ON ESTIMATING TEMPORAL AND SPATIAL CHANNEL PARAMETERS FOR DS-CDMA SYSTEM OVER MULTIPATH RAYLEIGH FADING CHANNELS

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ABSTRACT

In this paper, we develop an approximate maximum likelihood (ML) method for joint estimation of time delays and directions-of-arrivals (DOA's) for the direct-sequence code-devision multiple-access (DS-CDMA) systems over the multipath Rayleigh fading channels. In developing the proposed estimation scheme, we model the known training sequence of the desired user as the desired signal and the multiple access interferences (MAI)-plus-AWGN as unknown colored Gaussian noise uncorrelated with the desired signal. To reduce computational complexity, a fast algorithm realization is proposed taking advantage of the ridge structure of the objection function. Analytical and simulation results are presented to illustrate the performance in comparison with the Cramer-Rao bounds (CRBs). It is shown that the proposed algorithm is near-far resistant for both time delay and DOA estimations, with almost no performance loss compared with CRBs.

1. INTRODUCTION

The multipath channel is characterized by both the time delays and the DOA's of different propagation paths in wireless communication systems. How to estimate the time delays and the DOA's of a user's multipath signals arriving at a base station antenna array, and consequently, utilize the multipath components in the receiver is a major issue of in wireless communications.

The problem of joint estimation of time delays and the DOA's has received some attention in the literature [1, 2] due to its importance in practice. It is possible to exploit the difference in path delays to improve the angle estimation accuracy and vice-versa. An algorithm for estimating the time delays and the DOA's of multiple reflections of a known transmitted signal is proposed from signal processing perspective in [1]. In [2], a subspace based approach to estimate the time delays and the DOA's of multipath signals is presented for time division multiple access (TDMA) system. However, both approaches are developed for a single user system over the AWGN channels and there is no MAI problem involved.

In this work, we develop joint ML time delays and the DOA's estimation method for the DS-CDMA systems over multipath Rayleigh fading channels. The joint ML time delay and the DOA estimator for DS-CDMA system over multipath Rayleigh fading channels is derived after modeling the known training sequence of a desired user as the desired signal, the MAI-plus-AWGN as an unknown colored Gaussian noise uncorrelated with the desired signal, and exploiting the knowledge of the desired user's spreading signature. In order to reduce the computational complexity of the multidimensional numerical search associated with the MLE, we utilize the ridge structure of the objection function and propose a fast algorithm realization to estimate the parameters rapidly. Analytical and simulation results show that our proposed estimator is near-far resistant for both time delay and DOA, and it can reach the corresponding CRBs after several iterations.

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2. SIGNAL MODEL

We consider a DS-CDMA system with K users transmitting sequences of BPSK symbols through their respective multipath Rayleigh Fading channels. The kth user's channel is modeled as a multipath channel with L paths, each associated with a complex Gaussian fading coefficient $\{g_k(l)\}_{l=1}^{L}$. For each multipath component, there are different path delay and DOA associated with the signal of a desired user. Hence the received baseband signal at the output of a P-element antenna array due to the kth user is

$$\mathbf{y}_k(t) = \sum_{l=1}^{L} g_k(l)\, s_k(t-\tau_{kl})\mathbf{a}(\theta_{kl}), \tag{1}$$

where $s_k(t) = \sum_i b_k(i)c_k(t-iT)$, $k = 1, \cdots, K$ is the baseband signal for the kth user, T is the symbol interval, $b_k(i)$ is the ith symbol by the kth user and $c_k(t) = \sum_{q=0}^{N-1} c_k(q)\psi(t-qT_c)$ its normalized spreading signature, N is the processing gain, $\mathbf{c}_k = [\, c_k[0] \cdots c_k[N-1]\,]^T$ is a signature sequence assigned to the kth user, and $\psi(t)$ is a chip waveform of duration $T_c = T/N$. The path delays $\tau_{kl} \in [0, T)$ for $l = 1, 2, \cdots, L$, and $\mathbf{a}(\theta_{kl})$ is the steering vector corresponding to the lth path signal of the kth user, and the pth element of $\mathbf{a}(\theta_{kl})$ has the form $\mathbf{a}^p(\theta_{kl}) = \frac{1}{P}e^{j2\pi/\lambda(x_p \sin\theta_{kl} + y_p\cos\theta_{kl})}$, with $(x_p,\, y_p)$ the sensors locations for $p = 1, \cdots, P$.

