

# NEW DISPARITY MAP ESTIMATION USING HIGHER ORDER STATISTICS

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## ABSTRACT

This paper presents a new algorithm of disparity map estimation. Originality of this method lies in the process of dense disparity map estimation using the dynamic programming constrained by interest points and using Higher Order Statistics (HOS) criteria for matching noisy images. Experiments with noisy real images have validated our method and have clearly shown the improvement over the existing ones. The dense disparity map obtained is more reliable when compared to the similar Second-Order Statistics (SOS) based dynamic programming and HOS based correlation methods.

## 1. INTRODUCTION

Stereo is a useful method of obtaining depth information. The key problem in stereo is a search problem which finds the correspondence points between the left and right images [3][4][6][14], so that, given the camera model (i.e., the relationship between the right and left cameras of the stereo pair), the depth can be computed by triangulation [9][10]. If a pair of stereo images is rectified so that the epipolar lines are horizontal scanlines, a pair of corresponding points in the right and left images should be searched for only within the same horizontal scanlines. We call this search intra-scanlines search [8]. This intra-scanlines search can be treated as the problem of finding a matching path on a two dimensional (2D) search plane whose vertical and horizontal axes are the right and the left scanlines. A dynamic programming technique can handle this search efficiently [5].

Indeed, the geometric properties of the picture vary, one from the other because of geometric distortions; moreover objects visibility is often different in the two images [6][14]. The presence of noise or a variation of lighting conditions can also transform the photometric properties of the different pictures.

In order to solve these problems, we have developed a matching algorithm constrained by interest points [13]. But this technique does not consider noise effect.

There are some situations in which the images might be corrupted with noise. If the images are severely corrupted by additive correlated Gaussian noise of unknown covariance, Second-Order Statistics (SOS) based methods (like dynamic programming [5][13] or block matching...) do not work well. In this case, Higher Order Statistics (HOS) based methods are more advantageous since they are not affected by such noise [1][2].

Motivated by the noise insensitivity of Cumulant-based estimators, we have proposed a novel correlation method based on a Cumulant-matching criterion [12]. In addition, we propose a new constrained dynamic programming based on a Cumulant-matching criterion.

The objective of this paper is to develop a new algorithm for disparity map estimation which is insensitive to a Gaussian noise of unknown covariance. Here, we are interested to a couple of rectified images [11]. Paragraph 2 describes briefly the used HOS criterion. In paragraph 3, we recall the constrained dynamic programming method [13] and present the proposed one. Finally, paragraph 4 shows the obtained experimental results with a comparison of the proposed method, SOS-based constrained dynamic programming [13] and HOS-based correlation [12].

## 2. PROBLEM STATEMENT

Left and right noisy images are modeled as [1][2]:

$$IB_g(X) = I_g(X) + N_g(X) \quad (2-1)$$

$$IB_d(X) = I_d(X) + N_d(X) = I_g(X - d(X)) + N_d(X) \quad (2-2)$$

$IB_g$  and  $IB_d$ : noised left picture and right picture.

$I_g, I_d$ : left and right originals pictures.

$N_g$  and  $N_d$ : zero mean Gaussian noise of unknown covariance.

$d(X)$ : disparity vector.

With  $X = (x, y)$  so  $X - d(X) = (x - d_x(x, y), y - d_y(x, y))$ .

In the case of rectified pictures  $d_y(x, y) = 0$ .

Because Cumulants of colored Gaussian noise are zero [1][2], estimation techniques based on these statistics suppress the effect of additive Gaussian noise even if it is colored noise. In fact, third order Cumulants are

insensitive to any additive noise with a symmetric distribution.

