

# A PARAMETRIC APPROACH TO HOT CLUTTER CANCELLATION

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## ABSTRACT

Many reduced dimension STAP algorithms have been developed for airborne radar applications which rely on a stationary Doppler component of the interference in order to maintain acceptable performance. Two cases where this assumption is violated are when the ground clutter contains intrinsic clutter motion (ICM) and when hot clutter is present. In addition to the non-stationary Doppler component, hot clutter contains non-zero correlations in fast-time (across range bins) as well. This paper will present an algorithm designed to mitigate both ground clutter and hot clutter in the same step using a two dimensional vector autoregressive model to whiten the data in space, fast-time, and slow-time. This is an extension of the Space-Time AutoRegressive (STAR) filter that we have previously proposed. Using a simulated data set for circular array STAP augmented with synthetic hot clutter, we demonstrate that the extensions we present do result in a significant performance increase over the standard STAR filter. In addition we also show that the STAR filters have a narrower clutter notch than the optimized pre-Doppler filter when a finite sample support is used to train the filters.

## 1. INTRODUCTION

The value of space-time adaptive processing (STAP) algorithms for airborne radar interference suppression is limited by the computational cost of implementation as well as the amount of stationary training data available to train the filter weights. Due to the inherent low-rank nature of the interference, it is possible to design reduced dimension algorithms which can have near optimal performance. These “partially adaptive” STAP filters [1, 2] help to alleviate the problem of computational complexity as well as the sample support required to train the filter.

In [3] an alternative method for clutter and interference suppression was proposed that uses a vector autoregressive (AR) filter. An algorithm that uses this model to construct a structured subspace that is as orthogonal as possible to the clutter and interference has been proposed in [4]. This algorithm, referred to as Space-Time AutoRegressive (STAR) filtering, uses a projection onto the estimated subspace to whiten the primary data vector.

Most of the partially adaptive methods as well as the vector AR methods make an assumption that the covariance matrix from the clutter statistics is Toeplitz (*i.e.*, stationary from pulse to pulse). This assumption is violated in the presence of intrinsic clutter motion (ICM) or terrain scattered interference (also known as TSI,

hot clutter, or jammer multipath). It is the latter effect that will be addressed in this paper. We present an extension to the STAR filter that will, in the same step, cancel both ground clutter and hot clutter that is generated from the model of [5]. The extensions to the STAR filter will be as follows:

1. The vector AR filter coefficients will be updated after each new pulse is received. Although this will require additional sample support, tracking the non-stationary interference is necessary for good performance.
2. In order to best utilize the fast-time correlations, a two-dimensional vector AR filter will be used with matrix taps in both the slow-time (pulse) dimension, as well as the fast-time (range bin) dimension.

A performance evaluation will be made using ground clutter from a synthetic data set generated by MIT Lincoln Laboratory that simulates the output of a 20 element antenna array whose elements lie along a circular arc of  $120^\circ$  [6]. Circular array geometries are currently being considered for use in airborne surveillance radars since they can electronically scan the full  $360^\circ$  surroundings in much less time than a mechanically steered array. An experimental circular array is currently being developed by Raytheon as part of the UHF Electronically Scanned Array (UESA) program sponsored by the Office of Naval Research. The array is composed of 60 directional elements evenly spaced around the edge of the circular aperture, but nominally only 20 are used at any given time for transmit and receive. Hot clutter generated according to the “sandpaper” earth model will be added to the ground clutter data.

In the next section, we present the standard data model assumed for a three-dimensional STAP problem and develop the notation we will use throughout the paper. The STAR filtering technique is outlined in Section 3 as background for the extension of this filter. Section 4 will derive a three-dimensional STAR filter that can be used in the mitigation of hot clutter. The results of a series of numerical experiments comparing the STAR approach to standard STAP algorithms are presented in Section 5.

## 2. MATHEMATICAL MODEL

A target present in a particular range bin during some coherent processing interval (CPI) may be modeled as producing the following baseband vector signal (after pulse compression and demodulation) [1]:

$$\mathbf{x}_\ell(t) = b\mathbf{a}(\theta)e^{j\omega t} + \mathbf{n}_\ell(t) \in \mathbb{C}^m, \quad t = 1, \dots, N, \quad (1)$$

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where  $\ell$  is the range bin in which the target is located,  $b$  is the complex amplitude of the signal,  $\omega$  is the Doppler shift due to the relative motion between the array platform and the target,  $\mathbf{a}(\theta)$  is the response of the array to a unit amplitude plane wave arriving from direction  $\theta$  (azimuth and elevation angles), and  $\mathbf{n}_\ell(t)$  contains contributions from clutter, jamming, and thermal noise. In (1), we are assuming an array of  $m$  elements and a total of  $N$  transmitted pulses covering  $R$  range bins.

