PERFORMANCE BOUNDS FOR LINEAR BLIND AND GROUP-BLIND MULTIUSER DETECTORS

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ABSTRACT

In blind multiuser detection for CDMA systems, the receiver knows only the code of the user of interest, while in group-blind multiuser detection the receiver knows a subset of codes, e.g., the in-cell users in a basestation. This paper derives bounds for the performance of linear blind and group-blind multiuser detectors. The bounds are derived under a number of different system assumptions. The bounds show the theoretical gain by using group-blind detectors over blind detectors, and they also show that previously developed blind and group-blind detectors are relatively close to optimum among estimators using only second order moments. However, the bounds also show that a considerable gap exist to optimum detectors that are not restricted to second order moments.

1. INTRODUCTION

In blind multiuser detection for CDMA systems [1], the receiver knows only the code of the user of interest. Using some estimation technique, it then estimates a linear receiver that is able to reduce intereference from co-channel users. In [3] this principle was extextended to define *group-blind* multiuser detection, where the receiver is assumed to know a subset (group) of codes, but not all codes; for example known intra-cell users at a basestation with intercell interference, or a mobile station in a UTRA-TDD system.

In a recent paper [4, 5, 6] we have analyzed the performance of the blind and group-blind detectors developed in [2, 3]. However, this is the performance of a specific class of detectors. The question therefore remains open: what is the theoretically best performance that can be obtained when only one or a subset of codes is known? And how much does the knowledge of the additional codes in group-blind multiuser detection improve performance theoretically? In this paper we will try to answer these questions. The answer is not definite, but an answer that never-theless gives some insight into the problem. As will ne seen, the answer depends strongly on what knowledge the receiver is assumed to have, and what computational complexity can be allowed.

2. SYSTEM MODEL

We consider a simple, synchronous CDMA system with K users of which $\check{K}$ are known and $\tilde{K}$ are unknown ($\check{\cdot}$ indicates quantities belonging to the known users, $\tilde{\cdot}$ to the unknown users), where the received signal can be written as

$$\begin{aligned} \mathbf{r}[i] &= \sum_{k=1}^{K} b_k[i] A_k \mathbf{s}_k + \mathbf{n}[i] && (1)\\ &= \sum_{k=1}^{\check{K}} \check{b}_k[i] \check{A}_k \check{\mathbf{s}}_k + \sum_{k=1}^{\tilde{K}} \tilde{b}_k[i] \tilde{\mathbf{s}}_k + \mathbf{n}[i] && (2)\\ &= \check{\mathbf{S}}\check{\mathbf{A}}\check{\mathbf{b}}[i] + \tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}}[i] + \mathbf{n}[i]. && (3) \end{aligned}$$

where $\mathbf{S}$ are the codes A_i the amplitudes, and $b[i]$ the transmitted bits; $\mathbf{n}[i]$ is white Gaussian noise. We will assume that the codes are normalized. In many cases we will assume that the amplitudes are embedded in the codes, i.e. we will assume a signal model

$$\mathbf{r}[i] = \check{\mathbf{S}}\check{\mathbf{b}}[i] + \tilde{\mathbf{S}}\tilde{\mathbf{b}}[i] + \mathbf{n}[i] \qquad (4)$$

We will assume that the transmitted bits $b_k[i] \in \{+1, -1\}$ are iid with $P(b_k[i] = +1) = P(b_k[i] = -1) = \frac{1}{2}$. We will not assume any knowledge on the unknown codes $\tilde{\mathbf{S}}$.

3. PREFACE: BOUNDS FOR BLIND AND GROUP-BLIND DETECTORS

Consider the model (4) with $\check{\mathbf{S}}$ known and $\tilde{\mathbf{S}}$ unknown, and the transmitted bits unknown. Assume that M signal samples $\mathbf{r}[1] \ldots \mathbf{r}[M]$ are received. The question we seek to answer is: what is the error probability in detecting the bits $\check{\mathbf{b}}[1] \ldots \check{\mathbf{b}}[M]$? This question can be adressed in more generality, but in this paper we will consider only linear detectors:

$$\hat{\check{b}}_k[i] = \operatorname{sgn}\left(\hat{\mathbf{w}}_k^T \mathbf{r}[i]\right) \qquad (5)$$

