MODIFIED CHANNEL SUBSPACE METHOD FOR IDENTIFICATION OF SIMO FIR CHANNELS DRIVEN BY A TRAILING ZERO FILTER BANK PRECODER

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ABSTRACT

A modification of Moulines' blind second order statistical Channel Subspace approach is proposed for the identification of single input multiple output finite impulse response channels. The modification exploits the transmitter redundancy introduced by a trailing zero filter bank precoder. The method is shown to be robust to common zeros and channel order over-estimation errors.

1. INTRODUCTION

Using only second order statistics (SOS), it is not possible to blindly identify a single input single output (SISO) channel unless it is driven by a redundant precoder [1, 4] or is fractionally sampled [6, 5, 3]. Using a filter bank precoder to drive the channel has a number of advantages, including spreading the spectrum of the source symbols [2] and thereby mitigating the adverse effects of channel spectral nulls. The advantage of fractionally sampling the output of the SISO channel is that, unlike precoding, no reduction in the information rate is necessary. The disadvantage of fractional sampling is that no blind SOS method can identify the sub-channels if the sub-channels share a common zero.

This paper proposes to use both a precoder and fractionally sampled outputs to identify the channel. A subspace based channel identification algorithm is derived, and it is shown to exhibit better performance than schemes which rely on either a precoder or fractionally sampled outputs alone.

The rationale behind using both a precoder and fractionally sampled outputs is that they complement each other nicely. The fractionally sampled outputs means that the precoder does not need to introduce very much redundancy at all for it to be effective, while the precoder eliminates the problem of common zeros.

The signal model is illustrated in Fig. 1. The complex valued source symbols $s(n)$ are precoded by a filter bank precoder. It is assumed that the filter bank precoder introduces a number of trailing zeros, and hence is referred to as a trailing zero (TZ) precoder. These trailing zeros eliminate inter-block interference [4], greatly simplifying the channel identification procedure. The coded symbols then pass through a single input multiple output (SIMO) finite impulse response (FIR) channel. This SIMO channel results either from fractionally sampling the output of a SISO channel [3], or naturally by using multiple receive antennas. The aim of this paper is to derive a SOS subspace method for identifying the sub-channels in Fig. 1.

The organization of this paper is as follows. Section 2 derives a channel identification algorithm. It is an extension of the ideas in [4, 3]. Simulation results are presented in Section 3 which demonstrate the advantages of the proposed approach.

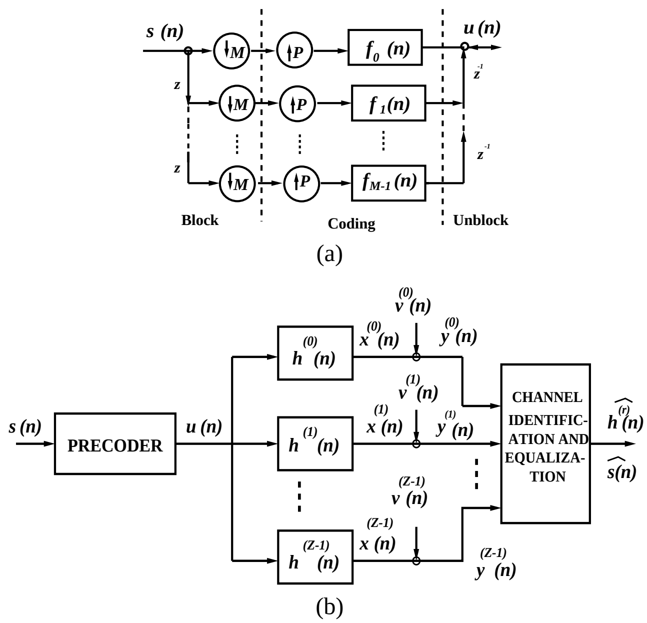

Fig. 1. (a) Precoder Structure, (b) TZ-Precoder with multiple FIR channels

2. TZ-CS: MODIFIED CHANNEL SUBSPACE APPROACH FOR BLIND CHANNEL IDENTIFICATION

Consider the discrete-time multi-rate transmitter and multichannel FIR model arrangement as shown in Fig.1. Note that this multichannel FIR system may have arisen from oversampling the output of a single sensor or by using a multiple receiver system. It consists of Z sub-channels each of order at most L. The zero mean complex valued input symbol stream $s(n)$ and the additive white noise are assumed to be stationary. The noise is assumed to be uncorrelated among channels.

