FUZZY ANISOTROPIC DIFFUSION FOR SPECKLE FILTERING

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ABSTRACT

An anisotropic diffusion filter contoled by fuzzy rules is presented. The proposed filter is based in the Perona-Malik technique, using a fuzzy reasoning to calculate de diffusion coefficient which controls the whole diffusion. The method has the advantage that it can be used for both smoothing and noise cleaning, as well as edge enhancement. This new aproach also allows ut to model the diffusion process through a rule base to have a better performance. Some examples are given to ilustrate the effectiveness of the proposed technique.

1. INTRODUCTION

Speckle is the term used for granular patterns that appears on some types of images, as, for example, ultrasonic images, due to the mottling, and it can be considered as a kind of multiplicative noise. Speckle degrades the quality of the image, and hence it reduces the ability of human observer to discriminate fine details, and it also makes further image processing more dificult. Ordinary filters, such as linear filters or median filters, do not work well for edge preserving smoothing of images corrupted with noise.

Perona and Malik [1] developed a multiscale smoothing and edge enhancement scheme which has proved to be a powerful tool for noise cleaning. Their anisotropic diffusion filtering method is mathematically formulated as a heat diffusion process, which smoothes region interiors, but not their interfaces. This work was further developed in [2, 3].

The main problem with anisotropic diffusion algorithms is that they need a large amount of iterations to reach its steady state. This means much time consumption and an important influence of border effects. In addition, in a very noisy enviroment, the resulting image after a large amount of iterations has not a real appearance. Consequently several proposals have appeared as feasible options to anisotropic diffusion [4, 5, 6].

In the present paper we introduce a new type of edge-preserving smoothing filter controlled by fuzzy rules. This proposed filter is based on Gerig's anisotropic filter [2, 1]. Our improved method is based on calculating the *diffusion coefficient* by a fuzzy inference, instead of by a decreasing function of the gradient, which allows us to have a more detailed control over the diffusion process.

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2. ANISOTROPIC FILTERING

The basic idea of anisotropic diffusion filtering is that smoothing and edge enhancement can be modelled as a diffusion process in wich a flow exists between adjacents cells containing substances such as gases or fluids. A diffusion process of the type needed for our problem can be described by a partial differential equation of the form:

$$\frac{\partial I(\vec{x},t)}{\partial t} = div(c(\vec{x},t)\nabla I(x,y,t)) \qquad (1)$$

where div denotes the *divergence operator* and ∇ denotes the gradient. The function $c(\vec{x},t)$ is called the *diffusion coefficient*. In conjuntion with the gradient it describes the *flow* between cells:

$$\Phi = c(\vec{x},t)\nabla I(\vec{x},t) \qquad (2)$$

This coefficent is defined as a monotonically decreasing function of the gradient magnitude, so that the flow increases within homogeneus regions where the gradient is small. Several functions has been proposed ([3, 2]).

In 2-D case (for simplicity) we can approximate equation 1 and write

$$I(x,y,t+\Delta t) \approx I(x,y,t) + \Delta t \frac{\partial I(x,y,t)}{\partial t} \qquad (3)$$
$$\approx I_t + \Delta t(\Phi_E - \Phi_W + \Phi_N - \Phi_S) \qquad (4)$$

where Φ_E, Φ_W, Φ_N and Φ_S are the local flow contributions.

3. FUZZY CONTROLLED ANISOTROPIC DIFFUSION FILTER

Fuzzy rules allow us to process directives described in terms of human-like reasoning. So, instead of using a well-defined function of the gradient in a certain neighbourhood to calculate the *diffusion coefficient*, $c(x, y, t)$, we will use a fuzzy aproximation, which will allow us to make operations such as *IF the difference beteewn pixels gray levels is small, THEN the diffusion coefficient is high*. Such a rule, wich is expresed in a plain linguistic form, can be translated into a more formal struture by a fuzzy operator.

Tha basic idea behind our filter is to use the fuzzy rule base to make the diffusion easier in those areas where the difference of the pixels is small, and to make it harder in the zones where the difference is high. By controlling the behaviour of the diffusion coefficient we control the whole diffusion process.

Fuzzy reasoning has proved its success in modelling the uncertainly that tipically occurs when both noise cancellation and detail preservation represent very critical issues. In the last few years, many approaches have been proposed, in particular, focussing in the area of nonlinear filtering [7].