The linear weight vector $\hat{\mathbf{w}}_k$ is a function of the received samples $\mathbf{r}[1] \ldots \mathbf{r}[M]$. The ultimate question to answer would then be: how can $\hat{\mathbf{w}}_k$ be chosen to minimize the error probability, and what is this error probability? However, we will simplify this question by dividing the estimation/detection process into two steps

1. Estimate $\hat{\mathbf{w}}_k$ so that it is as close as possible to a fixed linear detector, i.e., the MMSE detector or decorrelating detector (or combinations hereof).
2. Use this estimate in the detector and evaluate the resulting SINR.

Thus the problem reduces to finding an as accurate as possible estimate of the MMSE detector or decorrelating detector from the received samples $\mathbf{r}[1]\ldots\mathbf{r}[M]$. We can then concentrate on finding Cramer-Rao bounds for the estimate $\hat{\mathbf{w}}_k$. However, also this is not quite straight forward. The problem is the model to use for the bits $b_k[i]$. Our model assumption is that they are iid binomial $+1, -1$. Using this, it is indeed possible to calculate Cramer-Rao bounds, which will be done in section 4.1. However, this bound might not be very interesting, since the corresponding MLE would be extremely complex. Thus, the problem is the much more intricate problem of finding (Cramer-Rao) bounds for detectors when the complexity is constrained. Our solution to this is to assume that the receiver, in the estimation stage, has only limited information about $b_k[i]$. This information can be that $^{1)}$ the exact distribution $b_k[i]$ is not known, but only, for example, that the $b_k[i]$ have zero mean, $E\{b_k[i]^2\} = 1$ and are uncorrelated, or $^{2)}$ the bits $b_k[i]$ are assumed deterministic, possibly satisfying some constraints.

4. CRAMER-RAO BOUNDS FOR LINEAR DETECTORS

The MMSE detector is given by

$$\begin{aligned}\mathbf{w}_k &= \mathbf{C}_r^{-1}\mathbf{s}_k & (6)\\ \mathbf{C}_r &= E[\mathbf{r}\mathbf{r}^T] \;=\; \mathbf{S}\mathbf{A}^2\mathbf{S}^T + \sigma^2\mathbf{I} \\ &= \check{\mathbf{S}}\check{\mathbf{A}}^2\check{\mathbf{S}}^T + \tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T + \sigma^2\mathbf{I} & (7)\end{aligned}$$

while the decorrelating detector is given by

$$\begin{aligned}\mathbf{d}_k &= \mathbf{S}\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{e}_k \\ &= \begin{bmatrix}\check{\mathbf{S}} & \tilde{\mathbf{S}}\end{bmatrix}\begin{bmatrix}\check{\mathbf{S}}^T\check{\mathbf{S}} & \check{\mathbf{S}}^T\tilde{\mathbf{S}} \\ \tilde{\mathbf{S}}^T\check{\mathbf{S}} & \tilde{\mathbf{S}}^T\tilde{\mathbf{S}}\end{bmatrix}^{-1}\mathbf{e}_k & (8)\end{aligned}$$

The unknown part of these detectors is $\tilde{\mathbf{S}}$ (and $\tilde{\mathbf{A}}$). We can obtain the estimated detectors by inserting an estimate of $\tilde{\mathbf{S}}$ (and $\tilde{\mathbf{A}}$). Now, whatever (blind) method we use for estimation, there will alsways be ambiguities in the determination of $\tilde{\mathbf{S}}$. It is therefore important to understand to what extend (6) and (8) are invariant to ambiguities. It is easy to see that (6) is invariant to orthogonal transformations of $\tilde{\mathbf{S}}\tilde{\mathbf{A}}$, i.e., if $\mathbf{Q}$ is an arbitrary $\tilde{K}\times\tilde{K}$ orthogonal matrix, then if we insert $(\tilde{\mathbf{S}}\tilde{\mathbf{A}})_1 = \tilde{\mathbf{S}}\tilde{\mathbf{A}}\mathbf{Q}$ in (6) we obtain the same result for $\mathbf{w}_k$. The decorrelating detector, on the other hand, is invariant to transformations by an arbitrary non-singular $\tilde{K}\times\tilde{K}$ matrix $\mathbf{M}$, i.e., if we insert $\tilde{\mathbf{S}}_1 = \tilde{\mathbf{S}}\mathbf{M}$ in (8) we obtain the same $\mathbf{d}_k$. In other words: to form the decorrelating detector, we just need to know the subspace spanned by $\tilde{\mathbf{S}}$, but for the MMSE detector we also need to know the power in that subspace, what we can call the power ellipsoid. This power ellipsoid is related to the second order properties of $\tilde{\mathbf{b}}[i]$, and the MMSE detector therefore, not unsurprisingly, cannot be determined in a completely deterministic model (except with some twist, as we will see later). The decorrelating detector, on the other hand, is completely defined in deterministic model.