If  $C_{IB_gIB_gIB_g}$  and  $C_{IB_gIB_dIB_g}$  denotes third order auto-Cumulant and third Cross-Cumulant of image  $IB_g$  than :

$$\begin{aligned} C_{IB_gIB_gIB_g}(m, n) &= C_{I_gI_gI_g}(m, n) + C_{N_gN_gN_g}(m, n) \\ C_{IB_gIB_dIB_g}(m, n) &= C_{I_gI_dI_g}(m, n) + C_{N_gN_dN_g}(m, n) \end{aligned} \quad (2-3)$$

In the case of Gaussian noise :

$$\begin{aligned} C_{N_gN_gN_g}(m, n) &= 0 \\ C_{N_gN_dN_g}(m, n) &= 0 \end{aligned} \quad (2-4)$$

So:

$$\begin{aligned} C_{IB_gIB_gIB_g}(m, n) &= C_{I_gI_gI_g}(m, n) \\ C_{IB_gIB_dIB_g}(m, n) &= C_{I_gI_dI_g}(m, n) \end{aligned} \quad (2-5)$$

In these conditions and to get a good map disparity estimation, we propose a new matching technique based dynamic programming by minimizing the following criterion:

$$\hat{J}_{3ab}(d) = \sum_m \sum_n |\hat{C}_{IB_gIB_gIB_g}(m-d, n) - \hat{C}_{IB_gIB_dIB_g}(m, n)| \quad (2-6)$$

where:

$$C_{IB_gIB_gIB_g}(m, n) = E[IB_g(X)IB_g(X+m)IB_g(X+n)] \quad (2-7)$$

is the third order auto-Cumulant and

$$C_{IB_gIB_dIB_g}(m, n) = E[IB_g(X)IB_d(X+m)IB_g(X+n)] \quad (2-8)$$

is the third order Cross-Cumulant and  $\hat{C}$  denotes the estimated value of  $C$  computed as follows. Given  $N \times N$  segments of 2-D processes  $IB_g(X)$  and  $IB_d(X)$ , sample estimators of third-order auto-Cumulants and Cross-Cumulants are obtained, respectively, as [1][2].

$$\hat{C}_{IB_gIB_gIB_g}(m, n) = \frac{1}{N^2} \sum_{x=1}^N IB_g(x)IB_g(x+m)IB_g(x+n) \quad (2-9)$$

$$\hat{C}_{IB_gIB_dIB_g}(m, n) = \frac{1}{N^2} \sum_{x=1}^N IB_g(x)IB_d(x+m)IB_g(x+n) \quad (2-10)$$

$$\text{where } \sum_{x=1}^N = \sum_{x=1}^N \sum_{y=1}^N$$

### 3. PROPOSED DISPARITY ESTIMATION METHOD

#### 3.1. Dynamic Programming

We use dynamic programming to solve the problem of disparity estimation.

This technique is particularly adapted to optimization problems subject to constraints. The dynamic programming uses the local associations of features to condition the global optimization research. The

principal idea of this technique is to minimize a cost function in a bidimensional graph.

Indeed, the problem of obtaining correspondences between right and left epipolar lines can be solved as a path finding problem on a 2D plane.

For that, we have arranged the signals on the left and the right epipolar lines in two-array axis (see figure 1).

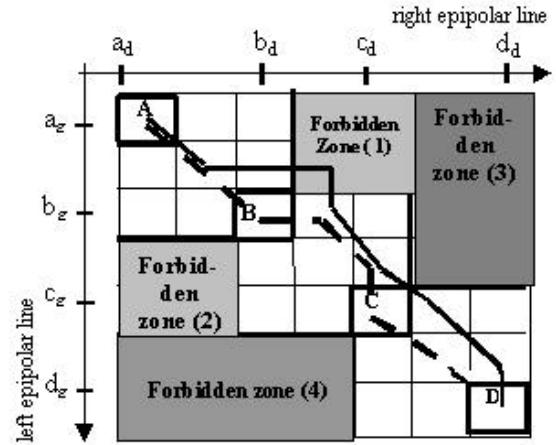


Figure 1. Dynamic programming constrained by interest points.

For that, we have associated a cost function to this array. For each element of the array a potential matching is allowed with a cost value. The goal is to find the optimal path between points A and D.

Order constraints and disparity domain enable the reduction of possible paths and allow the intra-line consistency [5][8]. The advantage of this technique is that it allows the subdivision of the matching problem into a set of under-problems (restriction to couples of epipolar lines). Each under-problem can be solved globally, thus avoiding error propagation problems on the same line. The main problem of this approach consists in the choice of the cost function and also in saving the inter-line consistency.