In Fig.1, the precoder maps blocks of M input symbols into blocks of P encoded symbols, which are then sent through the channel. As in [4], the input to the up-sampler of the mth branch is $s_m(n) := s(nM + m)$. It represents the m-th symbol in the nth block of M symbols. With the insertion of $P-1$ zeros, the corresponding upsampler's output is $u_m(n) := \sum_i s_m(i)\delta(n - iP)$ where $\delta(n)$ denotes Kronecker's delta. The transmitted data sequence is:$u(n) = \sum_{m=0}^{M-1} u_m(n) = \sum_i \sum_{m=0}^{M-1} s_m(i) f_m(n - iP)$.

In order to obtain a linear block data model, we make the following definitions:

$$\mathbf{s}(n) = [s_0(n), s_1(n), \ldots, s_{M-1}(n)]^T,$$
$$\mathbf{f}_m = [f_m(0), \ldots, f_m(M-1), 0, \ldots, 0]^T,$$
$$\mathbf{F}_0 = [\mathbf{f}_0 \ldots \mathbf{f}_{M-1}] \in \mathbb{C}^{P\times M},$$
$$\mathbf{x}^{(r)}(n) = [x^{(r)}(nP), x^{(r)}(nP+1), \ldots, x^{(r)}(nP+P-1)]^T,$$
$$\mathbf{v}^{(r)}(n) = [v^{(r)}(nP), v^{(r)}(nP+1), \ldots, v^{(r)}(nP+P-1)]^T$$

where $r = 0, \ldots, Z-1$. Also required are the $P \times P$ Toeplitz lower triangular matrices $\mathcal{H}_0^{(r)}$ with first column

$$[h^{(r)}(0), \ldots, h^{(r)}(L), 0, \ldots, 0]^T.$$

We make the following assumptions (cf. [4]):
$(\mathbf{a0})$ Sub-channels $h^{(r)}(l)$ are Lth order FIR with $h^{(r)}(0) \neq 0$.
$(\mathbf{a1})$ (P, M, L) are chosen to satisfy $P > L$, $P = M + L$.
$(\mathbf{a2})$ Precoder filters have L trailing zeros; i.e., $\{f_m(n)\}_{n=M}^{P} = 0, \forall m \in [0, M-1]$, and are linearly independent; i.e., $rank(\mathbf{F}_0) = M$, which guarantees one-to-one mapping and thus recovery of $s(n)$ from the coded symbols $u(n)$.
$(\mathbf{a3})$ There exists an $N \geq P$, such that the $M \times N$ matrix $\mathbf{S}_N := [\mathbf{s}(0), \ldots, \mathbf{s}(N-1)]$ has full rank M. Note that, as N tends to ∞, $(1/N)\mathbf{S}\mathbf{S}^H$ tends to the input correlation matrix $\mathbf{R}_{ss}$.