Estimation of $\mathbf{w}_k$ or $\mathbf{d}_k$ is thus a two step process: first $\tilde{\mathbf{S}}$ is estimated (non-uniquely), and the estimate is inserted in (6) or (8). Our approach will therefore be to calculate CRBs for $\tilde{\mathbf{S}}$ and use transformation of parameters for find the CRB for $\mathbf{w}_k$ or $\mathbf{d}_k$. Let $\mathbf{J}_S$ be the Fisher Information matrix for estimation of $\tilde{\mathbf{S}}$. The CRB for $\mathbf{w}_k$ can then be found as

$$\mathbf{J}_w^{-1} = \mathbf{H}^T\mathbf{J}_S^{-1}\mathbf{H} \tag{9}$$

where

$$H_{(i,j),k} = \frac{\partial w_k}{\partial \tilde{S}_{i,j}} \tag{10}$$

This is most easily found from the differential [6]

$$\Delta\mathbf{w}_1 = \mathbf{C}^{-1}\left(\tilde{\mathbf{S}}\Delta\tilde{\mathbf{S}}^T + \Delta\tilde{\mathbf{S}}\tilde{\mathbf{S}}^T\right)\mathbf{C}^{-1}\mathbf{s}_1 \tag{11}$$

from which

$$H_{(i,j),k} = [\mathbf{C}^{-1}\tilde{\mathbf{S}}]_{k,j}[\mathbf{C}^{-1}\mathbf{s}_1]_i + [\mathbf{C}^{-1}]_{k,i}[\tilde{\mathbf{S}}^T\mathbf{C}^{-1}\mathbf{s}_1]_j \tag{12}$$

Once $\mathbf{J}_w^{-1}$ has been found, the SINR can be found from [6]

$$\mathrm{SINR} = \frac{A_1^2(\mathbf{w}_1^T\mathbf{s}_1)^2}{\sum_{k=2}^K A_k^2(\mathbf{w}_1^T\mathbf{s}_k)^2 + \sigma^2\mathbf{w}_1^T\mathbf{w}_1 + \mathrm{tr}(\mathbf{J}_w^{-1}\mathbf{C}_r)} \tag{13}$$

Now, as stated, $\tilde{\mathbf{S}}$ generally cannot be determined uniquely, and $\mathbf{J}_S$ therefore may be singular. However, $\mathbf{w}_k$ is unique, and the CRB can be found by by using pseudo-inverse for $\mathbf{J}_S$ or fixing some parameters of $\tilde{\mathbf{S}}$.

4.1. Discrete Model

For simplicity, we will assume that the amplitudes of the known users are known, so that the model (4) applies. We assume, as is the actual model, that the bits are $+1, -1$ with equal probability. The PDF for a single observation can then be written

$$\frac{1}{2^K(2\pi\sigma^2)^{N/2}}\sum_{\mathbf{b}}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^N(x_i - \check{\mathbf{S}}_{i,:}\check{\mathbf{b}} - \tilde{\mathbf{S}}_{i,:}\tilde{\mathbf{b}})^2\right) \tag{14}$$

The Fisher Information matrix can be written as

$$J_{i,j} = E\left[\frac{\frac{\partial f(\mathbf{x};\theta)}{\partial\theta_i}\frac{\partial f(\mathbf{x};\theta)}{\partial\theta_j}}{f(\mathbf{x};\theta)^2}\right] \tag{15}$$