#### 3.2. Constrained Dynamic Programming

In order to achieve inter-line consistency we have used interest-points to further constrain the possible paths [13]. After extracting and matching interest point [7][15] they are used to define forbidden zone in the search plane :

The path is constrained to contain points B and C representing the couples of homologous interest points  $(b_d, b_g)$  and  $(c_d, c_g)$ . Assuming that the order constraint is verified, the correspondents of points between  $c_d$  and  $d_d$  are located between points  $c_g$  and  $d_g$  (and vice versa). This scheme defines two forbidden zones. Zone 3 permits to avoid the matching of the points of  $[c_d, d_d]$  interval with the  $[a_g, c_g]$  points. Zone 4 permits to avoid matching the points of  $[c_g, d_g]$  interval with the  $[a_d, c_d]$  points. Therefore, any horizontal, vertical or diagonal displacement that can browse a forbidden zone is

excluded. This restriction permits a best constrained matching of epipolar lines with a reduced execution time.

### 3.3. HOS-Based Dynamic Programming

In the previous methods, the cost function was based on a Second Order Statistics (SOS) criterion. The cost associated with the matching of pixels  $(x, y)$  and  $(x+d_x, y+d_y)$  is given by :

$$SAD(d) = \sum_{i=1}^{N_w} \sum_{j=1}^{M_w} |IB_g(x+i, y+j) - IB_d(x+d_x+i, y+d_y+j)| \quad (3-1)$$

where  $N_w \times M_w$  is the correlation window.

In order to take into account the presence of noise, we have introduced a cost function based on Higher Order Statistics (HOS) :

$$\hat{J}_{3abs}(d) = \sum_m \sum_n |\hat{C}_{IB_g IB_g IB_g}(m-d, n) - \hat{C}_{IB_g IB_d IB_g}(m, n)| \quad (3-2)$$

## 4. EXPERIMENTAL RESULTS AND DISCUSSION

We have processed 3 couples of noisy images (Fig. 2a, 2b, 3a, 3b, 4a and 4b) and we have compared the disparity field obtained with 3 different methods: HOS-based correlation, SOS-based constrained dynamic programming and the proposed HOS-based constrained dynamic programming. Correlation is a classical method for disparity estimation and HOS-based correlation thus gives us a reference for evaluating our results.

The disparity map obtained from the proposed method (fig. 2e, 3e and 4e) is clearly better than the disparity map obtained from constrained dynamic programming with SOS [13] (fig. 2d, 3d and 4d). It is also better than the disparity map obtained from HOS-based correlation matching [12] (fig. 2c, 3c and 4c).

In the presence of noise SOS-based dynamic programming produces artifacts in the disparity field which are propagated along epipolar lines. On the contrary, HOS-based dynamic programming overcomes noise presence and gives a more consistent disparity map.

## 5. CONCLUSION

A new HOS based dynamic programming algorithm for disparity map estimation has been proposed which enables an accurate analysis of noisy images to be made. The results presented have shown the relevance of our approach. Performance of various approaches was compared for different classes of test images and difficulties involved in the evaluation of stereo algorithms were addressed. Tested algorithms have proved to give a lower percentage of false matches as well as better accuracy of depth estimates. Yet, to completely validate the algorithm, we plan to evaluate our measure on synthetic images where the real disparity field is available and quantitative comparison

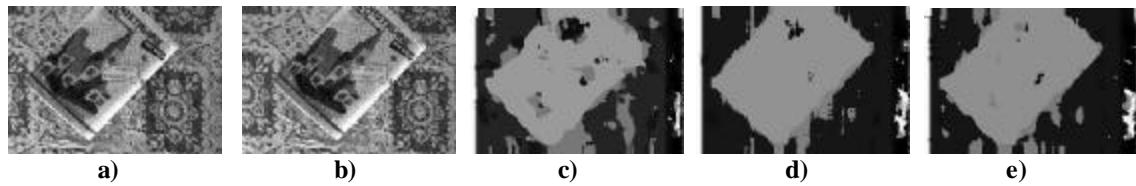
with ground truth can be made. 3D models obtained with the different methods could also be compared.

