

# STRUCTURE PRESERVING ERROR CONCEALMENT WITH DIRECTIONAL SMOOTHNESS MEASURE

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## ABSTRACT

We propose a *directional* smoothness measure for block-based error concealment through spatial correlation. Image structures revealed by consistent edge profiles are very important for subjective visual quality. We treat the problem of block reconstruction as consistent recovery of local image structures. The directional smoothness measure evaluates structural consistency along edge elongation and is used as the object function for block reconstruction. Corrupted DCT coefficients are recovered by smoothly extending various edge profiles from surrounding areas to missing blocks. The reconstruction is adaptive to local image structures. Consistent cross-edge sharpness and along-edge smoothness are maximally preserved during the reconstruction. The proposed concealment method demonstrates encouraging improvement both in the subjective image quality and in the reconstruction PSNR over conventional schemes. It is applicable to various spatial and spectral interleaving systems and a fast implementation is also proposed.

## 1. INTRODUCTION

The problem of recovering lost and damaged image data is often encountered when image/video streams are transmitted over noisy channels or congested networks. In this paper, we address the error concealment problem for block-based transform coded images by exploiting spatial correlation. Specifically, the discrete cosine transform (DCT) is considered. Block-based transform coding is widely used by current compression standards including JPEG, MPEG and H.261 where the coded bit stream is vulnerable to transmission error because insignificant bit error can cause significant quality degradation to the decoded images. However, image/video data contains sufficient spatial correlation that makes error concealment possible.

With different spatial and spectral interleaving designs, transmission errors can cause the loss of a few DCT coefficients to the loss of an entire block. Using available information in surrounding blocks to recover a lost one, the commonly used smoothness criterion [1, 2, 3] tends to blur the

image. A second-order derivative-based smoothness measure is introduced in [4] to alleviate the problem. As an alternative, edge-based spatial interpolation schemes [5] fill in a missing block with values that are consistent with the edges detected from surrounding pixels. However, these schemes are not able to take advantage of any correctly received coefficient. Moreover, the reconstruction can be misleading when the edge orientation inside the damaged block deviates from its neighbors or when edge detection is affected by quantization noise. Projection onto convex sets (POCS)[6] has also been used for recovering damaged blocks by iterative projections between spatial and spectral constraints. POCS is computationally expensive and unpleasant blocky effects have been observed in the recovered image in [6].

In contrast to the conventional approaches, we intend to pursue consistent image structures revealed by local edge profiles when recovering damaged blocks. We recognize the key role of structural consistency in achieving good visual quality and propose a new error concealment algorithm in this paper. A directional smoothness measure is derived to consistently extend the signal profiles from surrounding blocks to the damaged one. During the reconstruction, the smoothness along edge elongation as well as the sharpness across edges are maximally preserved. Hence the reconstruction is able to achieve good visual quality. In the following, we first introduce the directional smoothness measure for structural consistency in section 2. We discuss the recovery of lost DCT coefficients in section 3. Section 4 reports experimental results and performance improvement of the proposed method over the smoothness-based method in [4]. Final conclusions are given in section 5.

## 2. DIRECTIONAL SMOOTHNESS MEASURE

Edges play an important role in the subjective image quality because the human visual system is sensitive to the structural information revealed by edges. Over-smoothed edges blur an image while broken and falsely reconstructed edges cause unpleasant artifacts. Consistent edge profiles, i.e. con-

sistent cross-edge sharpness and along-edge smoothness, ensure consistent image structures as well as good visual quality. In order to preserve structural consistency, we measure the directional smoothness of image signal along the edge elongation. First-order and second-order directional derivatives are used here to serve the purpose.

Let  $\nabla f = [f_x, f_y]'$  denote the gradient vector of 2D function  $f$  and denote partial derivatives by subscripts. The magnitude and angular direction of the gradient are

$$\|\nabla f\| = \sqrt{f_x^2 + f_y^2}, \quad \theta = \tan^{-1}(f_y/f_x) \quad (1)$$

The first and second-order directional derivatives of  $f$  in the direction indicated by a unit vector  $\vec{n}$  are given by

$$f_{\vec{n}}^{(1)} = \vec{n} \cdot \nabla f, \quad f_{\vec{n}}^{(2)} = \begin{bmatrix} \vec{n} \cdot [f_{xx}, f_{xy}]' \\ \vec{n} \cdot [f_{yx}, f_{yy}]' \end{bmatrix} \quad (2)$$

If we take  $\vec{n}$  to be orthogonal to the local gradient  $\nabla f$ , i.e.  $\vec{n} = [\cos(\theta + \frac{\pi}{2}), \sin(\theta + \frac{\pi}{2})]'$  is the local tangent vector pointing along the edge elongation, we have

$$f_{\vec{n}}^{(1)} = 0, \quad f_{\vec{n}}^{(2)} = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \quad (3)$$