The total received data in a K users system is given by

$$\mathbf{y}(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} g_k(l)\, s_k(t-\tau_{kl})\mathbf{a}(\theta_{kl}) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{n}(t)$ is spatially and temporally i.i.d complex AWGN with zero mean and variance $2\sigma^2$. At the chip rate, the discrete-time version of the received data in (2) can be formulated as

$$\mathbf{y}(n) = \sum_{k=1}^{K}\sum_{l=1}^{L} g_k(l)\, s_k(n-\tau_{kl})\mathbf{a}(\theta_{kl}) + \mathbf{n}(n), \tag{3}$$

Within the nth processing interval of length T, which is commonly *not* aligned with the unknown delays, the chip-rate matched filtered and sampled N snapshots of data in (3) can be collected into a $P \times N$ matrix

$$\mathbf{Y}(n) \stackrel{\triangle}{=} [\mathbf{y}(nN) \cdots \mathbf{y}(nN+N-1)]$$
$$= \sum_{k=1}^{K}\sum_{l=1}^{L} g_k(l)\mathbf{a}(\theta_{kl}) \otimes \left[\mathbf{u}_k^{(r)T}(\tau_{kl}) b_k(n-1) + \mathbf{u}_k^{(l)T}(\tau_{kl}) b_k(n)\right] + \mathbf{N}(n),$$

where $\otimes$ stands for Kronecker product, matrix $\mathbf{N}(n)$ is spatially and temporally AWGN given by $\mathbf{N}(n) = [\mathbf{n}(nN), \cdots, \mathbf{n}(nN+N-1)]$, $(N \times 1)$ vectors $\mathbf{u}_k^{(r)}(\tau_{kl})$ and $\mathbf{u}_k^{(l)}(\tau_{kl})$ are partitioned temporal signature vectors of the kth user, parameterized by the time delay $\tau_{kl} = \nu_{kl} T_c + \gamma_{kl}$, where ν_{kl} and γ_{kl} are the integer and fractional parts of the delay parameter τ_{kl} modulo T_c (in our further analyses and simulations, we choose a normalized chip interval, $T_c = 1$) as follows

$$\begin{aligned} \mathbf{u}_k^{(r)}(\tau_{kl}) &= (1-\gamma_{kl})\,\mathbf{c}_k^{(r)}(\nu_{kl}) + \gamma_{kl}\,\mathbf{c}_k^{(r)}(\nu_{kl}+1), \\ \mathbf{u}_k^{(l)}(\tau_{kl}) &= (1-\gamma_{kl})\,\mathbf{c}_k^{(l)}(\nu_{kl}) + \gamma_{kl}\,\mathbf{c}_k^{(l)}(\nu_{kl}+1), \end{aligned}$$

vectors $\mathbf{c}_k^{(r)}(\nu_{kl})$ and $\mathbf{c}_k^{(l)}(\nu_{kl})$ are the right and left portion of signature vector $\mathbf{c}_k$ partitioned by integer part of delay ν_{kl}. That is $\mathbf{c}_k^{(r)}(\nu_{kl}) = [c_k[N-\nu_{kl}] \cdots c_k[N-1]\, 0 \cdots 0]^T$, $\mathbf{c}_k^{(l)}(\nu_{kl}) = [0 \cdots 0\, c_k[0] \cdots c_k[N-\nu_{kl}-1]]^T$.
Vectorize columns of data matrix $\mathbf{Y}(n)$ into an observation vector $\mathcal{Y}(n)$, we then have,

$$\mathcal{Y}(n) = vec(\mathbf{Y}(n))$$
$$= \sum_{k=1}^{K}\sum_{l=1}^{L} g_k(l) \left[\mathbf{u}_k^{(r)}(\tau_{kl}) b_k(n-1) + \mathbf{u}_k^{(l)}(\tau_{kl}) b_k(n)\right] \otimes \mathbf{a}(\theta_{kl}) + \mathcal{N}(n)$$
$$= \underbrace{\mathbf{H}_1(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1)\mathbf{G}_1(\mathbf{g}_1)\mathbf{z}_1(n)}_{desired signal} + \underbrace{\sum_{k=2}^{K}\mathbf{H}_k(\boldsymbol{\tau}_k,\boldsymbol{\theta}_k)\mathbf{G}_k(\mathbf{g}_k)\mathbf{z}_k(n) + \mathcal{N}(n)}_{MAI+noise:\ \mathcal{N}_I(n)}, \quad (4)$$

