EFFICIENT EXTRACTION OF EVOKED POTENTIALS BY COMBINATION OF WIENER FILTERING AND SUBSPACE METHODS

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ABSTRACT

A novel approach is proposed in order to reduce the number of sweeps (trials) required for the efficient extraction of the brain evoked potentials (EPs). This approach is developed by combining both the Wiener filtering and the subspace methods. First, the signal subspace is estimated by applying the singular-value decomposition (SVD) to an enhanced version of the raw data obtained by Wiener filtering. Next, estimation of the EP data is achieved by orthonormal projecting the raw data onto the estimated signal subspace. Simulation results show that combination of both two methods provides much better capability than each of them separately.

1. INTRODUCTION

The sensory brain evoked potentials (EPs) are electrical responses of the central nervous system to sensory stimuli applied in a controlled manner. The interest in these potentials arises from their utilization as clinical and research tools and for their contribution to the basic understanding of the functions of the brain [1]-[3], [5], [7]. Ensemble averaging and weighted ensemble averaging have been usually used to enhance the SNR [1]. Such techniques can be thought of as lowpass filtering of noise and a very large number of sweeps is required to obtain a suitable EP estimate. Wiener Filtering based techniques have also been extensively used for the enhancement and recovering of the EP [3], [7]. In one adaptive implementation of the Wiener filter, the noisy EPs are taken as the primary input while, the auxiliary reference input has been taken as constructed models of the EPs because the reference noise is in general not available. Various kinds of basis functions have been used to construct such models [4]. The performance of this approach is then dependent on how much the assumed model is close to the EP signal. In another approach, where multiple sweeps are available, the primary input is taken as the ensemble average while the reference input is taken as one sweep that is not included in the average, which keeps noise uncorrelation. Unfortunately, the Wiener filtering method deteriorates if both the signal and noise spectra are overlapped.

The subspace method implemented using the singular value decomposition (SVD) is commonly used for enhancing nonaveraged data [2], [6]. In this technique the space of the observed data is decomposed into signal and noise subspaces. Because both the signal and noise subspaces are orthogonal, orthonormal projecting the observed signal onto the signal subspace leads to the reduction of the noise. It is known that SVD can efficiently decompose the observed signal space if the signal-to-noise ratio (SNR) is relatively high. However, SNR may be as low as 0 to –10.0 dB, which deteriorates the performance of the subspace method.

In the present work, we propose to integrate both the Wiener filtering and the subspace methods for the extraction of the EP. This combination indeed reduces the number of sweeps required to achieve suitable extraction. In Section II we give a brief review for the Wiener filtering and the conventional subspace methods. In Section III, the proposed approach is described. Section IV presents extensive simulation results and finally Section V gives the conclusions.

2. CONVENTIONAL METHODS

2.1. Signal Model and Problem Formulation

Multiple sweeps of the observed EP can be modeled as

$$x_i(n) = s(n) + v_i(n) + z_i(n), \quad i = 1,2,..., L;\ 0 \le n \le N-1 \tag{1}$$

where $s(n)$ is the unknown EP signal which is deterministic for all i, the noise $v_i(n)$ is due to the spontaneous brain activity called electroencephalogram (EEG) which is temporally-correlated random process and its spectrum may be overlapped with the spectrum of the EP signal $s(n)$, and $z_i(n)$ is the sensors white noise. The objective is to extract the EP signal $s(n)$ given L sweeps (trials).

2.2. Wiener Filtering

In Wiener filtering [7], to enhance the ith sweep signal, the filter input $x_i(n)$ is the ith sweep signal that contains the EP plus the noise. The desired signal $d_i(n)$ is computed as the ensemble average of the L sweeps excluding the ith sweep, i.e.,

$$d_i(n) = \frac{1}{L-1} \sum_{j=1,\ j \ne i}^{L} x_j(n) \tag{2}$$

Excluding the ith sweep, which is the input of the filter, makes the noise in both the desired and the input uncorrelated. The filter output is given by

$$y_i(n) = \sum_{m=0}^{M-1} h_i(m) x_i(n-m) \tag{3}$$

where $h_i(m)$ is the impulse response of length M of the ith filter. The output $y_i(n)$ is an estimate of $s(n)$ if the filter impulse response is computed so as to minimize the mean square of the error signal $e_i(n) = d_i(n) - y_i(n)$. This optimum impulse response is known as the solution of the Wiener-Hopf equations given by

$$\sum_{m=0}^{M-1} h_i(m) r_{x_i}(m-k) = p(-k), \quad k = 0,1,..., M-1 \qquad (4)$$

where $r_{x_i}(m-k) =< x_i(n-k)x_i(n-m) >$ is the auto-correlation function of the input and $p(-k) =< x_i(n-k)d_i(n) >$ is the cross-correlation between the input and the desired in which <.> stands for the time average operator.