## ACKNOWLEDGMENTS

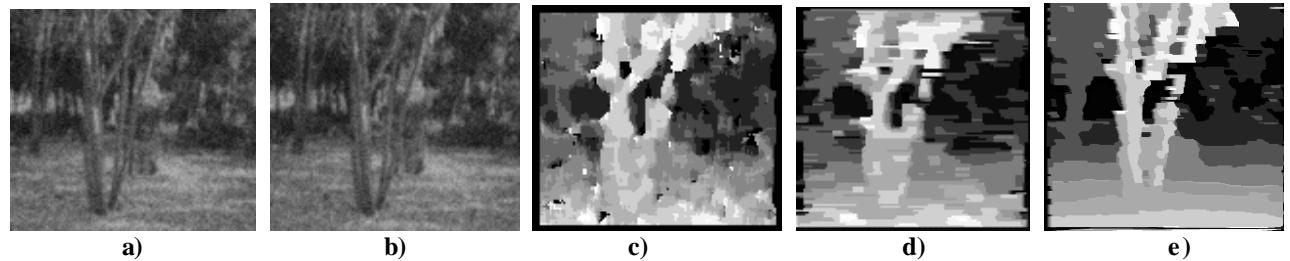
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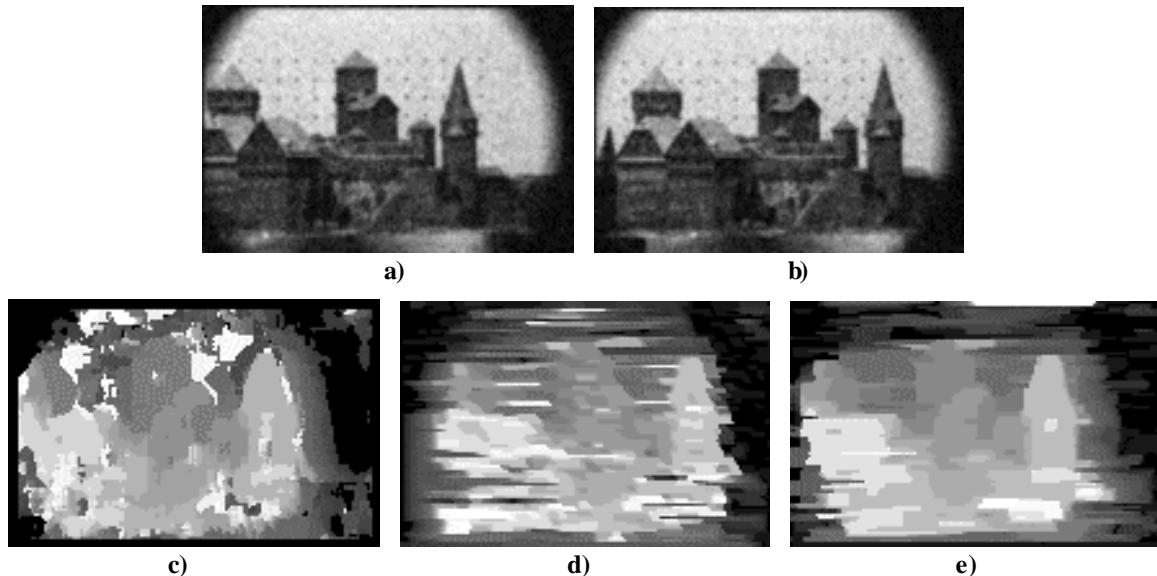
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**Figure 2.** **a)** Noisy image with SNR= 10 dB of “BOX”, **b)** right noisy image with SNR= 10 dB of “BOX”, dense disparity map obtained by, **c)** HOS-based correlation, **d)** SOS-based dynamic programming, **e)** HOS-based dynamic programming.



**Figure 3.** **a)** Noisy image with SNR= 10 dB of “TREES”, **b)** right noisy image with SNR= 10 dB of “TREES”, dense disparity map obtained by, **c)** HOS-based correlation, **d)** SOS-based dynamic programming, **e)** HOS-based dynamic programming.



**Figure 4.** **a)** Noisy image with SNR= 10 dB of “CASTEL”, **b)** right noisy image with SNR= 10 dB of “CASTEL”, dense disparity map obtained by, **c)** HOS-based correlation, **d)** SOS-based dynamic programming, **e)** HOS-based dynamic programming.