where $\mathbf{H}_k(\boldsymbol{\tau}_k,\boldsymbol{\theta}_k) = \left[\mathbf{h}_{k1}^{(r)}\,\mathbf{h}_{k1}^{(l)} \cdots \mathbf{h}_{kL}^{(r)}\,\mathbf{h}_{kL}^{(l)}\right]$, with $\mathbf{h}_{kl}^{(r)} = \mathbf{u}_k^{(r)}(\tau_{kl}) \otimes \mathbf{a}(\theta_{kl})$ and $\mathbf{h}_{kl}^{(l)} = \mathbf{u}_{kl}^{(l)}(\tau_{kl}) \otimes \mathbf{a}(\theta_{kl})$. $\mathbf{G}_k = \mathbf{g}_k \otimes \mathbf{I}_2$, with $\mathbf{g}_k = [g_k(1) \cdots g_k(L)]^T$, $\mathbf{z}_k(n) = [b_k(n-1)\ \ b_k(n)]^T$, $\mathcal{N}(n) = vec(\mathbf{N}(n))$, $\mathcal{N}_I(n) = \sum_{k=2}^{K}\mathbf{H}_k(\boldsymbol{\tau}_k,\boldsymbol{\theta}_k)\,\mathbf{G}_k(\mathbf{g}_k)\,\mathbf{z}_k(n) + \mathcal{N}(n)$, $\boldsymbol{\tau}_k = [\tau_{k1} \cdots \tau_{kL}]^T$, and $\boldsymbol{\theta}_k = [\theta_{k1} \cdots \theta_{kL}]^T$.
The focus of this work is to estimate channel parameters using data $\{\mathcal{Y}(n)\}$.

3. THE ML ESTIMATOR

To set the basis for our fast solution, we first derive the joint MLEs for the channel parameters (time delays and DOA's) of a desired user (user 1). When the number of active users in combination with number of multipath components is reasonably large, we can model the MAI-plus-noise as *colored* Gaussian noise. We then model the received data $\mathcal{Y}(n)$ as a complex Gaussian random vector with mean vector parameterized by $\boldsymbol{\tau}_1$, $\boldsymbol{\theta}_1$ and $\mathbf{g}_1$

$$E[\mathcal{Y}(n)] = \mathbf{H}_1(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1)\,\mathbf{G}_1(\mathbf{g}_1)\mathbf{z}_1(n),$$

and the second-order statistics summarized in matrices

$$E\left\{[\mathcal{Y}(n) - E(\mathcal{Y}(n))][\mathcal{Y}(m) - E(\mathcal{Y}(m))]^H\right\} \approx \mathbf{Q}\,\delta_{mn},$$
$$E\left\{[\mathcal{Y}(n) - E(\mathcal{Y}(n))][\mathcal{Y}(m) - E(\mathcal{Y}(m))]^T\right\} = \mathbf{0},$$

where δ_{mn} is a discrete Kronecker delta function, and

$$\mathbf{Q} = \sum_{k=2}^{K}\mathbf{H}_k(\boldsymbol{\tau}_k,\boldsymbol{\theta}_k)\,\mathbf{G}_k(\mathbf{g}_k)\,\mathbf{G}_k^H(\mathbf{g}_k)\mathbf{H}_k^H(\boldsymbol{\tau}_k,\boldsymbol{\theta}_k) + \sigma^2\,\mathbf{I}. \quad (5)$$