Based on (a0)-(a2), we can write the received block data model [4] for the rth channel as:

$$\mathbf{y}^{(r)}(n) = \mathbf{x}^{(r)}(n) + \mathbf{v}^{(r)}(n) = \mathcal{H}_0^{(r)}\mathbf{F}_0\mathbf{s}(n) + \mathbf{v}^{(r)}(n) \quad (1)$$

Let $\mathcal{H}_P^{(r)}$ denote the first M columns of $\mathcal{H}_0^{(r)}$ Then $\mathcal{H}_0^{(r)}\mathbf{F}_0 = \mathcal{H}_P^{(r)}\mathbf{F}$, where $\mathbf{F}$ is the full rank matrix formed from M rows of $\mathbf{F}_0$. The received block data model thus becomes [4]:

$$\mathbf{y}^{(r)}(n) = \mathcal{H}_P^{(r)}\mathbf{F}\mathbf{s}(n) + \mathbf{v}^{(r)}(n) \quad (2)$$

Stacking the outputs of the Z channels gives

$$\begin{pmatrix} \mathbf{y}^{(0)}(n) \\ \vdots \\ \mathbf{y}^{(Z-1)}(n) \end{pmatrix} = \begin{pmatrix} \mathcal{H}_P^{(0)} \\ \vdots \\ \mathcal{H}_P^{(Z-1)} \end{pmatrix} \mathbf{F}\mathbf{s}(n) + \begin{pmatrix} \mathbf{v}^{(0)}(n) \\ \vdots \\ \mathbf{v}^{(Z-1)}(n) \end{pmatrix}$$

$$\text{or} \qquad \mathbf{y}(n) = \mathcal{H}_P\mathbf{F}\mathbf{s}(n) + \mathbf{v}(n) = \mathbf{x}(n) + \mathbf{v}(n) \quad (3)$$

Given a block of data $\{\mathbf{y}(n)\}_{n=0}^{N-1}$, the objective here is to estimate the $Z(L+1) \times 1$ vector $\mathbf{h} = [\mathbf{h}^{(0)T}, \ldots, \mathbf{h}^{(Z-1)T}]^T$, where rth channel impulse response $\mathbf{h}^{(r)} = [h^{(r)}(0), \ldots, h^{(r)}(L)]^T$. As in [3], we choose to collect N consecutive data vectors $\{\mathbf{y}(n)\}_{n=0}^{N-1}$ in a matrix:

$$\mathbf{Y}_N := [\mathbf{y}(0), \ldots, \mathbf{y}(N-1)] = \mathcal{H}_P\mathbf{F}\mathbf{S}_N + \mathbf{V}_N \quad (4)$$

The covariance matrix of the received data is thus

$$\mathbf{R}_{yy} = E(\mathbf{Y}_N\mathbf{Y}_N^H) = \mathcal{H}_P\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathcal{H}_P^H + \mathbf{R}_{vv} \quad (5)$$

where $\mathbf{R}_{ss} = E(\mathbf{S}_N\mathbf{S}_N^H)$ and $\mathbf{R}_{vv} = E(\mathbf{V}_n\mathbf{V}_n^H)$. Note that $(a0)$ is enough to conclude that $\mathcal{H}_P$ is full rank i.e., rank $(\mathcal{H}_P) = M$. By assumption, $\mathbf{R}_{vv} = \sigma_v^2\mathbf{I}$ and $\mathbf{R}_{ss}$ is full rank . The EVD of $\mathbf{R}_{yy}$ is then expressed as

$$\mathbf{R}_{yy} = \mathbf{S}\text{diag}(\lambda_0, \ldots, \lambda_{M-1})\mathbf{S}^H + \sigma_v^2\mathbf{G}\mathbf{G}^H \quad (6)$$

where $\mathbf{S} = [\mathbf{S}_0, \ldots, \mathbf{S}_{M-1}]$ and $\mathbf{G} = [\mathbf{G}_0, \ldots, \mathbf{G}_{PZ-M-1}]$. The columns of $\mathbf{S}$ span the signal subspace, while those of $\mathbf{G}$, the noise subspace. The columns of $\mathcal{H}_P\mathbf{F}$ also span the signal subspace and thus by orthogonality, we have

$$\mathbf{G}_i^H\mathcal{H}_P\mathbf{F} = \mathbf{0}. \quad (7)$$

It can be shown that under the assumptions we have made, the channel $\mathbf{h}$ can be estimated up-to a scale factor. In practice, since the output data vectors are noisy, equation (7) is solved by minimizing the quadratic form:

$$q(\mathbf{h}) = \sum_{i=0}^{PZ-M-1} \mid \mathbf{G}_i^H\mathcal{H}_P\mathbf{F}|^2. \quad (8)$$