The resulting CRB can only be calculated numerically, but even then the complexity is very high. However, it can be seen that

$$\lim_{\sigma\to 0} J_{(i,j),(k,l)}\sigma^2 = \delta_{i,k}\delta_{j,l} \tag{16}$$

The CRB on $\mathbf{w}_k$ is then

$$\mathbf{J}^{-1} = \frac{\sigma^2}{M}\mathbf{H}^T\mathbf{H} \tag{17}$$

where $\mathbf{H}$ is given by (12). With some calculus we then get

$$\begin{aligned}&\frac{M}{\sigma^2}\mathbf{J}^{-1} = \\ &\quad \mathbf{C}^{-1}\tilde{\mathbf{S}}\tilde{\mathbf{S}}^T\mathbf{C}^{-1}\mathbf{s}_1^T\mathbf{C}^{-2}\mathbf{s}_1 + \mathbf{C}^{-2}\mathbf{s}_1^T\mathbf{C}^{-1}\tilde{\mathbf{S}}\tilde{\mathbf{S}}^T\mathbf{C}^{-1}\mathbf{s}_1 \\ &\quad + \mathbf{C}^{-1}\tilde{\mathbf{S}}\tilde{\mathbf{S}}^T\mathbf{C}^{-1}\mathbf{s}_1\mathbf{s}_1^T\mathbf{C}^{-2} + \mathbf{C}^{-2}\mathbf{s}_1\mathbf{s}_1^T\mathbf{C}^{-1}\tilde{\mathbf{S}}\tilde{\mathbf{S}}^T\mathbf{C}^{-1}\end{aligned} \tag{18}$$

The MLE corresponding to this model is extremely complex. It has to try all 2^{KM} possible bits. However, the present problem is somewhat similar to blind source separation, and some reasonably efficient (both computationlly and in terms of performance) exist [8] that suitably modified can be applied to (group) blind multiuser detection.

4.2. Estimation From the Covariance Matrix

The detectors developed in [2] and [3] are all based on estimating the covariance matrix, and then use this for estimating the detector. In [7] an asymptotic bound is given for estimation from the covariance matrix (or other statistics). In our case this bound is given as follows. Define

$$[\mathbf{J}]_{(i,j),(k,l)} = \frac{\partial \bar{\mathbf{C}}}{\partial [\tilde{\mathbf{S}}]_{i,j}}^T \bar{\mathbf{T}}^{-1} \frac{\partial \bar{\mathbf{C}}}{\partial [\tilde{\mathbf{S}}]_{k,l}}, \qquad (19)$$

with $\bar{\mathbf{C}} = \mathrm{vec}(\mathbf{C})$ and $\bar{\mathbf{T}} = \mathrm{Mat}(\mathbf{T})$, where $\bar{\mathbf{T}}$ is the covariance matrix of the covariance matrix estimate. The asymptotic covariance of the estimate of $\tilde{\mathbf{S}}$ is then lower bounded by $\mathbf{J}^{-1}$, and it is also shown in [7] that this bound is *asymptotically* tight. In [6] it was proven that the estimated covariance matrix is assymptotically Gaussian with covariance matrix

$$\begin{aligned} M[\mathbf{T}]_{i,j,k,l} &= M\mathrm{cov}\{[\hat{\mathbf{C}}]_{i,j}, [\hat{\mathbf{C}}]_{k,l}\} \\ &= [\mathbf{C}]_{i,k}[\mathbf{C}]_{j,l} + [\mathbf{C}]_{i,l}[\mathbf{C}]_{j,k} \\ &\quad -2\sum_{\alpha=1}^{K} [\mathbf{S}]_{i,\alpha}[\mathbf{S}]_{j,\alpha}[\mathbf{S}]_{k,\alpha}[\mathbf{S}]_{l,\alpha} \qquad (20) \end{aligned}$$

The derivatives needed in (19) are

$$\frac{\partial [\mathbf{C}]_{i,j}}{\partial [\tilde{\mathbf{S}}]_{m,n}} = \delta_{i,m}[\tilde{\mathbf{S}}]_{j,n} + \delta_{j,m}[\tilde{\mathbf{S}}]_{i,n} \qquad (21)$$

$$\frac{\partial \mathbf{C}}{\partial [\tilde{\mathbf{S}}]_{m,n}} = \mathbf{e}_m \tilde{\mathbf{s}}_n^T + \tilde{\mathbf{s}}_n \mathbf{e}_m^T \qquad (22)$$