The likelihood function for a given $\mathcal{Y}(n)$ is given by

$$L(\mathcal{Y}(n);\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q}) = \frac{1}{\pi^{NP}\,|\,\mathbf{Q}\,|}\exp\{-\mathcal{N}_I^H(n)\,\mathbf{Q}^{-1}\,\mathcal{N}_I(n)\},$$

where $|\cdot|$ stands for the determinant, and $\mathcal{N}_I(n) = \mathcal{Y}(n) - \mathbf{H}_1(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1)\,\mathbf{G}_1(\mathbf{g}_1)\mathbf{z}_1(n)$. Since $\mathcal{Y}(n)$ and $\mathcal{Y}(m)$ are approximately uncorrelated complex Gaussian vectors for $m \neq n$, we can write the likelihood function based on the observation $\mathcal{Y} = [\mathcal{Y}(1)\,\mathcal{Y}(2) \cdots \mathcal{Y}(M)]$ as

$$L\ (\mathcal{Y};\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q}) = \Pi_{n=1}^{M} L(\mathcal{Y}(n);\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q})$$
$$= \frac{1}{\pi^{MNP}\ |\,\mathbf{Q}\,|^M}\exp\{-\sum_{n=1}^{M}\mathcal{N}_I^H(n)\,\mathbf{Q}^{-1}\,\mathcal{N}_I(n)\}. \quad (6)$$

The log-likelihood function is (when ignoring constant terms)

$$\ell\ (\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q}) = -M\log|\,\mathbf{Q}\,|$$
$$-\operatorname{tr}\left\{\mathbf{Q}^{-1}\sum_{n=1}^{M}[\mathcal{Y}(n) - \mathbf{D}\mathbf{z}_1(n)]\,[\mathcal{Y}(n) - \mathbf{D}\mathbf{z}_1(n)]^H\right\}, \quad (7)$$

where $\mathbf{D} = \mathbf{H}_1(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1)\,\mathbf{G}_1(\mathbf{g}_1)$. The MLE is therefore given by

$$\left[\hat{\boldsymbol{\tau}}_1,\hat{\boldsymbol{\theta}}_1,\hat{\mathbf{g}}_1,\hat{\mathbf{Q}}\right] = \arg\max_{\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q}}\{\ell(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q})\}. \quad (8)$$

Maximizing $\ell(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{g}_1,\mathbf{Q})$ with respect to $\mathbf{Q}$ for a fixed $\mathbf{D}$ yields

$$\hat{\mathbf{Q}} = \frac{1}{M}\sum_{n=1}^{M}[\mathcal{Y}(n) - \mathbf{D}\,\mathbf{z}_1(n)]\,[\mathcal{Y}(n) - \mathbf{D}\,\mathbf{z}_1(n)]^H, \quad (9)$$

which, when substituted back into (7), yields the MLEs of channel parameters,

$$\left[\hat{\boldsymbol{\tau}}_1,\hat{\boldsymbol{\theta}}_1,\hat{\mathbf{g}}_1\right] = \arg\min_{\boldsymbol{\tau},\boldsymbol{\theta},\mathbf{g}} F$$
$$= \left|\frac{1}{M}\sum_{n=1}^{M}[\mathcal{Y}(n) - \mathbf{D}\mathbf{z}_1(n)][\mathcal{Y}(n) - \mathbf{D}\mathbf{z}_1(n)]^H\right| \quad (10)$$

Instead of minimizing F directly with respect to $\boldsymbol{\tau}_1$, $\boldsymbol{\theta}_1$ and $\mathbf{g}_1$, we minimize F first with respect to the unstructured matrix $\mathbf{D}$, and the unstructured estimate $\hat{\mathbf{D}}$ of $\mathbf{D}$ is then given by $\hat{\mathbf{D}} = \hat{\mathbf{R}}_{yz}\,\hat{\mathbf{R}}_{zz}^{-1}$, with $\hat{\mathbf{R}}_{yz} = \frac{1}{M}\sum_{n=1}^{M}\mathcal{Y}(n)\mathbf{z}_1^H(n)$ and $\hat{\mathbf{R}}_{zz} = \frac{1}{M}\sum_{n=1}^{M}\mathbf{z}_1(n)\mathbf{z}_1^H(n)$. Substituting $\hat{\mathbf{D}}$ into (9) yields

$$\hat{\mathbf{Q}} = \hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{yz}\,\hat{\mathbf{R}}_{zz}^{-1}\,\hat{\mathbf{R}}_{yz}^H.$$

with $\hat{\mathbf{R}}_{yy} = \frac{1}{M}\sum_{n=1}^{M}\mathcal{Y}(n)\,\mathcal{Y}^H(n)$. It now remains to minimize

$$\ell(\boldsymbol{\tau}_1,\boldsymbol{\theta}_1,\mathbf{G}_1,\hat{\mathbf{Q}}) = M\log|\,\hat{\mathbf{Q}}\,| + constant, \quad (11)$$