3.1. Generalized Diffusion Algorithm

We define the contributions of local flow as a function of the difference bewteen the central pixel and each of the 4-connected pixels in every orientation, and the diffusion coefficient for every direction [3, 2]:

$$\begin{aligned}
\Phi_E &= \frac{1}{\Delta x_2^2}\left[C_E(I(x+\Delta x, y, t) - I(x, y, t))\right] \\
\Phi_W &= \frac{1}{\Delta x_2^2}\left[C_W(I(x, y, t) - I(x-\Delta x, y, t))\right] \\
\Phi_N &= \frac{1}{\Delta y_2^2}\left[C_N(I(x, y+\Delta y, t) - I(x, y, t))\right] \\
\Phi_S &= \frac{1}{\Delta y_2^2}\left[C_S(I(x, y, t) - I(x, y-\Delta y, t))\right]
\end{aligned}$$

where C_N, C_S, C_E and C_W are the diffusion coefficients for each orientation (North, South, East and West[1]). The process to obtain them is explained later. Equation 4 describes the iterative process of the general anisotropic diffusion, as well as of our filter. At each new time step $t + \Delta t$, a new image is generated from previous image of step t.

3.2. Fuzzy Diffusion Coefficients

To calculate the four coefficients, we make use of fuzzy operators. We define two distances, D_1 and D_2. The former is the absolute difference between the pixel and the 4-connected pixel in the diffusion direction, i.e. for C_E, $D_1 = |I(x, y) - I(x+1, y)|$. $D2$ is the absolute difference between each 8-connected pixel in the diffusion direction, divided by 2, i.e. for C_E, $D_2 = \frac{1}{2}|I(x+1, y+1) - I(x+1, y-1)|$.[2] So, for each coefficient, i.e. in the considered direction, we use 4 pixels (see figure 1): the central one, the 4-connected and the two 8-connected. We will use these distances to make a

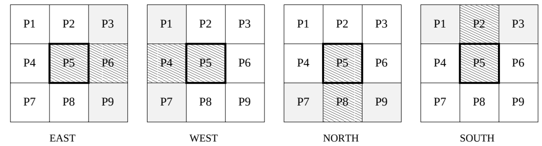

Fig. 1. Pixels involved in each flow

fuzzy inference to calculate C_i. The rules that we will use are as follows:

$$R_j(D_1, D_2): \quad \text{If } D_1 \text{ is } a_j \quad \text{and } D_2 \text{ is } b_j \quad \text{then } C_i \text{ is } c_j$$

Though several strategies have been reported [8], we will resort to the method proposed by Kosko [9] to carry out the fuzzy inferences. A crisp value is simply obtained by a defuzzification method, such as the centroid

$$F(y) = \frac{\sum_j a_j(D_1) b_j(D_2) c_j}{\sum_j a_j(D_1) b_j(D_2)} \tag{5}$$

where a_j and b_j are the antecedent sets and c_j is the centroid of the consequent set. We define two linguistic variables: *Luminance difference* and *diffusion coefficient*. Their fuzzy sets are shown in figures 2 and 3 respectively. We have chosen consistent, normal and complete fuzzy sets with PTS membership functions [10]. The distribution of the sets in the space is the result of heuristic experiments.

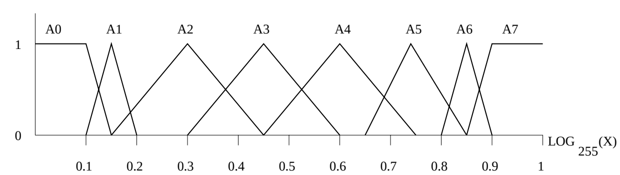

Fig. 2. Luminance differences (antecedent)

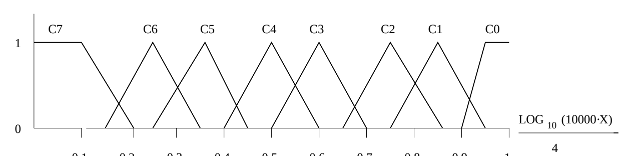

Fig. 3. Diffusion Coefficient (Consequent)

3.3. The Fuzzy Rulebase

As we have already said, the main behaviour of the filter depends on the behaviour of the diffusion coefficients. So, we can control several diffusion attributes such as speed, smoothness, and so forth, with the design of the fuzzy rulebase. The rule construction for image enhancement largely

[1] Paradoxically, North and South are upside down (figure 1). The reason of it, is the position of the y axis; its origin is in the top of the image.