Unfortunately, if both the spectra of the noise and the EP signal are overlapped, the filter cannot reject completely such noise. Therefore the Wiener filtering approach is capable of reducing only white noise and colored noise whose spectra are not overlapped with that of the EP signal.

2.3. Subspace Method

Singular value decomposition (SVD) of a given signal data matrix contains information about the signal energy, the noise level, the number of sources, and enable to divide data to signal and noise subspaces [6]. A brief review of SVD subspace method can be described as follows. In the matrix form the available data can be expressed as

$$X = [x_1, x_2, \cdots, x_L] \qquad (5)$$

where x_i is given by

$$x_i = [x_i(0),\ x_i(1), \cdots, x_i(N-1)]^T \qquad (6)$$

The SVD of matrix X for $L < N$ is given by

$$X = U\Sigma V^T \qquad (7)$$

where the matrices $U \in R^{N\times L}$, $V^T \in R^{L\times L}$ are orthonormal such that $U^TU = I_L$, $VV^T = I_L$ and $\Sigma = diag(\lambda_1,\ \lambda_1, \cdots, \lambda_L) \in R^{L\times L}$, with $\lambda_1 > \lambda_1 > \cdots \lambda_L > 0$. The columns of both U and V are called the right and left singular vectors of X, respectively. The diagonal entries of Σ are called the singular values of X, which give information about, the number of signals, signal energy and the noise level. If the signal-to-noise ratio is relatively high, the matrix X can be decomposed as

$$X = [U_s \ \ U_n]\begin{bmatrix}\Sigma_s & 0 \\ 0 & \Sigma_n\end{bmatrix}[V_s \ \ V_n]^T \qquad (8)$$

where Σ_s contains *s* largest singular values associated with *s* source signals and Σ_n contains $L - s$ singular values associated with the noise part. U_s and V_s contain *s* singular vectors associated with the signal part; and U_n and V_n contain $L - s$ singular vectors associated with the noise part. The subspace spanned by the columns of U_s is referred to as the signal subspace and the subspace spanned by the columns of U_n is referred to as the noise subspace. Both the signal and noise subspaces are orthogonal to each other thus we obtain the best least squares approximation of the observed noisy measurements. Therefore orthonormal projecting the noisy data onto the signal subspace leads to the reduction of the noise. This orthonormal projection is given by

$$Y = U_s(U_s^TU_s)^{-1}U_s^TX \qquad (9)$$

or simply by

$$Y = U_sU_s^TX \qquad (10)$$

due to the orthonormal property of the singular vectors.

It is worth to mention that SVD subspace method provides better performance in the case of relatively high SNR. This is because for high SNR, the signal singular values are substantially larger than the noise singular values and thus powering would widen the separation of the noise and signal singular values.

3. THE PROPOSED APPROACH

As mentioned, although the Wiener filtering method provides SNR improvement, the in-band noise cannot be removed in this single stage. However, due the fact that for real-world problems the SNR is relatively low, applying directly the conventional subspace method may provide also rather poor results. For this reason, we propose to apply the SVD to a matrix constructed from the filtered data $y_i(n)$. In this case the filtered data matrix can be written as (see (7) and (8))

$$Y = [y_1, y_2, \cdots, y_L] = \hat{U}\hat{\Sigma}\hat{V}^T \qquad (11)$$

where $y_i = [y_i(0),\ y_i(1), \cdots, y_i(N-1)]^T$. It should be noted that due to the SNR enhancement associated with the matrix Y compared with the raw EP data matrix X, the signal subspace referred to as $\hat{U}_s$ will be an enhanced estimate of the conventional subspace U_s obtained from the raw data. After the estimation of the signal subspace $\hat{U}_s$, the desired EP data can be obtained by orthonormal projecting the raw EP data X onto this signal subspace as

$$\hat{X} = \hat{U}_s\hat{U}_s^TX \qquad (12)$$

where $\hat{X}$ represents the matrix data of the desired EP of the overall approach. It is worth to note that the rank of the signal subspace is equal to the number of sources. For the deterministic EPs model, the number of sources is one and the first column of $\hat{U}_s$ represents the signal subspace. This is also valid when the EP signal *s*(*n*) from sweep to sweep is only amplified by different constant gains.

