AN EFFICIENT APPROACH FOR DESIGNING NEARLY PERFECT-RECONSTRUCTION COSINE-MODULATED AND MODIFIED DFT FILTER BANKS

Tapio Saramäki and Robert Bregović

Signal Processing Laboratory
Tampere University of Technology
P. O. Box 553, FIN-33101 Tampere, Finland
e-mail: ts@cs.tut.fi and bregovic@cs.tut.fi

ABSTRACT

Efficient two-step algorithms are described for optimizing the stopband response of the prototype filter for cosine-modulated and modified DFT filter banks either in the minimax or in the least-mean-square sense subject to the maximum allowable aliasing and amplitude errors. The first step involves finding a good start-up solution using a simple technique. This solution is improved in the second step by using nonlinear optimization. Several examples are included illustrating the flexibility of the proposed approach for making compromises between the required filter lengths and the aliasing and amplitude errors. These examples show that by allowing very small amplitude and aliasing errors, the stopband performance of the resulting filter bank is significantly improved compared to the corresponding perfect-reconstruction filter bank. Alternatively, the filter orders and, consequently, the overall delay can be significantly reduced to achieve practically the same performance.

1. INTRODUCTION

Among different classes of M-channel critically sampled filter banks, cosine-modulated [1]–[9] and modified DFT [10]–[11] filter banks have become very popular in many applications due to the following reasons. First, these banks can be generated using a single prototype filter by exploiting a proper transformation, making the overall implementation effective. Second, the overall synthesis can concentrate on optimizing only the prototype filter. This paper concentrates on designing cosine-modulated filter banks, but as has been pointed out in [11], the same prototype filter with a proper scaling can be used for both filter types mentioned above.

For designing the prototype filter different strategies can be applied. The design can be performed using constrained minimization [5], iterative methods [6], lattice factorizations [2], [3], [4] as well as by applying some other synthesis schemes [7], [8]. Some of these methods result in perfect-reconstruction (PR) filter banks whereas some in nearly PR filter banks.

For practical applications with lossy channel coding and quantization, the PR property is desirable but not necessary. In this case, the distortion caused by aliasing and amplitude errors to the signal is allowed provided that they are smaller than that caused by coding. Therefore, it is worth trying to release the PR condition with the ultimate goal being to achieve better filter bank properties.

This work was supported by the Academy of Finland, project No. 44876 (Finnish centre of Excellence program (2000-2005)).

This paper describes an efficient two-step approach for synthesizing prototype filters for nearly PR filter banks. In the first step, a proper prototype filter for a PR filter bank is generated using a systematic multi-step procedure described in [12]. In the second step, this filter is used as a start-up solution for solving the given constrained optimization problem. The optimization is carried out by using the second algorithm of Dutta and Vidyasagar [9], [13]. Several examples are included illustrating that by allowing small amplitude and aliasing errors, the filter bank performance can be significantly improved. Alternatively, the filter orders and the overall delay caused by the filter bank to the signal can be considerably reduced. This is very important in communication applications.

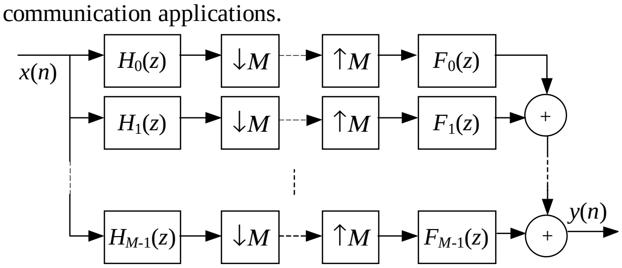

Figure 1. M-channel maximally decimated filter bank.

2. COSINE-MODULATED FILTER BANKS

A general M-channel critically sampled filter bank is shown in Figure 1 [1]. For this system the input-output relation in the z-domain is expressible as

$$Y(z) = T_0(z)X(z) + \sum_{l=1}^{M-1} T_l(z)\,X\left(ze^{-j2\pi l/M}\right), \tag{1a}$$

where

$$T_0(z) = \frac{1}{M}\sum_{k=0}^{M-1} F_k(z)H_k(z) \tag{1b}$$

and

$$T_l(z) = \frac{1}{M}\sum_{k=0}^{M-1} F_k(z)H_k\left(ze^{-j2\pi l/M}\right) \tag{1c}$$

for $l = 1, 2, \ldots, M-1$. Here, $T_0(z)$ is called the distortion transfer function and determines the distortion caused by the overall system for the unaliased component $X(z)$ of the input signal. The remaining transfer functions $T_l(z)$ for $l = 1, 2, \ldots, M-1$ are called the alias transfer functions and determine how well the aliased components $X(ze^{-j2\pi l/M})$ of the input signal are attenuated.