Let $\mathbf{G}_i^H\mathcal{H}_P\mathbf{F} = \mathbf{h}^T\mathcal{G}_i\mathbf{F}$ where $\mathcal{G}_i$ is the filtering matrix associated with $\mathbf{G}_i$ and can be obtained by back substitution. Therefore, $\mid \mathbf{G}_i^H\mathcal{H}_P\mathbf{F} \mid^2 = \mathbf{h}^H\mathcal{G}_i\mathbf{F}\mathbf{F}^H\mathcal{G}_i^H\mathbf{h}$ and equation (10) can thus be expressed as: $q(\mathbf{h}) = \mathbf{h}^H\mathbf{Q}\mathbf{h}$, where $\mathbf{Q} = \sum_{i=0}^{PZ-M-1}\mathcal{G}_i\mathbf{F}\mathbf{F}^H\mathcal{G}_i^H$ and the channel estimate can thus be formulated as

$$\mathbf{h} = \arg\min_{\|\mathbf{h}\|=1} \| \mathbf{h}^H\mathbf{Q}\mathbf{h} \|^2 . \quad (9)$$

Therefore, $\mathbf{h}$ is obtained as the eigenvector associated with the minimum eigenvalue of $\mathbf{Q}$. We call this method the TZ-CS method.

Remark 1: Whereas Moulines' method requires no common zeros to ensure that the corresponding $\mathcal{H}_P\mathbf{F}$ have full rank, the matrix $\mathcal{H}_P\mathbf{F}$ here have full rank due to trailing zeros introduced by the precoder. Intuitively, common zeros are not a problem because TZ-precoder can identify a SISO channel [4] without any restrictions on the location of channel zeros.

Remark 2: An important feature of TZ-CS is its robustness to channel order over-estimation. It is shown in [4], which considered the SISO case, that the channel identification is robust to channel order over-estimation.

Remark 3: Because $P > M$ therefore information rate is lower than transmission rate, which pays off in equalization. However, we can tradeoff increase in information rate (i.e. by setting M(and thus P) large) for increase in transceiver complexity.

Remark 4: (i) With $f_m(n) = exp(j2\pi mn/M)$, the filter bank precoder of Fig.1, reduces to the digital TZ-OFDM transmitter and (ii) using Hadamard Codes instead of complex exponentials, the filter bank of Fig.1 reduces to TZ-CDMA precoder that uses as filters the Hadamard basis with trailing zeros [4].

3. SIMULATION RESULTS

This section compares TZ-CS with CS and channel estimation method in [4]. Computer simulations were conducted with QPSK input signal. We simulated the oversampled output of two separate single receivers, with oversampling factor $L = 4$, ISI degree $Z = 4$. For CS temporal window length $N_w = 8$ and for TZ-CS we took $M = 4$ and $P = N_w = 8$. The channel coefficients are chosen as in [3] for first receiver, whereas for receiver 2 channel coefficients were obtained by artificially introducing common zeros among the sub-channels of set I. The zero distribution of these two sets of channels is plotted in Fig.2(a) and (b) with different symbols representing different zeros of different channels of channel sets I and II. As can be seen channel set I has no common zero, whereas, channel set II has a number of common zero shared by sub-channels.

