MULTIUSER DETECTION IN FLAT RAYLEIGH FADING CHANNEL USING AN APPROXIMATE MMSE ALGORITHM

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ABSTRACT

We study multiuser detection for synchronous DS-CDMA system in a flat Rayleigh fading channel. An approximate MMSE detector which is independent of the magnitude of the received signal is proposed. This detector is not only easier to implement but also easier to analyze than the exact MMSE detector. This is especially true for the case employing ML sequences. In this case we have developed a closed form expression for its performance. Numerical simulations for CDMA systems employing ML and random sequences are provided. These results demonstrate that the performance difference between the exact and approximate MMSE algorithms is very small which makes the later a very good approximation.

1. INTRODUCTION

Communication systems employing code division multiple access (CDMA) have received considerable attention over the past decade. Several suboptimum and lower complexity multiuser detection algorithms for DS-CDMA have been proposed. Analytical and numerical evidence has illustrated that these algorithms work very well in an additive Gaussian noise channels [1]. Compared with the numerous studies on multiuser detection in Gaussian channel, the studies on multiuser detection in the presence of Rayleigh fading are rather rare [2, 3, 4]. None of these previous studies analyze the bit error rate (BER) of the minimum mean square error (MMSE) detector. In this paper, we investigate MMSE multiuser detection and propose a new and simple approximate MMSE algorithm. The proposed algorithm provides considerable complexity reduction. It can also be useful for performance analysis of the exact MMSE detector.

This paper is organized as follows. In Section 2, an overview of the multiuser CDMA system and Rayleigh fading channel model is given. The MMSE detector is considered and an approximate algorithm relying only on the phase of the received signal is proposed in Section 3. In Section 4, the performance analysis of the approximate MMSE algorithm is provided. Numerical simulations which illustrate our analysis are presented in Section 5. Conclusions appear in Section 6.

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2. PROBLEM FORMULATION

A K-user synchronous CDMA system is modeled as [1]

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \quad t \in [0, T], \tag{1}$$

where T is the bit interval, $y(t)$ is the received signal, A_k is the complex received amplitude of the k^{th} user's signal, $b_k \in \{-1, 1\}$ is the bit transmitted by the k^{th} user, $s_k(t)$ is the deterministic signature waveform assigned to the k^{th} user which is normalized so as to have unit energy, $n(t)$ is white Gaussian noise and σ is the standard deviation of the noise.

One way of converting the received signal into a discrete time process is by first passing it through a bank of matched filters, each matched to the signature waveform of a different user, and then sampling the outputs of this matched filter bank at the end of each bit interval. It can be shown that for the model in (1) the vector of samples is a sufficient statistic and no performance loss is caused by this sampling procedure [1]. Let y_k denote the output of the filter matched to the k^{th} user and $\mathbf{y} = (y_1, \ldots, y_K)^T$, then

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n}, \tag{2}$$

where $\mathbf{R}$ is the $K \times K$ crosscorrelation matrix with the $k-j^{th}$ entry $\rho_{kj} = \int_0^T s_k(t) s_j^*(t) dt$, $\mathbf{A} = diag[A_1, \ldots, A_K]$, $\mathbf{b} = (b_1, \ldots, b_K)^T$ and $\mathbf{n} = (n_1, \ldots, n_K)^T$ with $n_k = \int_0^T n(t) s_k^*(t) dt$. In (2) $\mathbf{n}$ is a complex Gaussian vector with zero mean and covariance matrix equal to $2\sigma^2 \mathbf{R}$ and its real and imaginary components are independent.

Assuming the standard flat Rayleigh fading model, the real and imaginary components of A_k are also independent zero-mean Gaussian random variables so that the phase of A_k, $\angle A_k$, is uniformly distributed on $[0, 2\pi]$ while the magnitude of A_k, $|A_k|$, has the Rayleigh probability density function

$$f_{|A_k|}(r) = \begin{cases} \frac{r}{A} exp(-\frac{r^2}{2A^2}) & r \geq 0, \\ 0 & r < 0, \end{cases} \quad (3)$$

where A^2 is referred to as the energy of the received signal. We define the signal-to-noise ratio (SNR) as

$$SNR = 10log_{10}(\frac{A^2}{\sigma^2}).$$

In the following section we will describe MMSE multiuser detection algorithms and for simplicity we assume $\mathbf{R}$ and A are known to the receiver and $A_1, \ldots, A_k$ are independent.