For PR, it is required that $T_0(z) = z^{-N}$ with N being an integer and $T_l(z) = 0$ for $l = 1, 2, \ldots, M-1$. If these conditions are satisfied, then the output signal is a delayed version of the input signal, that is, $y(n)=x(n-N)$. It should be noted that PR is exactly achieved only in the case of lossless coding. For a lossy coding, PR is not achieved. Therefore, amplitude and aliasing errors being less than those caused by coding are allowed. In these nearly PR cases, the above-mentioned conditions should be satisfied within given tolerances.

For cosine-modulated filter banks, the impulse responses for the analysis transfer functions $H_k(z)$ in Figure 1 can be generated with the aid of a linear-phase FIR[1] prototype transfer function of the form

$$H_p(z)=\sum_{n=0}^{N} h_p(n) z^{-n}, \quad h_p(N-n)=h_p(n) \tag{2}$$

as follows [1], [2][2]:

$$h_k(n)=2h_p(n)\cos\left[\left(k+\frac{1}{2}\right)\frac{\pi}{M}\left(N-n+\frac{M+1}{2}\right)\right]. \tag{3}$$

The impulse responses for the synthesis filters are obtained by replacing $N-n$ by n in the above equation. Another alternative is to use the modulation scheme described in [3].

In the PR case, N, the order of the prototype filter, is restricted to be an odd integer equal to $2KM-1$, where M is the number of channels and K is an integer. In nearly PR cases, there is not such a limit. This contribution concentrates on the case where N is odd (the length $N+1$ is even). In this case, the frequency response of the prototype filter is expressible as

$$H_p(\mathbf{h}, e^{j\omega}) = e^{-jN\omega/2} H_p^{(0)}(\mathbf{h},\omega) , \tag{4a}$$

where

$$H_p^{(0)}(\mathbf{h},\omega) = 2\sum_{n=1}^{(N+1)/2} h_p[(N+1)/2-n]\cos[(n-1/2)\omega] \tag{4b}$$

and

$$\mathbf{h} = [h_p(0) \quad h_p(1) \quad\quad h_p[(N-1)/2]] \tag{4c}$$

denotes the adjustable parameter vector of the prototype filter.

The following two sections show how to optimize the stopband response of the prototype in the least-mean-square and minimax senses subject to the given allowable amplitude and aliasing errors.

3. LEAST-SQUARED-ERROR DESIGN

This section considers the design of the prototype filter in the least-mean-square sense.

3.1 Statement of the problem

The following optimization problem is considered: Given ρ, M, and N, find the coefficients of $H_p(z)$ to minimize

$$E_2 = \int_{\omega_s}^{\pi} \left|H_p(e^{j\omega})\right|^2 d\omega , \tag{5a}$$

where

$$\omega_s = (1+\rho)\pi/(2M) \tag{5b}$$

subject to

$$1-\delta_1 \le \left|T_0(e^{j\omega})\right| \le 1+\delta_1 \quad \text{for} \quad \omega\in[0,\pi] \tag{5c}$$

and for $l=1, 2, \ldots, M-1$

$$\left|T_l(e^{j\omega})\right| \le \delta_2 \quad \text{for} \quad \omega\in[0,\pi]. \tag{5d}$$

3.2 Proposed design method

In the above problem, Eq. (5a) can be expressed as

$$E_2(\mathbf{h}) = \sum_{\mu=1}^{(N+1)/2}\sum_{\eta=1}^{(N+1)/2} h_p\left[\frac{N+1}{2}-\mu\right] h_p\left[\frac{N+1}{2}-\eta\right]\Psi(\mu,\eta) , \tag{6a}$$

where

$$\Psi(\mu,\eta) = 4\int_{\omega_s}^{\pi}\cos[(\mu-1/2)\omega]\cos[(\eta-1/2)\omega]d\omega = \\ = \begin{cases} 2\pi-2\omega_s-\dfrac{2\sin[(2\mu-1)\omega_s]}{2\mu-1}, & \mu=\eta \\ -\dfrac{2\sin[(\mu+\eta-1)\omega_s]}{\mu+\eta-1}-\dfrac{2\sin[(\mu-\eta)\omega_s]}{\mu-\eta}, & \mu\ne\eta. \end{cases} \tag{6b}$$

The $\left|T_l(\mathbf{h}, e^{j\omega})\right|$'s for $l=0, 1, \ldots, M-1$, in turn, can be written as shown in Appendix A in [9].