The bound (19) can also be interpreted as a CRB. Using the fact that the covariance matrix estimate is asymptotically Gaussian, we can calculate an asymptotic CRB using the standard formulas for Gaussian noise:

$$\begin{aligned} [\mathbf{J}_{(i,j),(k,l)} &= \frac{1}{2}\mathrm{tr}\left[\bar{\mathbf{T}}^{-1} \frac{\partial \bar{\mathbf{T}}}{\partial [\tilde{\mathbf{S}}]_{i,j}} \bar{\mathbf{T}}^{-1} \frac{\partial \bar{\mathbf{T}}}{\partial [\tilde{\mathbf{S}}]_{k,l}}\right] \\ &\quad + \frac{\partial \bar{\mathbf{C}}}{\partial [\tilde{\mathbf{S}}]_{i,j}}^T \bar{\mathbf{T}}^{-1} \frac{\partial \bar{\mathbf{C}}}{\partial [\tilde{\mathbf{S}}]_{k,l}} \qquad (23) \end{aligned}$$

(notice that we just have one sample of the estimated covariance matrix)

In this case both the covariance matrix itself and the covariance of the covariance $\mathbf{T}$ depends on the unknown parameters. However, since we are estimating $\tilde{\mathbf{S}}$ from a single sample of $\hat{\mathbf{C}}$, the covariance of the covariance matrix $\mathbf{T}$ therefore cannot be estimated; in addition, $\mathbf{T}$ is actually a fourth order moment, so using this is not strictly a second order approach.

We can overcome these problems by modifying our model. Assume the detector does not know (or utilize) that $\mathbf{T}$ depends on $\tilde{\mathbf{S}}$; however, it still knows the noise might not be white. In that case, the Fisher information matrix becomes block diagonal, and the CRB for $\tilde{\mathbf{S}}$ does not depend on $\mathbf{T}$. We then arrive at (19).

4.3. Deterministic Model

Consider now the transmitted bits $\mathbf{b}[i]$ as deterministic, continuous variables. The problem is then the joint estimation of $\tilde{\mathbf{S}}$ and $\mathbf{b}[i]$, and we can calculate CRB for this problem. As mentioned in the introduction, to form the MMSE detector, $\tilde{\mathbf{S}}$ must be determined

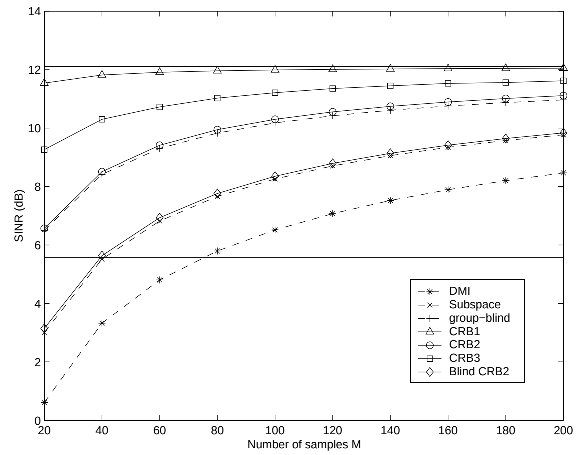

Fig. 1. Median performance. CRB1: Section 4.1, CRB2: Section 4.2, CRB3: Section 4.3.

except for an orthogonal rotation. However, in the deterministic model, only the subspace of $\tilde{\mathbf{S}}$ can be identified. We therefore modify the deterministic model somewhat: now assume that $\mathbf{b}[i]$ is assumed deterministic, but satisfying $\|\mathbf{b}[i]\|_2^2 = K$, i.e. that it is lying on a hypersphere (notice that the actual bits satisfy this). Then $\tilde{\mathbf{S}}$ can be identified except for an orthogonal rotation, and we can therefore calculate the CRB for the MMSE detector. Define