4. SIMULATION RESULTS

To examine the effectiveness of the proposed approach, extensive simulations have been carried out. Due to space limitations we present here only two illustrative examples. In these examples, two simulated models for the EP *s*(*n*) given in Figure 1 (top) and Figure 3.a are used with sampling frequency 6 *kHz* (i.e., *N* = 1500). The temporally correlated EEG activity *v*(*n*) is generated using the following autoregressive model [7]

$$\begin{aligned} v(n) = 1.5084\,v(n-1) - 0.1587v(n-2) - 0.3109\,v(n-3) \\ - 0.0510\,v(n-4) + u(n) \end{aligned} \qquad (13)$$

with *u*(*n*) representing a zero-mean white Gaussian noise. The white noise *z*(*n*) is modeled as a zero-mean white Gaussian noise. To generate 20 sweeps, 20 realizations of both *v*(*n*) and *z*(*n*) are added to *s*(*n*). The SNR's are 1.0 and 0.25 for colored and white noise, respectively, which implies that the total SNR is –7.0 dB. The filter length *M* is 128. From Figures 1, it is observed that both spectra of the colored noise and the EP signal are overlapped. From Figure 2.b, it is observed that the Wiener filter removes only the out-of-band noise. From Figure 2.c, it is apparent that the conventional subspace provides no efficient SNR improvement. Figure 2.d shows that the presented approach outperforms each of the Wiener filtering and subspace methods. Figures 3.a, b, c, d and e show the first sweep of the observed signals, the first sweep of the outputs of the Wiener filter, and the estimated EP signals using both the subspace method and the presented approach, for the second EP signal model, respectively.

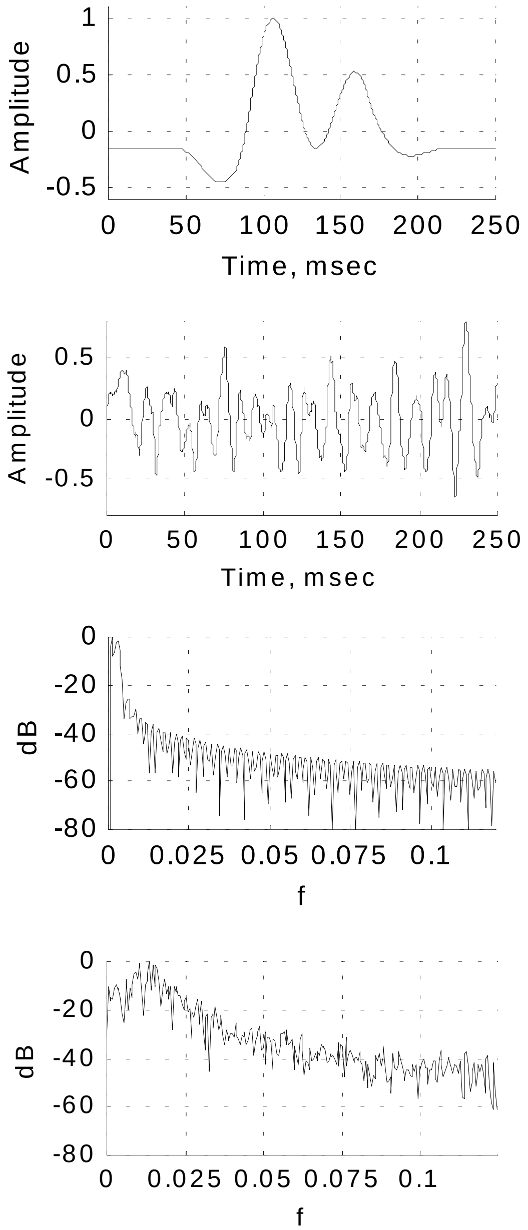
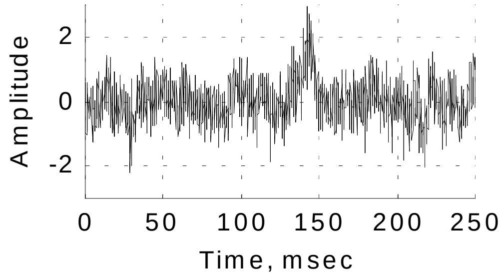


Figure 1. The first EP signal model, the additive colored noise and their spectra from top-to-bottom respectively.



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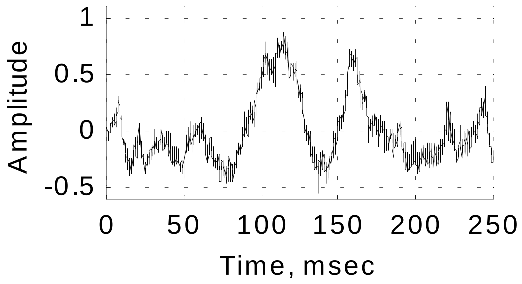