To solve this problem, we discretize the region $[0, \pi/M]$ into the discrete points $\omega_j\in[0, \pi/M]$ for $j=1, 2, \ldots, J_0$. In many cases, $J_0=N$ is a good selection to arrive at a very accurate solution. The resulting discrete problem is to find $\mathbf{h}$ to minimize

$$\rho(\mathbf{h}) = E_2(\mathbf{h}) , \tag{7a}$$

where $E_2(\mathbf{h})$ is given by Eq. (6), subject to

$$g_j(\mathbf{h}) \le 0 \quad \text{for} \quad j=1,2,\ldots,J , \tag{7b}$$

where[3]

$$J = \lfloor (M+2)/2 \rfloor J_0 , \tag{7c}$$

$$g_j(\mathbf{h}) = \left|\left|T_0(\mathbf{h}, e^{j\omega_j})\right|-1\right|-\delta_1 \quad \text{for} \quad j=1,2,\ldots,J_0 , \tag{7d}$$

and

$$g_{lJ_0+j}(\mathbf{h}) = \left|T_0(\mathbf{h}, e^{j\omega_j})\right|-\delta_2 \tag{7e}$$

for $l=1, 2, \ldots, \lfloor M/2 \rfloor$ and for $j=1, 2, \ldots, J_0$.

In the above, the region $[0, \pi/M]$, instead of $[0, \pi]$, has been used since the $|T_l(\mathbf{h}, e^{j\omega})|$'s are periodic with periodicity equal to $2\pi/M$. Furthermore, only the first $\lfloor (M+2)/2 \rfloor$ $|T_l(\mathbf{h}, e^{j\omega})|$'s have been used since $|T_l(\mathbf{h}, e^{j\omega})|=|T_{M-l}(\mathbf{h}, e^{j\omega})|$ for $l=1, 2, \ldots, \lfloor (M-1)/2 \rfloor$.

The above nonlinear optimization problem can be solved conveniently by using a two-step design procedure described in

[1] As shown in [6], non-linear-phase FIR filters can also be used as prototype filters. This contributions concentrates on the use of linear-phase filters.

[2] In [1], instead of the constant of value 2, the constant of value $\sqrt{2/M}$ has been used. The reason for this is that the prototype filter is implemented using special butterflies. The amplitude response of the resulting prototype filter approximates the value of $M\sqrt{2}$, instead of unity, at the zero frequency. For an approximately peak-scaled overall implementation, the scaling constants of values $1/(M\sqrt{2})$ and $1/\sqrt{2}$ are desired to be used in the final implementation for the $h_k(n)$'s and $f_k(n)$'s, respectively.

[3] $\lfloor x \rfloor$ stands for the integer part of x.

[9]. In the first step of this approach, a suboptimum start-up solution for the second step is generated in a manner to be briefly described in Section 5. For the second step, the optimization problem has been formulated above such that at least a good local optimum can be found by using the second algorithm of Dutta and Vidyasagar [13]. The details on how to apply this algorithm can be found in [9]. Due to the high nonlinearity of the problems under consideration, both steps are of great importance. This is because the convergence to a good overall solution implies both a good start-up solution and the use of a computationally efficient algorithm for further optimization.

A special case of the above problem is the case where $|T_0(\mathbf{h}, e^{j\omega})| \equiv 1$, that is, the amplitude distortion is zero. This implies that $\delta_1 \equiv 0$ in Eq. (7d). In practice, a good enough solution is obtained by using $\delta_1 = 10^{-16}$ in Eq. (7d).

4. MINIMAX DESIGN

This section considers the design of the prototype filter in the minimax sense.