Denote pixel location with  $(i, j)$ , the directional smoothness measure over an image block  $\{f_{i,j}\}$  is defined as

$$\begin{aligned} \Psi(f) &= \sum_{i,j} \|f_{\vec{n}}^{(2)}(i, j)\|^2 \\ &= \sum_{i,j} [\sin^2 \theta_{i,j} f_{xx}^2(i, j) + \cos^2 \theta_{i,j} f_{yy}^2(i, j) + f_{xy}^2(i, j) \\ &\quad - 2 \sin \theta_{i,j} \cos \theta_{i,j} f_{xy}(i, j) (f_{xx}(i, j) + f_{yy}(i, j))] \\ &= \sum_{i,j} [w_{i,j}^1 f_{xx}^2(i, j) + (1 - w_{i,j}^1) f_{yy}^2(i, j) + f_{xy}^2(i, j) \\ &\quad - 2w_{i,j}^2 f_{xy}(i, j) (f_{xx}(i, j) + f_{yy}(i, j))] \\ w_{i,j}^1 &= \sin^2 \theta_{i,j}, \quad w_{i,j}^2 = \sin \theta_{i,j} \cos \theta_{i,j} \end{aligned} \quad (4)$$

where we assume  $f_{xy} = f_{yx}$ . The discrete approximations of the partial derivatives are

$$\begin{aligned} f_{xx}(i, j) &= f_{i-1,j} - 2f_{i,j} + f_{i+1,j} \\ f_{yy}(i, j) &= f_{i,j-1} - 2f_{i,j} + f_{i,j+1} \\ f_{xy}(i, j) &= \frac{1}{4}[f_{i+1,j+1} - f_{i-1,j+1} - f_{i+1,j-1} + f_{i-1,j-1}] \end{aligned} \quad (5)$$

With directional derivatives, the proposed measure evaluates how smoothly the local edge profile propagates along the edge elongation. It provides a more faithful measurement of structural consistency. When the minimum value of the proposed measure is pursued to recover damaged blocks, we suppress the signal variation along edge directions and consequently allow various edge profiles (sharpness and orientation) to be inherited from surrounding image areas and smoothly extended to damaged blocks. Note the first three summation terms in the proposed measure (4) correspond to

the quadratic variation in [4] when  $\theta = \frac{\pi}{4}$ , i.e. the diagonal edge case. Compared to the smoothness measure used in [4], the adaptive weighting  $w^{1,2}$  in the proposed measure promotes adaptive treatment for blocks with different local structures.

### 3. RECONSTRUCTION BY DIRECTIONAL SMOOTHNESS MEASURE

Block-based DCT coding system divides an image into  $M \times M$  blocks and performs block DCT. Transform coefficients are quantized and coded. Let  $\{a_{k,l}\}_{k,l=0}^{M-1}$  represent the quantized block DCT coefficients and  $\{f_{i,j}\}_{i,j=0}^{M-1}$  be the pixel values obtained by inverse DCT on  $\{a_{k,l}\}$ , then the DCT transform pair can be written as [4]

$$\mathbf{a} = T' \mathbf{f}, \quad \mathbf{f} = T \mathbf{a} \quad (6)$$

where  $\mathbf{f}$  and  $\mathbf{a}$  are the vector representations of pixel values and transform coefficients. Refer to [4] for details on obtaining the transform matrix  $T$ . Let  $\mathbf{a}_c$  and  $\hat{\mathbf{a}}_l$  be the vector representations of correctly received coefficients and the presumed estimate of damaged ones. Following (6), the reconstructed vector of  $\mathbf{f}$  is given by

$$\hat{\mathbf{f}} = T_c \mathbf{a}_c + T_l \hat{\mathbf{a}}_l \quad (7)$$

where  $T_c$  and  $T_l$  are matrices composed of the columns of  $T$  corresponding to  $\mathbf{a}_c$  and  $\hat{\mathbf{a}}_l$ . We use the directional smoothness measure as the object function to recover  $\hat{\mathbf{a}}_l$ . The reconstruction problem is to find the best estimate of damaged coefficients  $\hat{\mathbf{a}}_l$  such that the recovered image  $\hat{\mathbf{f}}$  minimizes the directional smoothness measure  $\Psi(\hat{\mathbf{f}})$  over the damaged block as defined in (4). To find the solution, we first rewrite  $\Psi(\hat{\mathbf{f}})$  in matrix form. Note if we use  $\hat{\mathbf{f}}_{\mathbf{xx}}$  to denote the vector representation of the partial derivatives  $\{f_{xx}(i, j)\}$  arranged in the same order as in  $\hat{\mathbf{f}}$ , then from (5) we have the linear form