3. MMSE MULTIUSER DETECTION

The MMSE detector can be found from optimum linear estimation theory by choosing the $K \times K$ matrix $\mathbf{M}$ that achieves

$$\min_{\mathbf{M} \in R^{K \times K}} E[\|\mathbf{Ab} - \mathbf{My}\|^2].$$

It has been shown that the MMSE detector should make decisions according to the rule [5]

$$\hat{\mathbf{b}} = sgn(Re\{\mathbf{A}^* \mathbf{My}\}), \quad (4)$$

where

$$\mathbf{M} = [\mathbf{R} + 2\sigma^2(\mathbf{AA}^*)^{-1}]^{-1}. \quad (5)$$

Conventional coherent receivers ultimately obtain $\angle A_k$ through carrier recovery and timing synchronization techniques [6]. But the decision rule for the MMSE detector in (4) also requires the knowledge of $|A_k|$. Thus estimation of $|A_k|$ is necessary for the exact MMSE detector. Estimation methods which use training sequences reduces transmission efficiency and increases the necessary processing [7].

Since the sign of the real component will be invariant to the magnitude of the variable, it follows that only the calculation of $\mathbf{M}$ in (4) requires the knowledge of $|A_k|$. We suggest replacing $\mathbf{AA}^*$ with its average value $2A^2\mathbf{I}$ in (5), where $\mathbf{I}$ is the identity matrix. Thus our proposed algorithm approximates $\mathbf{M}$ as

$$\mathbf{M} = (\mathbf{R} + \sigma^2 A^{-2}\mathbf{I})^{-1}. \quad (6)$$

Obviously the approximate MMSE detector is easier to implement. Since $\mathbf{R}$ and A are known, the matrix $\mathbf{M}$ need to be calculated only once. Its complexity is also lower. Further, the algorithm should be more robust to poor estimates. Later in this paper, we will show that the approximate MMSE detector can achieve performance very close to the exact MMSE detector and much better than the single-user matched filter (MF) and the decorrelator.

4. PERFORMANCE ANALYSIS

The single-user MF makes decisions directly as

$$\hat{\mathbf{b}} = sgn(Re\{\mathbf{A}^*\mathbf{y}\}).$$

From (3.61) and (3.66) in [1], the BER of the single-user MF is

$$\begin{aligned} P_{e,k} &= E\left[Q\left(\frac{|A_k|}{\sigma_k} + \sum_{j \neq k} \frac{Re\{A_j\}\rho_{kj}}{\sigma_k}\right)\right] \\ &= E\left[Q\left(\frac{|A_k|}{\sqrt{\sigma_k^2 + A^2 \sum_{j \neq k} \rho_{kj}^2}}\right)\right] \\ &= \int_0^\infty \frac{r}{A} exp(-\frac{r^2}{2A^2}) Q\left(\frac{r}{\sqrt{\sigma_k^2 + A^2 \sum_{j \neq k} \rho_{kj}^2}}\right) dr \\ &= \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{\sigma_k^2}{A^2} + \sum_{j \neq k} \rho_{kj}^2}}\right). \end{aligned} \quad (7)$$

All linear multiuser detectors, including the exact and approximate MMSE detector, can be formulated as

$$\begin{aligned} \tilde{y}_k &= (\mathbf{My})_k \\ &= \tilde{A}_k b_k + \sum_{j \neq k} \tilde{A}_j b_j + \tilde{n}_k \\ &= \tilde{A}_k b_k + \sum_{j \neq k} \frac{|\tilde{A}_k|}{|\tilde{A}_j|} \tilde{A}_j b_j \tilde{\rho}_{kj} + \tilde{n}_k, \end{aligned} \quad (8)$$

where $\tilde{A}_j = (\mathbf{MR})_{kj} A_j$, $\tilde{\rho}_{kj} = |\tilde{A}_j|/|\tilde{A}_k|$, $\tilde{n}_k \sim \mathcal{N}(0, 2\tilde{\sigma}_k^2)$ and $\tilde{\sigma}_k = \sigma(\mathbf{M^H RM})_{kk}^{1/2}$. Comparing $\tilde{y}_k$ from (8) with y_k from (2), we notice that the two equations have exactly the same form and all corresponding variables have the same type of distribution. Thus the formula for the BER of the single-user MF in (7) can be used to calculate the BER of the approximate MMSE detector by replacing ρ_{kj} with $\tilde{\rho}_{kj}$ and σ_k with $\tilde{\sigma}_k$.