4.1 Statement of the problem

The following optimization problem is considered: Given ρ, M, and N, find the coefficients of $H_p(z)$ to minimize

$$E_\infty = \max_{\omega \in [\omega_s, \pi]} \left| H_p(e^{j\omega}) \right|, \tag{8}$$

where ω_s is given by Eq. (5b), subject to the conditions of Eqs. (5c) and (5d).

4.2 Proposed design method

The main difference compared to the above least-squared-error design is that now, in addition to the discretization performed in Subsection 3.2, the stopband region $[\omega_s, \pi]$ has to be discretized into points $\omega_i \in [\omega_s, \pi]$ for $i = 1, 2, \ldots, I$. In many cases, $I = 20N$ is a good selection. The resulting discrete minimax problem is to find $\mathbf{h}$ to minimize

$$\rho(\mathbf{h}) = \hat{E}_\infty(\mathbf{h}) = \max_{1 \le i \le I} \{ f_i(\mathbf{h}) \} \tag{9a}$$

subject to

$$g_j(\mathbf{h}) \le 0 \quad \text{for} \quad j = 1, 2, \ldots, J, \tag{9b}$$

where

$$f_i(\mathbf{h}) = \left| H_p(\mathbf{h}, e^{j\omega_i}) \right| \quad \text{for} \quad i = 1, 2, \ldots, I \tag{9c}$$

and J and the $g_j(h)$'s are given by Eqs. (7c), (7d), and (7e).

A two-step procedure similar to that used for least-squared-error design can be applied. The start-up solution is determined as will be described in Section 5. In the second step, the Dutta-Vidyasagar algorithm is again applied. The special case $|T_0(\mathbf{h}, e^{j\omega})| \equiv 1$ can also be solved like for the least-squared-error problem.

5. INITIAL STARTING-POINT SOLUTIONS

Good start-up solutions can be generated for both of the above problems by using systematic multi-step procedures described in [4], [9], [12] for generating PR filter banks in such a way that the stopband behavior of the prototype filter is optimized in the least-mean-square or the minimax sense. These procedures have been constructed in such a way that they are unconstrained optimization procedures. To achieve this, the basic unknowns have been selected such that the PR property is satisfied independent of the values of the unknowns. Compared to other existing design methods, these synthesis procedures are faster and allow us to synthesize filter banks of significantly higher filter orders than the other existing design schemes.

For the PR case, the order of the prototype filter is restricted to be $N = K \cdot 2M - 1$, where M is the number of filters in the analysis and synthesis banks and K is an integer. If the desired order does not satisfy this condition, then a good initial solution is found by first designing the PR filter with the order of the prototype filter being selected such that K is the smallest integer making the overall order larger than the desired one. Then, the first and the last impulse-response values are dropped out until achieving the desired order.

6. EXAMPLES

For comparison purposes, several filter banks have been optimized for $M = 32$ channels and $\rho = 1$, that is, the stopband edge of the prototype filter is located at $\omega_s = \pi/32$. The results are summarized in Table I. In all cases under consideration, the order of the prototype filter is $K \cdot 2M - 1$, where K is an integer and the stopband response has been optimized in either the minimax or least-mean-square (LSQ) sense. δ_1 shows the maximum deviation of the amplitude response of the reconstruction error $T_0(z)$ from unity, whereas δ_2 is the maximum amplitude value of the worst-case aliasing transfer function $T_l(z)$. The boldface numbers indicate those parameters that have been fixed in the optimization. E_∞ and E_2 give the maximum stopband amplitude value of the prototype filter and the stopband energy, respectively.

Table I
Comparison between filter banks with $M = 32$ and $\rho = 1$. Boldface numbers indicate those parameters that have been fixed in the optimization.