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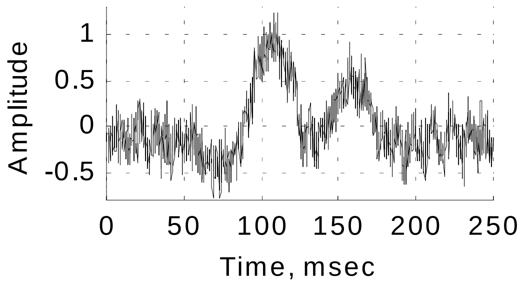

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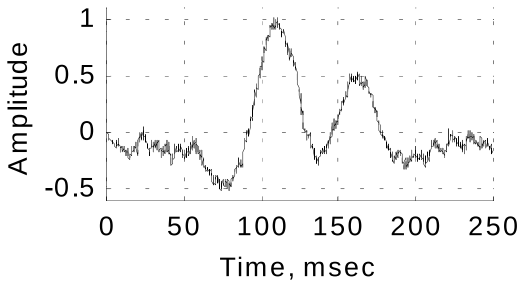
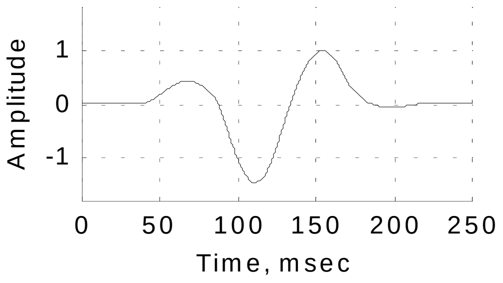


d

Figure 2. Results in the case of the first EP signal model: **a**, one sweep of the noisy EP; **b**, one sweep of the Wiener filter outputs; **c**, the enhanced EP signal using the conventional subspace method; **d**, the enhanced EP signal using the approach presented.

Results of the second EP signal model confirm also the efficiency of the presented approach. In both EP examples, the first column of the right singular values is taken as the signal subspace. Comparing the estimated EP signal of the approach presented with that of the conventional subspace method shows that the latter is still noisy while the former is a good replica of the noise-free EP signal.



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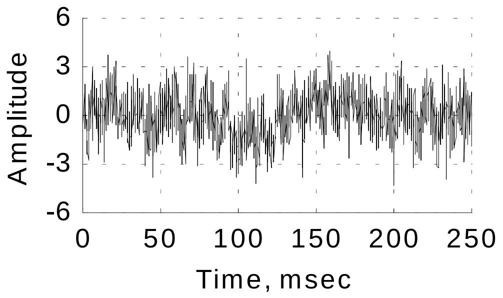

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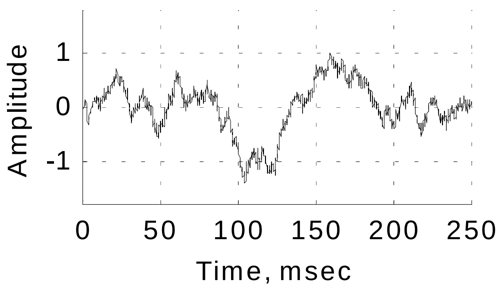

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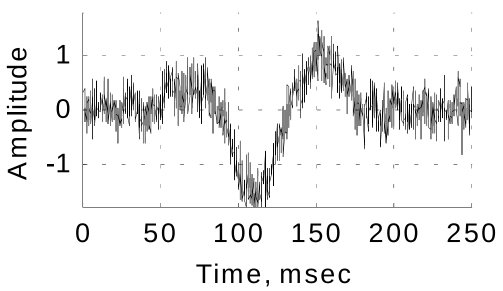

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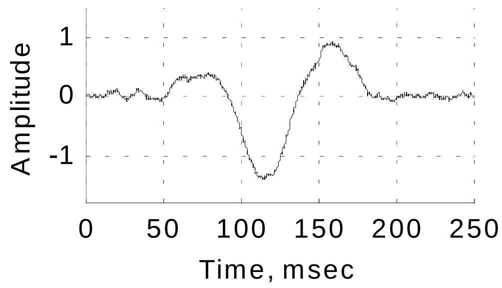

e

Figure 3. Results in the case of the second EP signal model: **a**; the noise-free EP; **b**, one sweep of the noisy EP; **c**, one sweep of the Wiener filter outputs; **d**, the enhanced EP signal using the conventional subspace; e, the enhanced EP signal using the approach presented.

5. CONCLUSION

An approach has been described for the efficient extraction of the brain evoked potentials. In this approach, for the estimation of the signal subspace, the SVD is applied to an enhanced version of the raw data obtained using the Wiener filtering technique. The final enhanced evoked potential data are obtained by orthonormal projecting the raw data onto the estimated signal subspace. The approach can be used to reduce the number of sweeps required for reliable extraction of the EP signal from huge noise.

6. REFERENCES

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