If we stack the  $N$  array outputs and the  $R$  range bin vectors into a single  $mNR \times 1$  snapshot, we may re-write (1) as

$$\chi_{3D} = \begin{bmatrix} \chi_0 \\ \vdots \\ \chi_{R-1} \end{bmatrix} = bs(\theta, \omega, \ell) + \begin{bmatrix} \eta_0 \\ \vdots \\ \eta_{R-1} \end{bmatrix}, \quad (2)$$

where

$$\begin{aligned} \chi_\ell &= \begin{bmatrix} \mathbf{x}_\ell(1) \\ \vdots \\ \mathbf{x}_\ell(N) \end{bmatrix} \\ \mathbf{s}(\theta, \omega, \ell) &= \mathbf{v}_{fast}(\ell) \otimes \mathbf{v}_{slow}(\omega) \otimes \mathbf{a}(\theta) \\ \mathbf{v}_{fast}(\ell) &= [\cdots \ 0 \ 1 \ 0 \ \cdots]^T \\ \mathbf{v}_{slow}(\omega) &= [1 \ e^{j\omega} \ \cdots \ e^{j(N-1)\omega}]^T \end{aligned}$$

and  $\otimes$  represents the Kronecker product. The vector  $\eta_\ell$  contains the stacked vector samples of the clutter and interference for range bin  $\ell$ , and has an unknown covariance matrix denoted by

$$\mathcal{E}\{\eta_\ell \eta_\ell^*\} = \mathbf{R}.$$

The clutter is neither temporally nor spatially white; in fact, the rank of  $\mathbf{R}$  is typically much less than  $mN$ . The rank ( $\rho$ ) of  $\mathbf{R}$  is important because it determines how many secondary data samples are required to accurately estimate  $\mathbf{R}$ . According to [7], the number of required samples is on the order of  $2\rho$  to  $5\rho$ . The fully adaptive approach to whitening this type of data is to multiply the data by the inverse of an estimate of the matrix  $\mathbf{R}$ . Because the size of this matrix can become quite large, its low rank nature is exploited to derive reduced dimension whitening algorithms. The next section will summarize the work in [4] to present the STAR filter.

### 3. SPACE-TIME AUTOREGRESSIVE FILTERING

Following the derivation in [4], the STAR approach assumes that a set of  $L$  matrices  $\mathbf{H}_0, \mathbf{H}_1, \dots, \mathbf{H}_{L-1}$  of dimension  $m' \times m$  exist that satisfy

$$\sum_{i=0}^{L-1} \mathbf{H}_i \mathbf{n}(t+i) = 0, \quad t = 1, \dots, N-L+1, \quad (3)$$

for the interference and clutter in the primary range bin. We may also write (3) in the following two different ways:

$$\underbrace{[\mathbf{H}_0 \ \cdots \ \mathbf{H}_{L-1}]}_{\mathbf{H}^*} \underbrace{\begin{bmatrix} \mathbf{n}(1) & \cdots & \mathbf{n}(N-L+1) \\ \vdots & \cdots & \vdots \\ \mathbf{n}(L) & \cdots & \mathbf{n}(N) \end{bmatrix}}_{\mathbf{N}} = 0 \quad (4)$$

or

$$\mathcal{H}^* \eta = 0, \quad (5)$$

where

$$\mathcal{H}^* = \begin{bmatrix} \mathbf{H}_0 & \cdots & \mathbf{H}_{L-1} & & \\ & \mathbf{H}_0 & \cdots & \mathbf{H}_{L-1} & \\ & & \ddots & & \\ & & & \mathbf{H}_0 & \cdots & \mathbf{H}_{L-1} \end{bmatrix}. \quad (6)$$

We assume that equations (4) and (5) also hold for the secondary data as well:

$$\mathbf{H}^* \mathbf{N}_k = 0 \quad (7)$$

$$\mathcal{H}^* \eta_k = 0, \quad (8)$$

for  $k = 1, \dots, N_s$ , where  $N_s$  is the number of secondary data snapshots used to train the filter.

