_{\mathcal{F} \in \mathbb{C}^{2JN \times 2K}} E\{\| \mathbf{b}(t) - \mathcal{F}^H \mathbf{y}(t) \|^2\}, \quad (4)$$

where $E\{\cdot\}$ denotes statistical expectation, $(\cdot)^H$ denotes conjugate transpose, and

$$\begin{aligned}
\mathbf{b}(t) &= \begin{bmatrix} \mathbf{b}_1^T(t), & \cdots, & \mathbf{b}_K^T(t) \end{bmatrix}^T, \\
\mathbf{b}_k(t) &= \begin{bmatrix} b_k(2t-1), & b_k(2t) \end{bmatrix}^T.
\end{aligned}$$

The solution to (4) is standard, given by

$$\mathcal{F}_M = \mathbf{R}_{yy}^{-1} \mathbf{R}_{yb},$$

where

$$\begin{aligned}
\mathbf{R}_{yy} &= E\left\{\mathbf{y}(t)\mathbf{y}^H(t)\right\}, \\
\mathbf{R}_{yb} &= E\left\{\mathbf{y}(t)\mathbf{b}(t)\right\} = \sqrt{\rho_1}\mathbf{G}\mathbf{P}, \\
\mathbf{G} &= \begin{bmatrix} \mathbf{G}_1, & \cdots, & \mathbf{G}_K \end{bmatrix}, \quad \mathbf{G}_k = \begin{bmatrix} \mathbf{g}_k, & \bar{\mathbf{g}}_k \end{bmatrix}, \\
\mathbf{P} &= \operatorname{diag}\left\{ \sqrt{\rho_1}, \ \sqrt{\rho_1}, \ \cdots, \ \sqrt{\rho_K}, \ \sqrt{\rho_K} \right\},
\end{aligned}$$

where we assumed $\mathbf{b}(t)$ and $\mathbf{v}(t)$ are independent of each other and $\mathbf{b}(t)$ is drawn from some unit-energy constellation. If only the first user is of interest, the MMSE receiver reduces to

$$\mathbf{F}_M = \sqrt{\rho_1}\mathbf{R}_{yy}^{-1}\mathbf{G}_1.$$

4. CAPON RECEIVER

The MMSE receiver requires the CSI. In the sequel, we introduce an alternative receiver, referred to as the Capon receiver, which

- does not require training;
- needs only the spreading codes and timing of the desired user but not the others;
- is able to suppress all kinds of interference, irrespective of their origin, that is, MAI, ISI, spatial self-interference, inter-cell interference, narrow-band interference, etc.;

- is computationally efficient and also facilitates adaptive implementation.

The idea is to design a receiver $\mathbf{F}_c$ which minimizes the receiver output power, while passing the desired signal without distortion (unit-gain):

$$\mathbf{F}_c \triangleq [\ \mathbf{f}_c, \quad \bar{\mathbf{f}}_c\ ] = \arg \min_{\mathbf{F} \in \mathbb{C}^{2JN \times 2}} E\left\{ \| \mathbf{F}^H \mathbf{y}(t) \|^2 \right\}$$
$$= \arg \min_{\mathbf{F} \in \mathbb{C}^{2JN \times 2}} \operatorname{tr}\left\{ \mathbf{F}^H \mathbf{R}_{yy} \mathbf{F} \right\},$$
$$\text{subject to} \quad \mathbf{f}^H \mathbf{g}_c = 1 \text{ and } \bar{\mathbf{f}}^H \bar{\mathbf{g}}_c = 1,$$

where $\mathbf{f}$ and $\bar{\mathbf{f}}$, respectively, are the first and second column of the dummy matrix $\mathbf{F}$, while $\mathbf{g}_c$ and $\bar{\mathbf{g}}_c$ are estimates (to be determined next) of the true parameters g and $\bar{\mathbf{g}}$, respectively. Similar criterion has been used to design blind receivers for CDMA systems without SPC [9], [10]. Solving the above constrained minimization problem yields

$$\mathbf{f}_c = \left(\mathbf{g}_c^H \mathbf{R}_{yy}^{-1} \mathbf{g}_c \right)^{-1} \mathbf{R}_{yy}^{-1} \mathbf{g}_c, \tag{5}$$

$$\bar{\mathbf{f}}_c = \left(\bar{\mathbf{g}}_c^H \mathbf{R}_{yy}^{-1} \bar{\mathbf{g}}_c \right)^{-1} \mathbf{R}_{yy}^{-1} \bar{\mathbf{g}}_c, \tag{6}$$

$$V(\mathbf{g}_c, \bar{\mathbf{g}}_c) \triangleq \min_{\mathbf{F} \in \mathbb{C}^{2NJ \times 2}} \operatorname{tr}\left\{ \mathbf{F}^H \mathbf{R}_{yy} \mathbf{F} \right\}$$
$$= (\mathbf{g}_c^H \mathbf{R}_{yy}^{-1} \mathbf{g}_c)^{-1} + (\bar{\mathbf{g}}_c^H \mathbf{R}_{yy}^{-1} \bar{\mathbf{g}}_c)^{-1}.$$