[2] This last distance is obtained from the calculation of terms such as $I(x + \frac{1}{2}, y)$ and $I(x - \frac{1}{2}, y)$ from the approximation of the gradient [3]

$D_1 \setminus D_2$	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
A_0	C_0	C_0	C_1	C_1	C_2	C_3	C_4	C_6
A_1	C_0	C_1	C_1	C_1	C_2	C_3	C_4	C_6
A_2	C_1	C_1	C_1	C_1	C_2	C_3	C_4	C_6
A_3	C_1	C_1	C_1	C_2	C_2	C_4	C_5	C_7
A_4	C_2	C_2	C_2	C_2	C_3	C_4	C_5	C_7
A_5	C_3	C_3	C_3	C_4	C_4	C_5	C_5	C_7
A_6	C_4	C_4	C_4	C_5	C_5	C_5	C_6	C_7
A_7	C_6	C_6	C_6	C_7	C_7	C_7	C_7	C_7

Table 1. Rule Set for the Fuzzy Anisotropic Diffusion

depends on the application domain. We can build rules sets, obtaining different behaviours of the same filter. For our rulebase, we have tried to model a large diffusion for small differences, and a strong restriction to diffusion when the difference is large enough (table 1).

4. EXPERIMENTAL RESULTS

The proposed method has been applied to several images of size 256x256 and 8 bits per pixel, corrupted by multiplicative noise ($\sigma_s^2 = 0.04$) and additive gaussian noise ($\sigma_g^2 = 100$) according to the equation

$$I_s = I_0 + I_0 \Delta r(\sigma_s^2) + \eta(\sigma_g^2) \tag{6}$$

Two original images are shown in Fig. 5(a), Fig. 6(a), and the corrupted ones in Fig. 5(b) and Fig. 6(b).

Fig. 4 shows the output image after 0, 2, 5, 8, 12 and 20 iterations respectively. In the last one, we have already gotten a good quality image. It shows one of the advantages of the Fuzzy Anisotropic Diffusion (FAD) filtering: it requires a smaller amount of iterations, which makes the influence of the border efects smaller, and it makes the algorithm faster as well.

To evaluate the quality of the restored images, the FAD filter is compared to a classical median filter and to a non-fuzzy anisotropic filter. The same noise parameters as before are used. To calculate the diffusion coefficient for the Perona-Malik technique the next equation is used [1]:

$$c(x,y,t) = \frac{1}{(1 + |\nabla I(x,y,t)|/K)^2} \tag{7}$$

with different values of constant K ($K = 2$ and $K = 5$). To obtain stable solutions in all the experiments, we have chosen a value of the time step $\Delta = 0.2$ [1].

The images restored by the FDA filters are in Fig. 5(c) and Fig. 6(c). The results of the Median filter are in Fig. 5(d) and Fig. 6(d). Finally, the results for the anisotropic diffusion (AD) filter are in Fig. 5(e) and Fig. 6(e) for $K = 2$ and in Fig. 5(f) and Fig. 6(f) for $K = 5$.

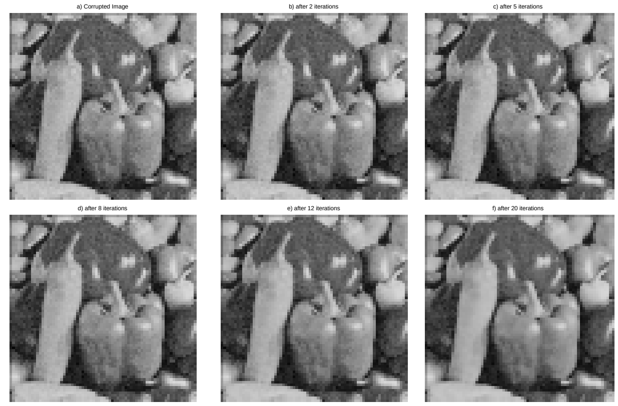

Fig. 4. Evolution of the Fuzzy Diffusion Algorithm. a) Corrupted image. b) After 2 iterations. c) after5 iterations. d) after 8 iterations e) after 12 iterations f) after 20 iterations

From all these images, it can be observed that the visual quality of Fig. 5(c) and Fig. 6(c) (the ones filtered by the FAD filter) are better than the others. The new filter is capable to eliminate the noise without distorting the edges. Although the AD filter with $K = 5$ also removes noise effectively, many small details have disappeared. The whole scene has an artificial look, which is more evident in the shade zones, where new edges appear.

For all of them, the RMS error has been calculated, showing again a better performance of the FAD filter. For the first image (fig 5) the error values are: 0.1138 (corrupted image), 0.0379 (FAD), 0.0751 (Median Filter), 0.0937 (AD, with K=2) and 0.0523 (AD, with K=5). For the second image (fig 6) the error values are: 0.4950 (corrupted image), 0.0379 (FAD), 0.0638 (Median Filter), 0.0827 (AD, with K=2) and 0.0392 (AD, with K=5).