The matrix  $\mathcal{H}$  is  $mN \times m'(N-L+1)$ . If (3) holds and  $m'$  and  $L$  are chosen so that  $m'(N-L+1)$  is large enough, the columns of  $\mathcal{H}$  form a basis for the space orthogonal to the clutter and interference subspace. Although this relationship will not hold in practice due to the presence of thermal noise, a least squares solution is applied to approximate the subspace. This suggests the following space-time filter be used for interference rejection:

$$\mathbf{w}_{AR}(\theta, \omega) = \mathbf{P}_{\mathcal{H}} \mathbf{s}(\theta, \omega), \quad (9)$$

where  $\mathbf{P}_{\mathcal{H}}$  is the projection onto the columns of  $\mathcal{H}$ :

$$\mathbf{P}_{\mathcal{H}} = \mathcal{H} (\mathcal{H}^* \mathcal{H})^{-1} \mathcal{H}^*. \quad (10)$$

We refer to the implementation of STAP with the weight vector of (9) as Space-Time AutoRegressive (STAR) filtering. The STAR filter weights are “adaptive” in the sense that  $\mathcal{H}$  must be estimated from the secondary data prior to computation of  $\mathbf{w}_{AR}$ .

### 4. 3D-STAR FILTER

In this section, a 3D-STAR filter is derived that can mitigate both ground clutter and hot clutter in the same step. This filter has two main differences from the STAR filter of the previous section. First, the 3D-STAR filter will update its coefficients after each pulse is received. Second, it will have fast-time degrees of freedom (DOFs) available for hot clutter mitigation. These extra DOFs are used to take advantage of the fact that hot clutter has non-zero correlations from one range bin to the next. These DOFs are then used to help mitigate main beam jamming signals. In the first part of this section, we derive a (slow) time varying STAR (TVSTAR) filter, and in the second part we derive a three dimensional (two dimensional vector AR) STAR filter.

When hot clutter or ICM is present, the model for the clutter in (3) is no longer valid, as the spatial covariance changes from pulse to pulse. If the model in (3) is applied in such a non-stationary environment, a large filter order ( $L$ ) will be necessary to account for the slow-time variations in the data. A better approach in this case is to let the space-slow-time vector AR filter be

$$\mathcal{H}_{TV}^* = \begin{bmatrix} \mathbf{H}_0(1) & \cdots & \mathbf{H}_{L-1}(1) \\ & \ddots & \\ & & \mathbf{H}_0(n) & \cdots & \mathbf{H}_{L-1}(n) \end{bmatrix}, \quad (11)$$

where  $n = N-L+1$ . Each block row is a set of new coefficients based on dropping the data from the oldest pulse and adding the data from the most recent pulse (*i.e.*, computing new coefficients for each sub-CPI). This time varying STAR filter may also be used

when trying to account for ICM.

In order to utilize the fast-time correlation of the data, an extra dimension needs to be added to the STAR filter. We will assume for a moment that the interference is stationary across the pulses. This filter will model the fast-time and slow-time correlations with a two-dimensional vector AR filter. For a set of  $LJ$  matrices of size  $m' \times m$ , assume that the clutter obeys the model

$$\sum_{j=0}^{J-1} \sum_{i=0}^{L-1} \mathbf{H}_{i,j} \mathbf{n}_{k+j}(t+i) = \mathbf{0}, \quad t = 1, \dots, N-L+1, \quad k = 1, \dots, P-J+1, \quad (12)$$

where  $k = 0$  is the range bin of interest and  $P$  is the number of fast-time samples used. This may also be expressed as

$$\sum_{j=0}^{J-1} \mathcal{H}_j^* \mathbf{n}_{k+j} = \mathbf{0} \quad k = 1, \dots, P-J+1, \quad (13)$$

where  $\mathcal{H}_j$  is the matrix defined in (6) with a subscript  $j$  to indicate which fast-time sample it is associated with. From this point we may again take into account the slow-time variations caused by the hot clutter by replacing  $\mathcal{H}_j$  with the slow-time varying filter  $\mathcal{H}_{TV,j}$ .

Rewriting this sum with the time varying filter we get

$$\mathbb{H}^* \eta_{3d}(k) = \mathbf{0}, \quad (14)$$

where

$$\eta_{3d}(k) = \begin{bmatrix} \eta_k \\ \vdots \\ \eta_{k+P-1} \end{bmatrix} \quad \mathbb{H}^* = \begin{bmatrix} \mathcal{H}_{TV,0}^* & \cdots & \mathcal{H}_{TV,J-1}^* \\ & \ddots & \\ & & \mathcal{H}_{TV,0}^* & \cdots & \mathcal{H}_{TV,J-1}^* \end{bmatrix}$$