To obtain $\mathbf{g}_c$ and $\bar{\mathbf{g}}_c$, we maximize $V(\mathbf{g}_c, \bar{\mathbf{g}}_c)$ (in order to maximize the signal component at the receiver output) with respect to the unknowns or, equivalently, minimize

$$\mathbf{g}_c^H \mathbf{R}_{yy}^{-1} \mathbf{g}_c + \bar{\mathbf{g}}_c^H \mathbf{R}_{yy}^{-1} \bar{\mathbf{g}}_c. \tag{7}$$

We thus obtain a channel estimate [by replacing $\mathbf{g}_1$ and $\bar{\mathbf{g}}_1$ in (7) by their expressions in (1) and (2)] as given in (8), shown on top of the next page, where $\underline{\mathbf{h}}$ is a dummy vector that is used to distinguish from the true channel vector $\mathbf{h}$. The solution to (8), subject to the standard constraint $\|\mathbf{h}_c\| = 1$, is the eigenvector that corresponds to the smallest eigenvalue of $\mathbf{\Omega}$. To summarize, the Capon receiver consists of the following steps:

1. Estimate the data covariance matrix $\mathbf{R}_{yy}$ from the received data $\{\mathbf{y}(t)\}_{t=0}^{T}$;
2. Compute $\mathbf{h}_c$ as the minimum eigenvector of $\mathbf{\Omega}$;
3. Compute $\mathbf{g}_c$ and $\bar{\mathbf{g}}_c$ using (1) and (2), respectively;
4. Calculate the Capon receiver $\mathbf{F}_c = [\ \mathbf{f}_c,\ \bar{\mathbf{f}}_c\ ]$ using (5) and (6), respectively.

5. PERFORMANCE ANALYSIS

In this section, we present several analytical results regarding the proposed Capon receiver. Proofs of these results are omitted due to space limitation. Interested readers are referred to [8] for details.

A fundamental question of the proposed blind channel estimator is identifiability, i.e., the conditions under which the channel can be uniquely and perfectly (within a scaling factor) recovered when SNR$= \infty$. This is addressed by the following result.

Theorem 1 (Identifiability) *[8]: The channel parameter* $\mathbf{h}$ *is perfectly and uniquely (within a scalar) identifiable at SNR*$= \infty$ *if a) the spreading codes* $\mathbf{c}_{11}$ *and* $\mathbf{c}_{21}$ *for the first user are linearly independent of each other; b)* $\mathbf{c}_{11}$ *and* $\mathbf{c}_{21}$ *are linearly independent of the spreading codes of all other users; and c)* $\mathbf{h}_n = [h_{1n}, h_{2n}]^T \neq \mathbf{0}$ *at least for some* n.

The above conditions are satisfied in almost all practical systems. The next result quantifies the channel estimation error at high but finite SNRs.

Theorem 2 *[8]: Under the identifiability conditions,* $\mathbf{h}_c$ *in (8) satisfies*

$$\mathbf{h}_c = \frac{\mathbf{h}}{\|\mathbf{h}\|} + \epsilon, \quad \text{with} \quad \lim_{\sigma_w^2 \to \infty} \|\epsilon\| = 0.$$

An explicit expression for ϵ at high SNR is given in [8]. Finally, we compare the performance of the MMSE and Capon receiver in terms of the output SINR. To obtain a compact expression for the output SINR, we break the covariance matrix $\mathbf{R}_{yy}$ into three parts:

$$\mathbf{R}_{yy} \triangleq \mathbf{R}_s + \bar{\mathbf{R}}_s + \mathbf{R}_i,$$

where

$$\mathbf{R}_s = \rho_1 \mathbf{g}_1 \mathbf{g}_1^H, \quad \bar{\mathbf{R}}_s = \rho_1 \bar{\mathbf{g}}_1 \bar{\mathbf{g}}_1^H.$$
$$\mathbf{R}_i = \sum_{k=2}^{K} \rho_k \mathbf{G}_k \mathbf{G}_k^H + \sigma_w^2 \mathbf{I}_{2JN}, \tag{9}$$

Note that to facilitate our analysis, we assume here the channel noise is white [cf. (9)] even though the Capon receiver does no impose this restriction. Let $\mathbf{F} \triangleq [\,\mathbf{f},\ \bar{\mathbf{f}}\,] \in \mathbb{C}^{2JN \times 2}$ denote a generic receiver. The output SINR is given by

$$\text{SINR} = \frac{\mathbf{f}^H \mathbf{R}_s \mathbf{f} + \bar{\mathbf{f}}^H \bar{\mathbf{R}}_s \bar{\mathbf{f}}}{\mathbf{f}^H (\mathbf{R}_{yy} - \mathbf{R}_s) \mathbf{f} + \bar{\mathbf{f}}^H (\mathbf{R}_{yy} - \bar{\mathbf{R}}_s) \bar{\mathbf{f}}}.$$

The MMSE receiver is optimum in the sense that it minimizes the output SINR [11]. The next result states that the proposed blind Capon receiver achieves similar output SINR to that of the MMSE receiver, particularly when the input SNR is high.