with respect to channel parameters only. After some manipulations, we can show that the MLE of $\mathbf{G}_1$ for fixed non-linearly entered channel parameters $\boldsymbol{\tau}_1$ and $\boldsymbol{\theta}_1$ is

$$\hat{\mathbf{G}_1} = (\mathbf{H}_1^H \,\hat{\mathbf{Q}}^{-1}\, \mathbf{H}_1)^{-1}\mathbf{H}_1^H \,\hat{\mathbf{Q}}^{-1}\, \hat{\mathbf{R}}_{yz}\, \hat{\mathbf{R}}_{zz}^{-1}, \qquad (12)$$

with $\mathbf{H}_1 \triangleq \mathbf{H}_1(\boldsymbol{\tau}_1, \boldsymbol{\theta}_1)$ being parameterized by channel parameters. Substituted the above $\hat{\mathbf{G}}_1$ into (11), yields

$$\left[\hat{\boldsymbol{\tau}}_1, \hat{\boldsymbol{\theta}}_1\right] = \arg\max_{\boldsymbol{\tau}_1, \boldsymbol{\theta}_1} J(\boldsymbol{\tau}_1, \boldsymbol{\theta}_1)$$

$$= \arg\max_{\boldsymbol{\tau}_1, \boldsymbol{\theta}_1} -\left|\mathbf{I} + \hat{\mathbf{Q}}^{-1}(\mathbf{H}_1\hat{\mathbf{G}}_1 - \hat{\mathbf{D}})\hat{\mathbf{R}}_{yy}(\mathbf{H}_1\hat{\mathbf{G}}_1 - \hat{\mathbf{D}})^H\right| \qquad (13)$$

Getting the MLEs of time delays and DOAs from (13) involves a $2L$ dimensional search over parameter space $[\boldsymbol{\tau}_1, \boldsymbol{\theta}_1]$, hence, is computationally prohibitive even for moderate values of L.

4. A FAST ESTIMATION SCHEME

We next present a rapid algorithm realization that approximates the MLE and is computationally efficient starting by initializing the estimations for a single path. In figure 1, we show the mesh and contour plots of the objective function in (13) in a 2-D parameter space. The ridge structure of the objective function is revealed. As being pointed out in our previous work [3, 4], there exists an efficient way of estimating τ_{11} and θ_{11}. The parameters for the strongest path of the desired user can be obtained as follows. We first fix the value of θ_{11} to any value, say $\theta_{11} = 0°$, search only along the axis of τ to get $\hat{\tau}_{11}$ as in [4]. Once the $\hat{\tau}_{11}$ is obtained, we then come back to refine the value of θ_{11}. That is

$$\hat{\tau}_{11} = \arg\max_{\tau, \theta_{11}=0} J(\boldsymbol{\tau}_1, \boldsymbol{\theta}_1)$$

$$\hat{\theta}_{11} = \arg\max_{\theta_{11}, \tau_{11}=\hat{\tau_{11}}} J(\boldsymbol{\tau}_1, \boldsymbol{\theta}_1)$$

We can find the second strongest path's parameter estimations in a similar way after removing the strongest path's effect by projecting the received data to the null space of the space spanned by $\mathbf{H}_1(\tau_{11}, \theta_{11})$. Repeat this process until the parameters for all L paths are obtained.

The refined parameter estimations for the lth path are then obtained via iteration. Specifically, we fix the parameters of $L-1$ paths as the previous estimations and search for the more accurate estimation of τ_{1l} and θ_{1l} using the steps described above. The iteration is continued until there is no more improvement in the objection function in (13). The number of iterations in most scenarios is between $5-10$.

5. SIMULATION RESULTS

The simulation results are presented to demonstrate the performance of the proposed algorithm. For comparison, the CRBs for the joint estimation problem are also presented.