Assuming that there is target energy in the  $k = 0$  range bin then there will also be target energy in the vectors  $\eta_{3d}(0)$ ,  $\eta_{3d}(-1)$ ,  $\dots$ ,  $\eta_{3d}(-P+1)$  which may not be used for training the filter. In order to define the algorithm to find the filter coefficients let

$$\tilde{\mathbf{H}}(t)^* = [\mathbf{H}_{0,0}(t) \quad \cdots \quad \mathbf{H}_{L-1,J-1}(t)] \quad (15)$$

$$\mathbf{g}_k(t) = \begin{bmatrix} \mathbf{n}_k(t) \\ \vdots \\ \mathbf{n}_k(t+L-1) \end{bmatrix} \quad (16)$$

$$\mathcal{G}_k(t) = \begin{bmatrix} \mathbf{g}_k(t) & \cdots & \mathbf{g}_{k+P-J}(t) \\ \vdots & & \vdots \\ \mathbf{g}_{k+J-1}(t) & \cdots & \mathbf{g}_{k+P-1}(t) \end{bmatrix} \quad (17)$$

$$\mathbf{G}(t) = [\mathcal{G}_1(t) \quad \cdots \quad \mathcal{G}_{N_s}(t)]. \quad (18)$$

The filter coefficients can then be found by the following least squares criterion:

$$\hat{\tilde{\mathbf{H}}}(t) = \arg \min_{\tilde{\mathbf{H}}(t)} \left\| \tilde{\mathbf{H}}(t)^* \mathbf{G}(t) \right\|_F^2 \quad t = 1, \dots, N-L \quad (19)$$

subject to the constraint that  $\mathbf{H}(t)^* \mathbf{H}(t) = \mathbf{I}$ . From this point the  $m'$  left singular vectors corresponding to the smallest singular values of each  $\mathbf{G}(t)$  matrix will be used to compute the  $N-L+1$  sets of filter coefficients which define  $\mathbb{H}$ . The three dimensional

weight vector will be

$$\mathbf{w}_{AR3D} = \mathbf{P}_{\mathbb{H}} \mathbf{s}_{3D}(\theta, \omega), \quad (20)$$

where

$$\mathbf{s}_{3D}(\theta, \omega) = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \otimes \mathbf{s}(\theta, \omega). \quad (21)$$

## 5. NUMERICAL RESULTS

Since the prototype UESA circular array has yet to be field tested, a data package has been created by MIT Lincoln Laboratory to simulate the output of the array due to ground clutter in a standard operating scenario. Instead of 60 elements, the simulated data assumes an array of 54 elements uniformly spaced around a circle of 5.93m diameter. Only  $m = 20$  of the elements are assumed to be used for transmit and receive during one CPI. The antenna elements are assumed to have a cosine-shaped response with a  $-30$  dB back lobe for both the azimuth and elevation dimensions. The airborne platform is moving with a velocity of 100 m/s above a 4/3 earth model at an altitude of 9000 m. The operating frequency of the radar is taken to be 435 Mhz, the radar bandwidth and sampling frequency are 3.75 Mhz, the pulse-repetition frequency is 300 Hz, and  $N = 18$  pulses are assumed to be transmitted during one CPI. Data are generated for 9325 range gates between 20-400 km with a clutter-to-white-noise power ratio of 45 dB at a range of 100 km.

Hot clutter is included into the data by adding a term of the form

$$\mathbf{j}_k = b_j \begin{bmatrix} \mathbf{c}_k(1) \\ \vdots \\ \mathbf{c}_k(N) \end{bmatrix},$$

where  $b_j$  is the amplitude of the jammer,

$$\mathbf{c}_k(t) = \mathbf{a}(\theta_j) z_k + \sum_{i=1}^{\ell} \mathbf{b}_i z_{k-i}$$

is the contribution of the hot clutter for a single pulse at range  $k$ ,  $\ell$  is the longest multipath delay,  $\theta_j$  is the direction of arrival of the jammer signal,  $z_k$  is the jammer waveform (white in both slow and fast-time), and  $\mathbf{b}_i$  is a random vector that approximates the sum of the spatial steering vectors for each of the multipath signals. When present, the jammer-to-clutter power ratio is assumed to be 10 dB. When secondary data are used to estimate the clutter covariance or STAR filter parameters, equal amounts of data from range gates on either side of the target range gate are used.

The true clutter covariance matrix used to generate the data is known for 20 of the 9325 range bins, and thus the maximum achievable SINR can be calculated at these ranges. The results here are for a target at 350 km. All of the figures show the SINR loss in dB as a function of normalized Doppler for a look direction of  $0^\circ$  azimuth. Each figure shows the best possible SINR (solid line) as a reference for the other curves. Comparisons are made between the STAR algorithms presented herein as well as the optimized 3D pre-Doppler algorithm [5]. The optimized 3D pre-Doppler algorithm is implemented using three pulses in each sub-CPI as well as diagonal loading to improve performance. The STAR algorithms are implemented with  $m' = 20$ . All of the algorithms use a training data length of  $N_s = 60$  snapshots.