In the general case it appears difficult to determine the expression for $\tilde{\rho}_{kj}$ and $\tilde{\sigma_k}$. For a DS-CDMA system with K users each employing a maximal-length (ML) sequence of length N as its signature waveform, it is easy to prove that $\mathbf{R} = \mathbf{I}_K(1, -1/N)$ where $\mathbf{I}_K(\alpha, \beta)$ denotes a $K \times K$ matrix with diagonal elements all equal to α and other elements all equal to β. In the case of the approximate MMSE detector,

$$\begin{aligned} \mathbf{M} &= (\mathbf{R} + \sigma^2 A^{-2}\mathbf{I})^{-1} \\ &= \mathbf{I}_K\Bigg(\frac{1 + \frac{\sigma^2}{A^2} - \frac{K-2}{N}}{(1 + \frac{\sigma^2}{A^2})^2 - (1 + \frac{\sigma^2}{A^2})\frac{K-2}{N} - \frac{K-1}{N^2}}, \\ &\qquad \frac{\frac{1}{N}}{(1 + \frac{\sigma^2}{A^2})^2 - (1 + \frac{\sigma^2}{A^2})\frac{K-2}{N} - \frac{K-1}{N^2}}\Bigg). \end{aligned}$$

Using (7) and property of the matrix $\mathbf{I}_K(\alpha, \beta)$, after some algebra we obtain a closed-form formula for the BER of the approximate MMSE detector which is presented in (11). Compared with the BER formulas given in [1], we see in fact both the single-user MF and the decorrelator are limiting cases of the approximate MMSE detector. When $\sigma \to \infty$, the approximate MMSE detector approaches the single-user MF. When $\sigma \to 0$, the approximate MMSE detector approaches the decorrelator.

The asymptotic efficiency is another useful performance measure which is defined as [2]

$$\eta = \lim_{\sigma \to 0} \frac{\sigma^2}{A^2[1/(1-2P_e)^2 - 1]}. \tag{9}$$

From its definition, the asymptotic efficiency is only good for the high SNR region. It measures the relative energy that a multiuser system would require to achieve BER equal to that in the ideal system employing orthogonal signature waveforms with the same background noise. Immediately we conclude that for the ideal system employing orthogonal signature waveforms $\eta = 1$.

Now for the approximate MMSE detector, when $\sigma \to 0$, (11) yields

$$\eta = \frac{(N+1)(N-K+1)}{N(N-K+2)}. \tag{10}$$

In the special case of $N = K$, we have $\eta = (N+1)/2N$. If N is large, then $\eta \approx 0.5$. This result implies that there is always a $3dB$ gap between the performance of the approximate MMSE detector and the ideal performance when $K/N = 1$. However, we can show that this gap decreases very fast as K decreases. Thus for large N, the approximate MMSE detector can achieve near-optimum performance even if K/N is very close to 1.

5. SIMULATION RESULTS

In this section, we investigate the performance of the proposed approximate MMSE algorithm using numerical simulations. A slowly varying, flat Rayleigh fading channel is consiered here. The bandwidth efficiency of DS-CDMA system is defined as the ratio of number of users to length of spreading sequence, K/N. All presented results are calculated on the basis of 1,000,000 Monte Carlo trials.

First we will investigate CDMA systems employing ML sequences. Figure 1 examines our formula for the BER of the proposed approximate MMSE algorithm. In this figure,

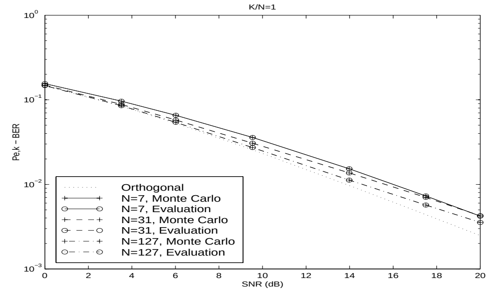

Fig. 1. Comparison between Monte Carlo and evaluation

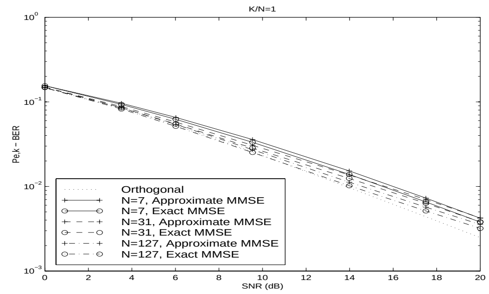

Fig. 2. Performance for ML sequence CDMA (I)

the curves obtained from Monte Carlo simulations and the curves obtained by evaluating the formula previously presented in (11) are visually indistinguishable. This example gives us confidence in the correctness of (11). Figure 2 compares the performance of the exact and approximate MMSE detectors with different values of N, while fixing the bandwidth efficiency K/N to be 1. Our results show that the performance of the approximate MMSE detector is very close to the performance of the exact MMSE detector in all cases. Now we fix N and vary K, i.e. vary the bandwidth efficiency K/N. Figure 3 and 4 compare the performance of the single-user MF, the decorrelator, the approximate and exact MMSE detector. The performance of an ideal CDMA system employing orthogonal signature waveforms is also presented as a reference. From these figures, we observe that, again in all cases, the approximate MMSE detector achieves performance very close to the exact MMSE detector and outperforms the other two linear multiuser algorithms. Further, as expected, in the low SNR region the performance of both the approximate and exact MMSE approach that of the single-user MF. In the high SNR region the performance of both the approximate and exact MMSE approach that of the decorrelator.