Criterion	K	N	δ_1	δ_2	E_∞	E_2
LSQ	**8**	**511**	**0**	**0** **−∞ dB**	$1.2 \cdot 10^{-3}$ −58 dB	$7.4 \cdot 10^{-9}$
Minimax	**8**	**511**	**0**	**0** **−∞ dB**	$2.3 \cdot 10^{-4}$ −73 dB	$7.5 \cdot 10^{-8}$
LSQ	**8**	**511**	**10^{-4}**	$2.3 \cdot 10^{-6}$ −113 dB	$1.0 \cdot 10^{-5}$ −100 dB	$5.6 \cdot 10^{-13}$
Minimax	**8**	**511**	**10^{-4}**	$1.1 \cdot 10^{-5}$ −99 dB	$5.1 \cdot 10^{-6}$ −106 dB	$3.8 \cdot 10^{-11}$
LSQ	**8**	**511**	**0**	$9.1 \cdot 10^{-5}$ −81 dB	$4.5 \cdot 10^{-4}$ −67 dB	$5.4 \cdot 10^{-10}$
LSQ	**8**	**511**	**10^{-2}**	$5.3 \cdot 10^{-7}$ −126 dB	$2.4 \cdot 10^{-6}$ −112 dB	$4.5 \cdot 10^{-14}$
LSQ	**6**	**383**	**10^{-3}**	**10^{-5}** **−100 dB**	$1.7 \cdot 10^{-4}$ −75 dB	$8.8 \cdot 10^{-10}$
LSQ	**5**	**319**	**10^{-2}**	**10^{-4}** **−80 dB**	$8.4 \cdot 10^{-4}$ −62 dB	$2.7 \cdot 10^{-9}$

The first two banks in Table I are PR filter banks where the stopband performance has been optimized in the least-mean-square sense and in the minimax sense, respectively. The third and fourth designs are the corresponding nearly PR banks designed in such a way that the reconstruction error is restricted to be less than or equal to 10^{-4}. For these designs as well as for the fifth and sixth solutions in Table I, no constraints on the levels of the aliasing errors have been imposed. Some characteristics of the first and third designs are depicted in Figs. 2 and 3, respectively. From these figures as well as from Table I, it is seen that the nearly PR filter banks provide significantly improved filter bank performances at the expense of a small reconstruction error and very small aliasing errors. When comparing the second and fourth solutions in Table I, it is seen that the same is true for the corresponding minimax designs.

When comparing the first and fifth solutions in Table I, it is observed that even an optimized nearly PR filter bank without amplitude error provides a considerably better performance than the PR filter bank. Furthermore, it is seen that the performance of the nearly PR filter bank significantly improves when a higher reconstruction error is allowed (the sixth design in Table I).

For the last two solutions in Table I, the orders of the prototype filters have been decreased and they have been optimized subject to the given reconstruction and aliasing errors. Some of the characteristics of last design are depicted Figs. 4. When comparing this solution with the first PR design of Table I (see also Fig. 2), it is observed that the same or even a better filter bank performance can be achieved with a significantly lower filter order (319 compared with 511) when small amplitude and aliasing errors are allowed.

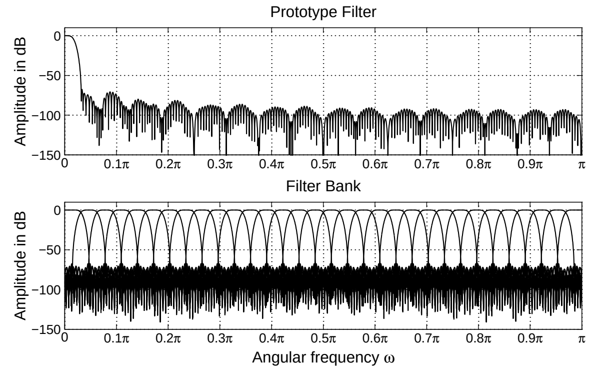

Figure 2. PR filter bank of $M = 32$ filters of length $N + 1 = 512$ for $\rho = 1$. The least-mean-square error design has been used.

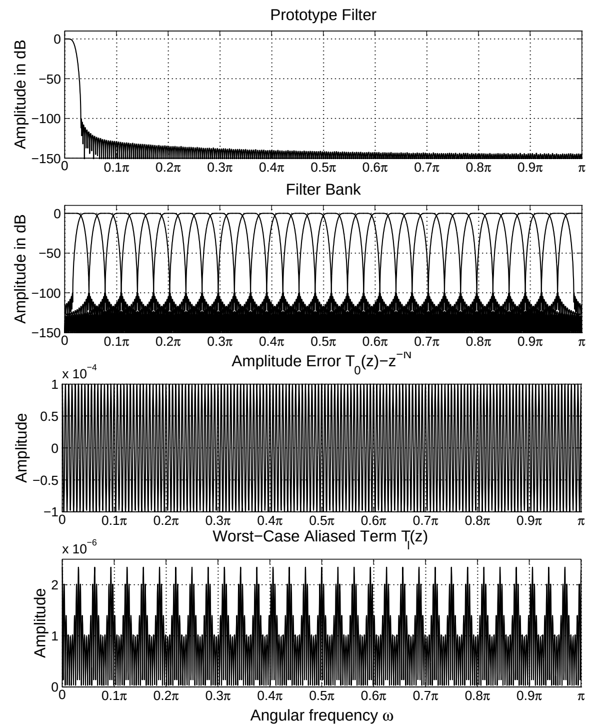

Figure 3. Filter bank of $M = 32$ filters of length $N + 1 = 512$ for $\rho = 1$ and $\delta_1 = 0.0001$. The least-mean-square error design has been used.