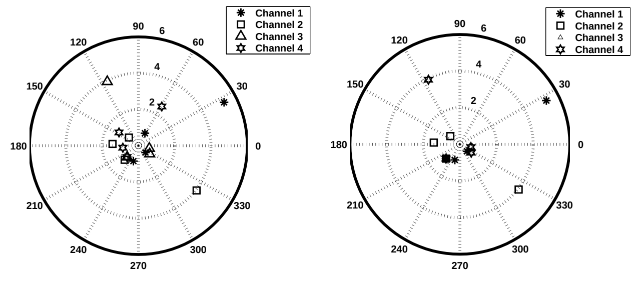

Fig. 2. Zero distribution of the channels: (a) first channel set, (b) second channel set

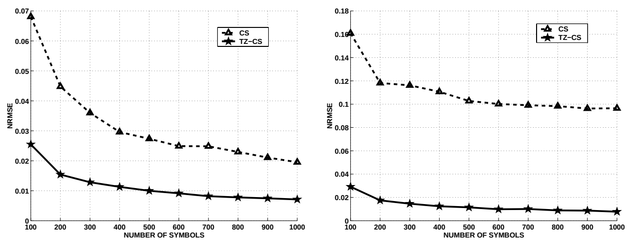

Fig. 3. Comparison of the TZ-CS and CS methods for N=100-1000 symbols

The normalized root-mean-square error (NRMSE) is defined as:

$$NRMSE = \frac{1}{\parallel \mathbf{h} \parallel} \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \parallel \hat{\mathbf{h}}_{(i)} - \mathbf{h} \parallel^2} \qquad (10)$$

where Nt is the number of Monte Carlo runs (100 in our case) and $\hat{\mathbf{h}}_i$ is the estimate of the channel from ith run.

In the first simulation study, we fixed the SNR to 25 dB and varied the number of symbols from 100-1000. Fig.3(a) and (b) shows the NRMSEs of the channel estimates from the TZ-CS and CS methods for channel sets I and II respectively. We can see that CS is unable to identify channel set II with common zero, whereas, the new method is successful in identifying both channel sets. TZ-CS performs better than the CS method even with no common zeros and requires much less number of data samples.

In the second simulation study, we fixed the number of symbols to be 100 and varied the SNR from 10-40 dB. Fig.4(a) gives the NRMSEs of the channel estimates from these two methods for channel set I. It is clear that TZ-CS exhibits good performance at low SNR.

In the third simulation study, we tested the robustness of TZ-CS when channel order is over-estimated. Fixing the SNR = 25 dB and with true channel length (L) of 3 for channel set I, we manually varied the length estimate to 4 (i.e., $\overline{L}$=4). It is apparent from Fig.4(b) that TZ-CS is robust to channel order over-estimation error without any severe degradation of estimation accuracy, however, CS is very sensitive and thus not a reliable approach when channel order is over-estimated.

We can tradeoff increase in information rate (i.e. by setting

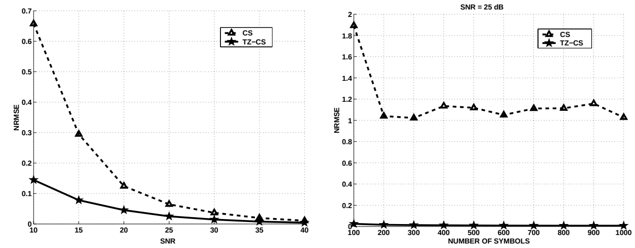

Fig. 4. Comparison of the TZ-CS and CS methods (a) for SNR=10-40 dB, (b) when channel order is overestimated

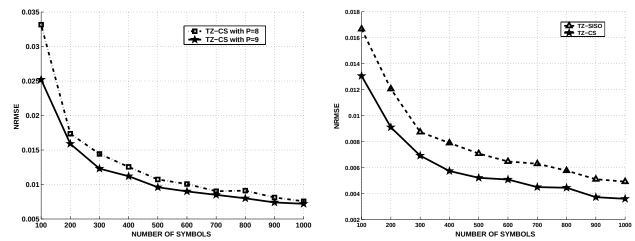

Fig. 5. (a) Comparison of the TZ-CS for P=8 and P=9, at SNR=25 dB, (b) Comparison of TZ-CS and TZ-SISO, for SNR=25 dB and N=100-1000 symbols

M(and thus P) large) for increase in transceiver complexity, however, this results in performance degradation of TZ-CS. This is shown in Fig.5 (a) by comparing performance of TZ-CS with P=8 (M=4) and P=9 (M=5), at SNR=25dB.