$$P_{e,k} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{\sigma^2}{A^2}\frac{(1+\frac{\sigma^2}{A^2}-\frac{K-2}{N})^2-(1+\frac{2\sigma^2}{A^2}-\frac{K-2}{N})\frac{K-1}{N^2}}{(1+\frac{\sigma^2}{A^2}-\frac{K-2}{N}-\frac{K-1}{N^2})^2} + \frac{K-1}{N^2}\left(\frac{\frac{\sigma^2}{A^2}}{1+\frac{\sigma^2}{A^2}-\frac{K-2}{N}-\frac{K-1}{N^2}}\right)^2}}\right). \tag{11}$$

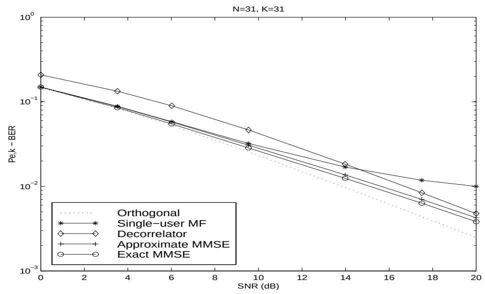

Fig. 3. Performance for ML sequence CDMA (II)

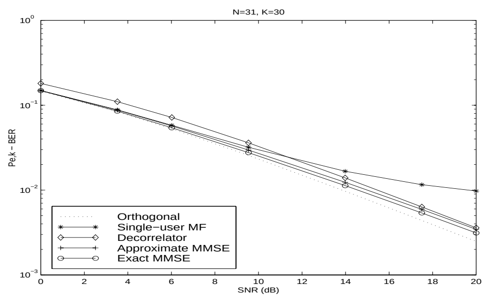

Fig. 4. Performance for ML sequence CDMA (III)

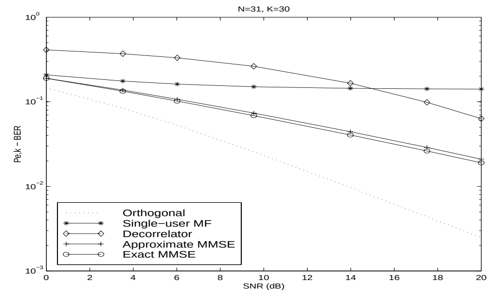

Fig. 5. Performance for random sequence CDMA (I)

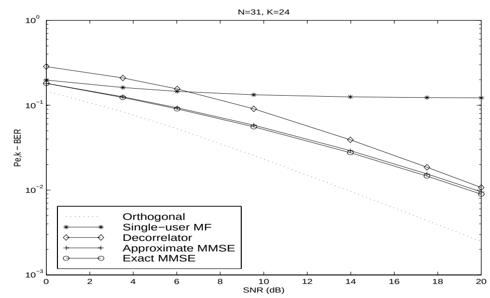

Fig. 6. Performance for random sequence CDMA (II)

Some practical CDMA systems, such as IS-95, employ very long PN sequences. However, no attempt is made to design signature waveforms with low crosscorrelation, and the long period enable the approximation of randomly selected sequences for the purposes of analysis.

Next, we will investigate CDMA systems employing random sequences. All results are obtained by averaging the performance of 100 different realizations of the crosscorrelation matrix $\mathbf{R}$. Figure 5 and 6 compare the performance of various linear multiuser algorithms in this case. All our conclusions drawn from Figure 3 and 4 still hold in these figures. These results indicate that the accuracy of approximation for the proposed algorithm is still satisfying.

Figure 1 through 6 are representative of many cases we have tried. All these cases suggest that the proposed algorithm provides a very good approximation to the exact MMSE detector.

6. CONCLUSIONS

In this paper, we have formulated and analyzed the problem of multiuser reception for synchronous DS-CDMA system in a flat Rayleigh fading channel. We propose an approximate MMSE multiuser detection algorithm. The proposed algorithm has lower complexity and is more robust to poor estimates. In the context of employing ML sequences, we develop a closed form expression for the BER of this algorithm and use it to analyze the asymptotic efficiency. The BER of the approximate MMSE detector employing ML and random sequences is also investigated using numerical methods. We show that it can achieve performance very close to the exact MMSE detector and much better than the single-user MF and the decorrelator.

7. REFERENCES

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