$$\bar{\mathbf{r}} = [\mathbf{r}^T[1] \dots \mathbf{r}^T[M]]^T \qquad (24)$$

$$\bar{\mathbf{b}} = [\mathbf{b}^T[1] \dots \mathbf{b}^T[M]]^T \qquad (25)$$

$$\bar{\mathbf{B}}[i] = [b_1[i]\mathbf{I}, b_2[i]\mathbf{I} \dots b_K[i]\mathbf{I}] \qquad (26)$$

$$\bar{\mathbf{B}} = [\mathbf{B}^T[1] \dots \mathbf{B}^T[M]]^T \qquad (27)$$

$$\bar{\mathbf{b}} = [b_1[1], b_2[1] \dots b_K[1] \dots b_1[M] \dots b_K[M]]^T \qquad (28)$$

$$\bar{\mathbf{S}} = \mathbf{I} \otimes \mathbf{S} = \mathrm{diag}(\mathbf{S}, \mathbf{S} \dots \mathbf{S}) \qquad (29)$$

$$\bar{\mathbf{s}} = [s_{1,1}, s_{2,1} \dots s_{N,1}, s_{1,2} \dots s_{N,K}]^T \qquad (30)$$

And similarly for known and unknown users. Since the bits are contrained to have unit norm, we reparametrize them as

$$\mathbf{b}^T[m] = \left[\pm\sqrt{K - \sum_{i=2}^{K} b_i^2[m]}, b_2[m], b_3[m] \dots b_K[m]\right]$$

and define $\phi[m] = [b_2[m], b_3[m], \dots, b_K[m]]^T$. We can write the received vector in two ways

$$\bar{\mathbf{r}} = \bar{\mathbf{S}}\bar{\mathbf{A}}\bar{\mathbf{b}} = \check{\bar{\mathbf{S}}}\check{\bar{\mathbf{A}}}\check{\bar{\mathbf{b}}} + \tilde{\bar{\mathbf{S}}}\tilde{\bar{\mathbf{A}}}\tilde{\bar{\mathbf{b}}} + \bar{\mathbf{n}} \qquad (31)$$

$$\bar{\mathbf{r}} = \check{\bar{\mathbf{B}}}\check{\bar{\mathbf{A}}}\check{\bar{\mathbf{s}}} + \tilde{\bar{\mathbf{B}}}\tilde{\bar{\mathbf{A}}}\tilde{\bar{\mathbf{s}}} + \bar{\mathbf{n}} \qquad (32)$$

We can then write the Fisher Information matrix as

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{12}^T & \mathbf{F}_{22} \end{bmatrix} \qquad (33)$$

where

$$\mathbf{F}_{11} = \frac{\partial \bar{\mathbf{b}}}{\partial \phi}^T \bar{\mathbf{A}}\bar{\mathbf{S}}^T\bar{\mathbf{S}}\bar{\mathbf{A}} \frac{\partial \bar{\mathbf{b}}}{\partial \phi} \qquad (34)$$

$$= \quad \mathrm{diag}\left(\frac{\partial \mathbf{b}[1]}{\partial \phi[1]}^T \mathbf{A}\mathbf{S}^T\mathbf{S}\mathbf{A}\frac{\partial \mathbf{b}[1]}{\partial \phi[1]}\right. \tag{35}$$

$$\left.\dots \frac{\partial \mathbf{b}[M]}{\partial \phi[M]}^T \mathbf{A}\mathbf{S}^T\mathbf{S}\mathbf{A}\frac{\partial \mathbf{b}[M]}{\partial \phi[M]}\right) \tag{36}$$

$$\mathbf{F}_{12} = \frac{\partial \bar{\mathbf{b}}}{\partial \phi}^T \bar{\mathbf{A}}\bar{\mathbf{S}}^T\tilde{\bar{\mathbf{B}}} \tag{37}$$

$$= \begin{bmatrix} \frac{\partial \mathbf{b}[1]}{\partial \phi[1]}^T \mathbf{A}\mathbf{S}^T\tilde{\mathbf{B}}[1] \\ \vdots \\ \frac{\partial \mathbf{b}[M]}{\partial \phi[M]}^T \mathbf{A}\mathbf{S}^T\tilde{\mathbf{B}}[M] \end{bmatrix} \tag{38}$$

$$\mathbf{F}_{22} = \tilde{\bar{\mathbf{B}}}^T\tilde{\bar{\mathbf{B}}} \tag{39}$$