Figures 1 and 2 have the jammer located at  $-20^\circ$  azimuth. As noted earlier, the STAR algorithm can only achieve good performance if  $L$  is large, thus increasing the computational cost. Figure 1 illustrates this fact. Figure 2 shows the performance improvement of the time varying STAR filter (11) for  $L = 2$  over the STAR filter (9) for  $L = 5$ . Also shown is the optimized pre-Doppler algorithm which has a much wider clutter notch than the other filters.

Figures (3) and (4) have the jammer located at  $-1^\circ$  azimuth to illustrate performance when there is a jammer in the main beam. Figure (3) shows that poor performance results when using only one fast-time tap as in standard two dimensional STAP. The parameters of the filters are the same as in Figure (2). Figure (4) shows the performance of the three dimensional algorithms using  $P = 2$  fast-time taps. The 3D-STAR filter is implemented with  $L = 2$  and  $J = 2$ . A dramatic improvement in performance is seen over using a single fast-time tap for both of the algorithms but the pre-Doppler filter still has a wider clutter notch than 3D-STAR.

## 6. CONCLUSIONS

We have presented an extension to the space-time autoregressive (STAR) filter for the hot clutter scenario. The extensions included making the filter time varying across pulses and adding fast-time degrees of freedom to mitigate main beam jamming signals. These two extensions, the time varying STAR (TVSTAR) filter and the 3D-STAR filter, are able to provide good performance in the presence of hot clutter. In addition we have shown that the STAR based algorithms provide an improvement in performance over the reduced dimension pre-Doppler algorithm. This improvement occurs primarily around the clutter notch which results in a better minimum detectable velocity.

## 7. REFERENCES

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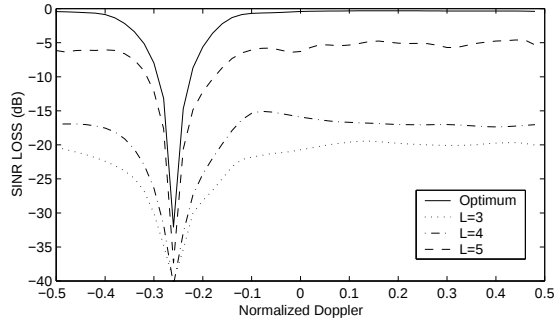


Fig. 1. SINR loss for the STAR algorithm as a function of the filter order  $L$

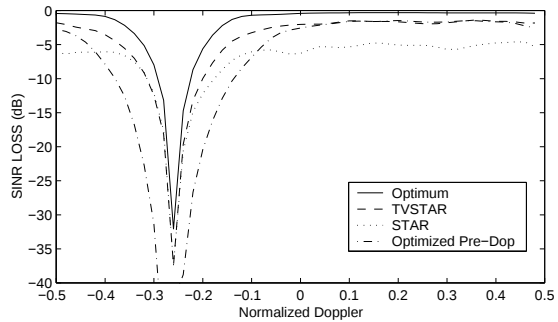


Fig. 2. SINR loss for the STAR, TVSTAR, and optimized pre-Doppler

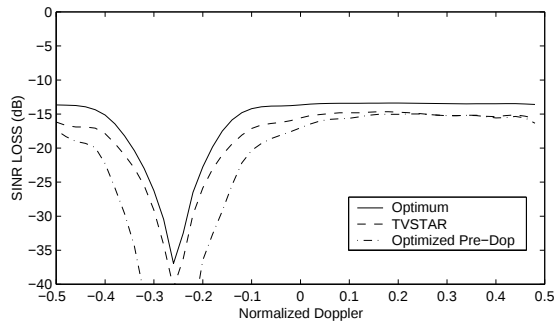


Fig. 3. SINR loss with a main beam jammer and one fast-time tap

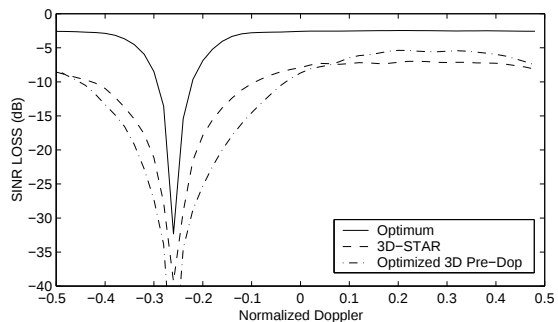


Fig. 4. SINR loss with a main beam jammer and two fast-time taps