TZ-filter bank precoder has been successfully applied to the problem of SISO FIR channel identification without channel zero locations (abbreviated to TZ-SISO in this paper) in [4]. The work in this paper extends the work in [4] to the identification of SIMO FIR channels. One can argue that identification of TZ-precoder driven SIMO FIR channels can be divided into identifying a set of TZ-precoder driven SISO FIR channels between the input and each of the outputs using [4]. However, each of these FIR channels would be identified using only the measurements of its output and not the full set of observations which contains more information. Obviously TZ-CS is expected to result in improved performance to channel estimates as it exploits additional information from the outputs of other channels. This is shown in fig 5(b), where channel 1 of the channel set I is identified using TZ-SISO and using TZ-CS (using observation outputs from other channels of the channel set I). The improved performance of TZ-CS is clear.

Based on the multichannel block data model equation (3), we extended the ZF and MMSE equalizers derived in [4] to the SIMO channel in Fig.1. We found that the better channel estimates obtained by the TZ-CS method led to better source symbol estimates obtained with either ZF or MMSE equalizers. The results are shown in Fig. 6 and are now explained. For channel set II, the phase pattern of the receiver outputs is plotted in Fig. 6(a). Using the channel estimates via TZ-CS method for 100 snapshots of data multichannel ZF and MMSE equalizers are implemented. The equalized phase patterns using multichannel ZF and MMSE equal-

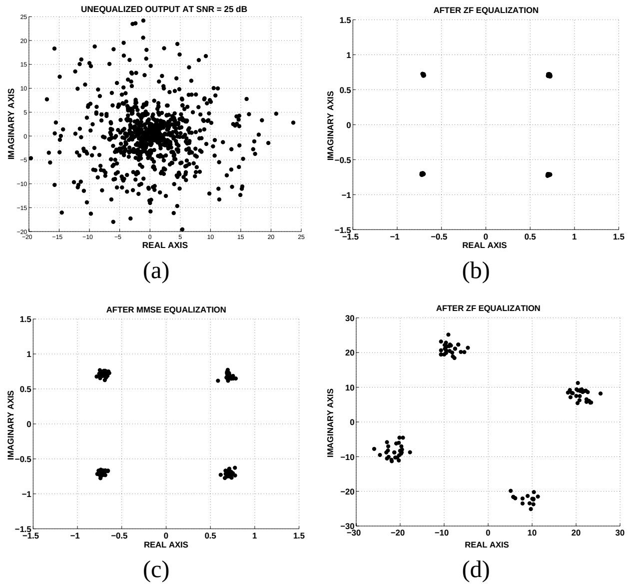

Fig. 6. Comparison of phase patterns: (a)before equalization (c) after ZF equalization using TZ-CS estimates; (b) after MMSE equalization using TZ-CS estimates and (d) after ZF equalization using CS estimates

izers are given in Fig 6(b) and (c). In comparison with Fig.6(a), the constellation is clearly much improved. Fig 6. (d) gives an insight of the channel estimates by CS method where phase pattern is obtained by using CS channel estimates in the ZF equalizer for TZ-CS. It is clear that CS channel estimates are unacceptable and equalization is impossible.

4. CONCLUSION

This paper presented a modification of Moulines' CS method for identifying SIMO FIR channels driven by a trailing zero (TZ) precoder. It was demonstrated that by exploiting the input redundancy created by the TZ precoder, the resulting blind multichannel subspace method (TZ-CS) outperforms Moulines' CS method in terms of robustness to common zeros and channel order overestimation. Moreover, the TZ-CS method requires shorter data lengths and lower SNR than the CS method does to achieve the same level of performance.

5. REFERENCES

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