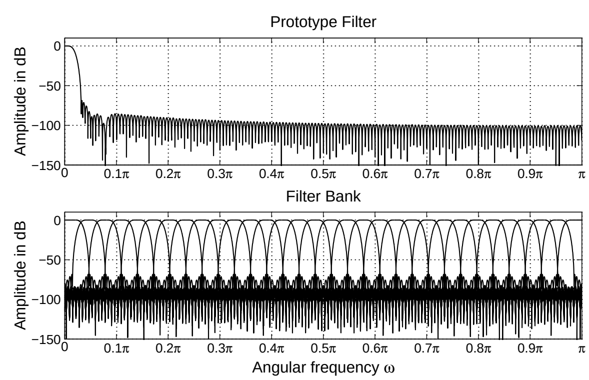

Figure 4. Filter bank of $M = 32$ filters of length $N + 1 = 320$ for $\rho = 1$, $\delta_1 = 0.01$, and $\delta_2 = 0.0001$. The least-mean-square error design has been used.

7. REFERENCES

[1] H. S. Malvar, *Signal Processing with Lapped Transforms*. Norwood: Artec House, 1992.

[2] H. S. Malvar, "Extended lapped transforms: Properties, applications, and fast algorithms," *IEEE Trans. Signal Proc.*, vol. 40, pp. 2703−2714, Nov. 1992.

[3] R. D. Koilpillai and P. P. Vaidyanathan, "Cosine-modulated FIR filter banks satisfying perfect reconstruction," *IEEE Trans. Signal Proc*, vol. 40, pp. 770−783, April 1992.

[4] T. Saramäki, "Designing prototype filters for perfect-reconstruction cosine-modulated filter banks," in *Proc. IEEE Int. Symp. Circuits Syst.*, San Diego, CA, May 1992, pp. 1605−1608.

[5] T. Q. Nguyen, "Digital filter bank design quadratic constrained formulation," *IEEE Trans. Signal Proc.*, vol. 43, pp. 2103−2108, Sept. 1995.

[6] H. Xu, W.-S. Lu, and A. Antoniou, "Efficient iterative design method for cosine-modulated QMF banks," *IEEE Trans. Signal Proc.*, vol. 44, pp. 1657−1668, July 1996.

[7] C. Creusere and S. Mitra, "A simple method for designing high-quality prototype filters for *M*-band pseudo QMF banks," *IEEE Trans. Signal Proc.*, vol. 43, pp. 1005−1007, April 1995.

[8] A. Mertins, "Subspace approach for the design of cosine-modulated filter banks with linear-phase prototype filter," *IEEE Trans. Signal Proc.*, vol. 46, pp. 2812−2818, Oct. 1998.

[9] T. Saramäki, "A generalized class of cosine-modulated filter banks," in *Proc. First Int. Workshop on Transforms and Filter Banks*, Tampere, Finland, Feb. 1998, pp. 336–365.

[10] Y.-P. Lin and P. P. Vaidyanathan, "Linear phase cosine modulated maximally decimated filter banks with perfect reconstruction," *IEEE Trans. Signal Proc.*, vol. 42, pp. 2525−2539, Nov. 1995.

[11] T. Karp and N. J. Fliege, "Modified DFT filter banks with perfect reconstruction," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 1404−1414, Nov. 1999.

[12] R. Bregović and T. Saramäki, "A systematic technique for designing prototype filters for perfect reconstruction cosine modulated and modified DFT filter banks," to appear in *Proc. IEEE Int. Symp. Circuits Syst.*, Sydney, Australia, May 2001.

[13] S. R. K. Dutta and M. Vidyasagar, "New algorithms for constrained minimax optimization," *Mathematical programming*, vol. 13, pp. 140−155, 1977.