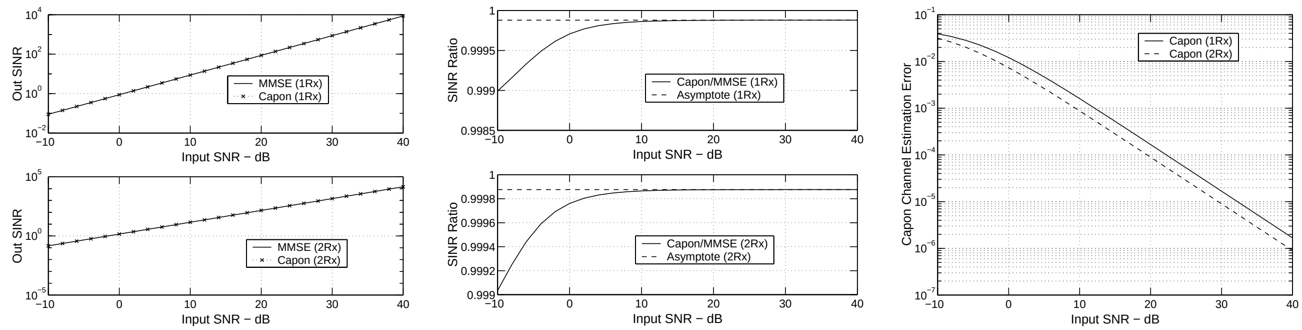
Theorem 3 *[8]: Under the identifiability conditions, the receiver output SINR for the Capon receiver and MMSE receiver are related by*

$$\lim_{\sigma_w^2 \to \infty} \frac{\text{SINR}_{\text{MMSE}}}{\text{SINR}_{\text{Capon}}} = 1 + \delta,$$

where δ *is very small for small* σ_w^2.

Again, an explicit expression for δ is derived in [8].

$$\mathbf{h}_c = \arg \min_{\underline{\mathbf{h}} \in \mathbb{C}^{2N\times 1}} \underline{\mathbf{h}}^H \underbrace{\left[\left(\mathbf{I}_N \otimes \mathbf{D}_1\right)^H \mathbf{R}_{yy}^{-1} \left(\mathbf{I}_N \otimes \mathbf{D}_1\right) + \left(\mathbf{I}_N \otimes \bar{\mathbf{D}}_1\right)^H \mathbf{R}_{yy}^{-1} \left(\mathbf{I}_N \otimes \bar{\mathbf{D}}_1\right) \right]}_{\mathbf{\Omega}} \underline{\mathbf{h}}, \quad (8)$$



(a) (b) (c)

Fig. 1. Performance of the Capon and MMSE receivers vs. input SNR with $N = 1$ and $N = 2$ receive antennas. (a) Output SINR. (b) Output SINR ratio. (c) Channel estimation error $\left\| \mathbf{h}_c - \|\mathbf{h}\|^{-1}\mathbf{h} \right\|$.

6. SIMULATION RESULTS

A simulation study of the Capon and MMSE receivers is depicted in Figure 1, where we simulated a 10-user CDMA system that employs the Alamouti's space-time coding scheme ($M = 2$), BPSK constellation, Gold codes of spreading gain $J = 31$, and one or two receive antennas. A near-far scenario was simulated, with the power of all interfering users being 10 dB larger than that of the desired one. The simulated channel experiences Rayleigh flat-fading. The results were obtained by using the true covariance matrix $\mathbf{R}_{yy}$ for both the Capon and MMSE receiver. In particular, Figure 1(a) depicts the output SINR as a function of the input SNR for both receivers; it is noted that the output SINR for both receivers are very close for a wide range of the input SNR. Figure 1(b) shows the ratio of the output SINR for both receivers; we see that at high input SNRs, the ratio converges to a a number very close to 1, as predicted by Theorem 3. Figure 1(c) shows the channel estimation error of Capon; it is seen that the error vanishes quickly as the SINR increases, as predicted by Theorem 2. Also observed in Figures 1(a) to 1(c) is the improved performance achieved by the system with two receive antennas.

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