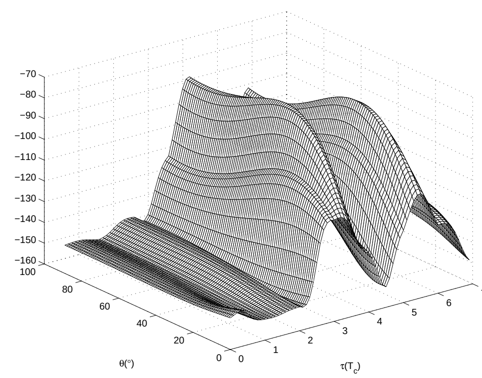
We initially assume that the system has $K = 3$ users, each user has $L = 2$ multipaths arriving at a half-wavelength uniformly spaced linear array with $P = 2$, so that $\mathbf{a}_{kl} = [1\, e^{-j\pi\sin(\theta_{kl})} \,\cdots\, e^{-j\pi(P-1)\sin(\theta_{kl})}]^T$. Gold sequence of length $N = 7$ is used. The multipath fading coefficients for the desired user are normalized [5] so that $|g_1(1)| = |g_1(2)| = 1$. The interference user's multipath fading coefficients also have the same absolute value, $|g_{k_1}(l_1)| = |g_{k_1}(l_2)|$ for $k_1, k_2 \in \{2,3\}$ and $l_1, l_2 \in \{1,2\}$. The signal to noise ratio (SNR) of the desired user in dB is defined as $SNR_1 = 10\log_{10}\frac{\|\mathbf{g}_1\|^2}{2\sigma^2}$, and the near-far ratio (NFR) in dB is defined as $NFR = 10\log_{10}\frac{\|\mathbf{g}_k\|^2}{\|\mathbf{g}_1\|^2}$. We fix the time delays and DOA's of the desired user as $\boldsymbol{\tau}_1 = [3.4 \quad 5.2]T_c$ and $\boldsymbol{\theta}_1 = [16° \quad 34°]$. The observation time is fixed and $M = 100$ bits are used for channel estimation, and the root mean square error (RMSE) is obtained from 500 Monte Carlo runs.

Figure 3 and figure 4 present the time delay and DOA estimation accuracy as a function of NFR. The SNR of the desired user is fixed as $SNR_1 = 10$dB. Clearly, the proposed algorithm approach to the CRB after $5-10$ iterations, and the difference between the RMSE and CRB is less than 3dB.

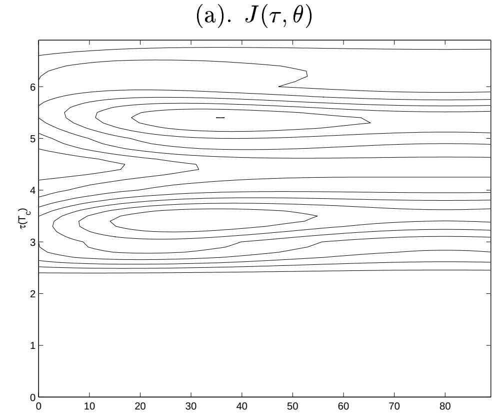
Figure 5 and figure 6 demonstrate the dependence of the time delay and DOA estimation accuracy on the SNR_1. Compared to the CRB, the difference is less than 3dB.

6. CONCLUSIONS

An iterative maximum likelihood (ML) approach for joint time delays and DOA's estimation is developed for DS-CDMA systems over multipath Rayleigh fading channels. The proposed fast estimation algorithm utilizes the ridge structure of the objection function, and greatly speeds up the joint ML estimation process.



(a). $J(\tau, \theta)$



(b). $J(\tau, \theta)$

Figure 1: Mesh and contour plots of the objective function $J(\tau, \theta)$ in 2-D parameter space. with $SNR_1 = 10dB$. Parameter used are: $K = 3$, $N = 7$, $P = 2$, $\boldsymbol{\tau}_1 = [3.4 \quad 5.4]T_c$ and $\boldsymbol{\theta}_1 = [35° \quad 55°]$.

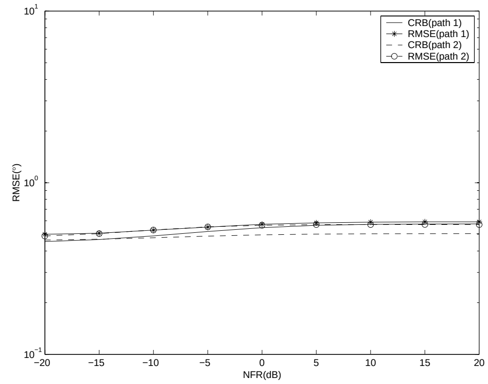


Figure 2: RMSE of DOA's as a function of NFR, with $SNR_1 = 10dB$. Parameter used are: $K = 3$, $N = 7$, $P = 2$, $\boldsymbol{\tau}_1 = [3.4 \ \ 5.2]T_c$ and $\boldsymbol{\theta}_1 = [16^\circ \ \ 34^\circ]$.