The inverse is then given by

$$\mathbf{F}^{-1} = \begin{bmatrix} \mathbf{F}_{11}^{-1} + \mathbf{F}_{11}^{-1}\mathbf{F}_{12}\mathbf{D}^{-1}\mathbf{F}_{12}^T\mathbf{F}_{11}^{-1} & \mathbf{F}_{11}^{-1}\mathbf{F}_{12}\mathbf{D}^{-1} \\ \mathbf{D}^{-1}\mathbf{F}_{12}^T\mathbf{F}_{11}^{-1} & \mathbf{D}^{-1} \end{bmatrix} \tag{40}$$

where

$$\mathbf{D} = \tilde{\bar{\mathbf{B}}}^T\tilde{\bar{\mathbf{B}}} - \sum_{m=1}^{M}\tilde{\mathbf{B}}^T[m]\mathbf{S}\mathbf{A}\frac{\partial \mathbf{b}[m]}{\partial \phi[m]} \times \tag{41}$$

$$\left(\frac{\partial \mathbf{b}[m]}{\partial \phi[m]}^T \mathbf{A}\mathbf{S}^T\mathbf{S}\mathbf{A}\frac{\partial \mathbf{b}[m]}{\partial \phi[m]}\right)^{-1}\frac{\partial \mathbf{b}[m]}{\partial \phi[m]}^T \mathbf{A}\mathbf{S}^T\tilde{\mathbf{B}}[m] \tag{42}$$

Notice that the CRB for $\tilde{\mathbf{S}}$ is given by $\mathbf{D}^{-1}$. Here

$$\frac{\partial \mathbf{b}[m]}{\partial \phi[m]} = \begin{bmatrix} \mathbf{b}'[m] \\ \mathbf{I} \end{bmatrix} \tag{43}$$

$$\mathbf{b}'[m] = \pm\frac{1}{\sqrt{K - \sum_{i=2}^{K} b_i^2[m]}}[b_2[m]\dots b_K[m]] \tag{44}$$

$$= -b_1[m][b_2[m]\dots b_K[m]]. \tag{45}$$

With this we get

$$\frac{\partial \mathbf{b}[m]}{\partial \phi[m]}^T \mathbf{A}\mathbf{S}^T = \mathbf{A}_{K-1}\mathbf{S}_{K-1}^T - A_1 b_1[m]\mathbf{b}_{K-1}[m]\mathbf{s}_1^T. \tag{46}$$

This inverse should be averaged over all bit sequences. This can either be done numerically or asymptotically ($M \rightarrow \infty$). In the present paper we do the avaraging numerically.

5. NUMERICAL RESULTS

The bounds were calculated for a system with a spreading gain of 16 and 6 known users and 3 unknown users. All users had power one, and the SNR was 15dB. Random codes were used, and the statistics were calculated over 100 random code sets. The bounds are compared in Fig. 1 and 2 with the performance of the estimators developed in [2] and [3] using the performance analysis of [4, 5]. All bounds are for the group-blind detectors, except 'blind CRB2'.

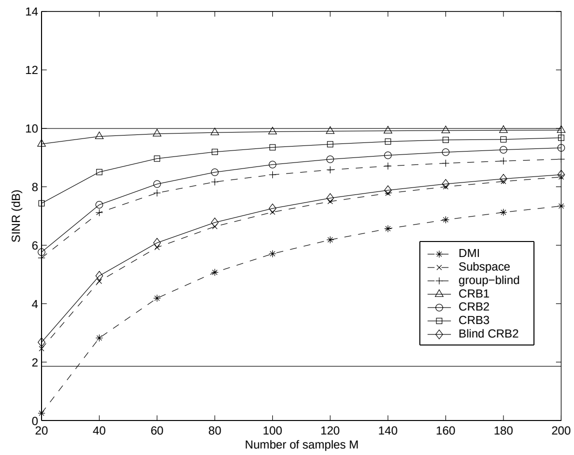

Fig. 2. 10-percentile performance (i.e. 'worst case' performance). CRB1: Section 4.1, CRB2: Section 4.2, CRB3: Section 4.3.

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