5. CONCLUSIONS

In this paper an improved anisotropic diffusion filter contoled by fuzzy rules is presented. Experimental results have shown that the FAD filter has a better performance than the Perona-Malik filter in images corrupted by both speckle and aditive gaussian noise. As the diffusion is controlled by a set of rules, it is easier to adapt this diffusion to the conditions of the problem, without changing the whole filter. The future work will be oriented to the introduction of techniques for learning and tuning the rule-base from data [11].

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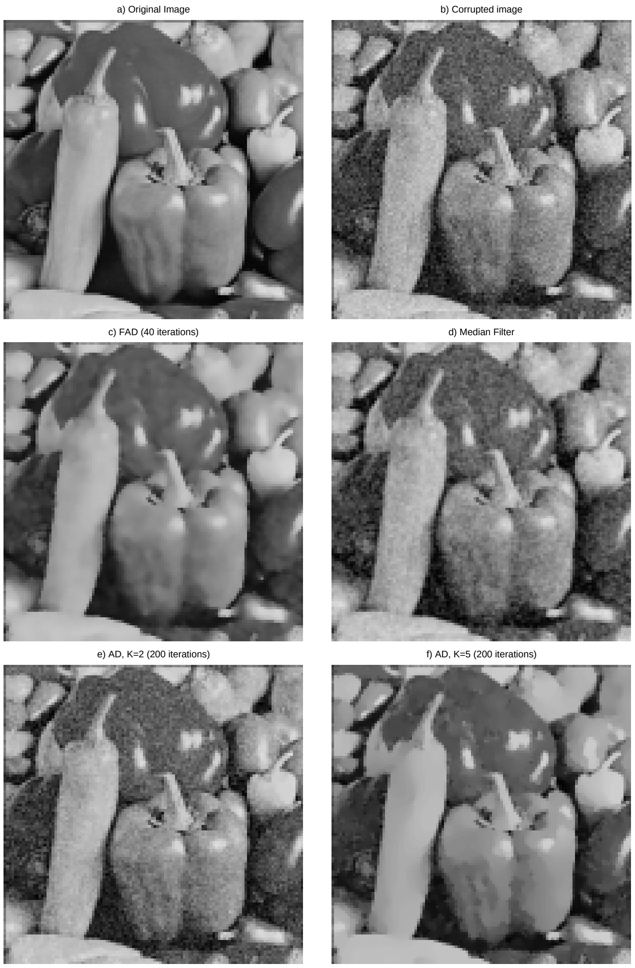

Fig. 5. Comparison of diferent methods of noise smoothing.

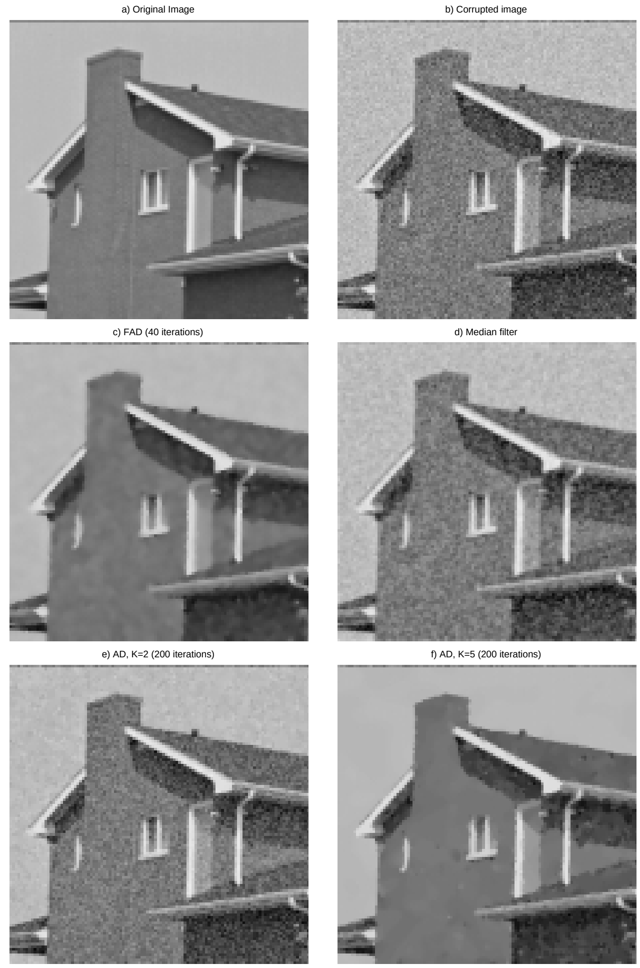

Fig. 6. Comparison of diferent methods of noise smoothing.

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