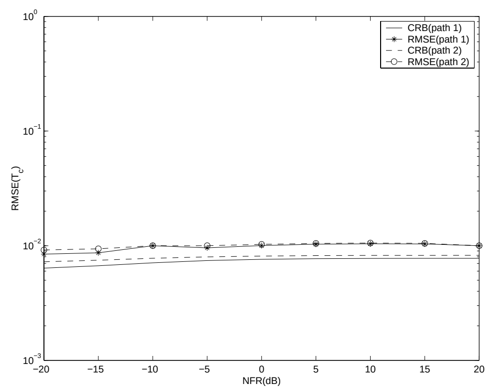


Figure 3: RMSE of time delays as a function of NFR, with $SNR_1 = 10dB$. Parameter used are: $K = 3$, $N = 7$, $P = 2$, $\boldsymbol{\tau}_1 = [3.4 \ \ 5.2]T_c$ and $\boldsymbol{\theta}_1 = [16^\circ \ \ 34^\circ]$.

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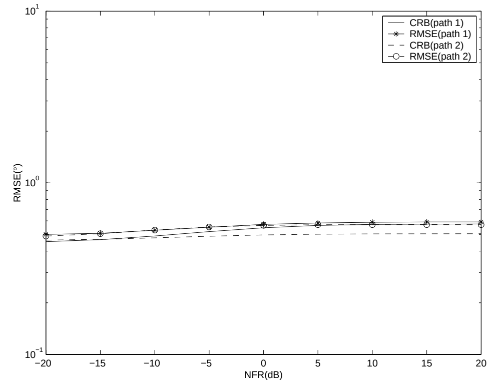

Figure 4: RMSE of DOA's as a function of NFR, with $SNR_1 = 10dB$. Parameter used are: $K = 3$, $N = 7$, $P = 2$, $\boldsymbol{\tau}_1 = [3.4 \ \ 5.2]T_c$ and $\boldsymbol{\theta}_1 = [16^\circ \ \ 34^\circ]$.

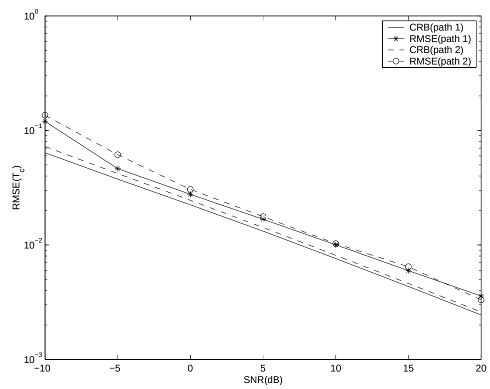

Figure 5: RMSE of time delays as a function of SNR_1, with $NFR = 0$dB. Parameter used are: $K = 3$, $N = 7$, $P = 2$, $\boldsymbol{\tau}_1 = [3.4 \ \ 5.2]T_c$ and $\boldsymbol{\theta}_1 = [16^\circ \ \ 34^\circ]$.

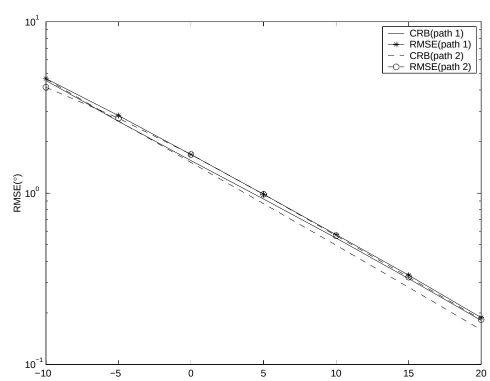

Figure 6: RMSE of DOA's as a function of SNR_1, with $NFR = 0$dB. Parameter used are: $K = 3$, $N = 7$, $P = 2$, $\boldsymbol{\tau}_1 = [3.4 \ \ 5.2]T_c$ and $\boldsymbol{\theta}_1 = [16^\circ \ \ 34^\circ]$.