$$\hat{\mathbf{f}}_{\mathbf{xx}} = A_1 \hat{\mathbf{f}} + b_1 \quad (8)$$

$A_1$  is the matrix for the second order differential operation in the horizontal direction and  $b_1$  is a vector composed of zeros and the boundary pixel values used to compute the derivatives inside the damaged block. Similarly we define vectors  $\hat{\mathbf{f}}_{\mathbf{yy}}$  and  $\hat{\mathbf{f}}_{\mathbf{xy}}$  for  $\{f_{yy}(i, j)\}$  and  $\{f_{xy}(i, j)\}$ , and have

$$\hat{\mathbf{f}}_{\mathbf{yy}} = A_2 \hat{\mathbf{f}} + b_2, \quad \hat{\mathbf{f}}_{\mathbf{xy}} = A_3 \hat{\mathbf{f}} + b_3 \quad (9)$$

where  $A_i$ 's are matrices for differential operations and  $b_i$ 's are vectors composed of boundary pixels involved in the differentiation. Let  $W_{1,2}$  be diagonal matrices with  $w_{i,j}^{1,2}$  being the diagonal elements, the smoothness measure can

then be written in matrix form

$$\begin{aligned}
\Psi(\hat{\mathbf{f}}) &= (A_1\hat{\mathbf{f}} + b_1)'W_1(A_1\hat{\mathbf{f}} + b_1) + (A_2\hat{\mathbf{f}} + b_2)'(I - W_1) \\
&\quad \cdot (A_2\hat{\mathbf{f}} + b_2) - 2(A_3\hat{\mathbf{f}} + b_3)'W_2((A_1 + A_2)\hat{\mathbf{f}} + (b_1 + b_2)) \\
&= \hat{\mathbf{f}}'A\hat{\mathbf{f}} - b'\hat{\mathbf{f}} + c \\
A &= A'_1W_1A_1 + A'_2(I - W_1)A_2 - 2A'_3W_2(A_1 + A_2) \\
b &= -2A'_1W_1b_1 - 2A'_2(I - W_1)b_2 + 2(A'_1 + A'_2)W_2b_3 \\
&\quad + 2A'_3W_2(b_1 + b_2) \\
c &= b'_1W_1b_1 + b'_2(I - W_1)b_2 - 2b'_3W_2(b_1 + b_2)
\end{aligned} \tag{10}$$

From (7) and (10),  $\Psi$  is a quadratic function of  $\hat{\mathbf{a}}_l$ , and the optimal estimate is given by

$$\begin{aligned}
\frac{\partial\Psi}{\partial\hat{\mathbf{a}}_l} &= (T'_l(A + A')T_l)\hat{\mathbf{a}}_l + T'_l(A + A')T_c\mathbf{a}_c - T'_lb = 0 \\
\hat{\mathbf{a}}_l &= (T'_l(A + A')T_l)^{-1}T'_l(b - (A + A')T_c\mathbf{a}_c)
\end{aligned} \tag{11}$$

In the above discussion, we assume that the weighting matrices  $W_{1,2}$  are known and the surrounding blocks are uncorrupted. To obtain  $W_{1,2}$ , we notice the definition in (4) indicates that the weighting functions  $w_{i,j}^{1,2}$  are slowly varying functions in the vicinity of edges which is also a consequence of structural consistency present in natural images. Low order polynomial interpolation can be used to interpolate the weighting for the damaged block. We first find the polynomials  $P_{1,2}(i, j)$  that best fit  $w_{i,j}^{1,2}$  in surrounding areas  $\Omega_N$  by solving the following least square problem,

$$\min \sum_{(i,j) \in \Omega_N} g(\|\nabla f_{i,j}\|)(P_{1,2}(i, j) - w_{i,j}^{1,2})^2 \tag{12}$$

$P_{1,2}(i, j)$  are then used to interpolate the weighting  $w_{i,j}^{1,2}$  inside the missing block.  $g(x)$  is an increasing function of  $x$  and we set  $g(x) = x^2$  in the experiment. This means that the weighting functions are determined mostly by the information from the vicinity of edges. The preference for edge information does not affect the reconstruction in smooth areas because smooth areas can be recovered from any direction. Through polynomial interpolation, the weighting terms  $w^{1,2}$  with the underlying edge orientation  $\theta$  are recovered as functions slowly varying over the damaged block and smoothly extending across block boundaries. Thus, we resolve the situation where edge orientation in the damaged block is different from the neighbors. Our experiment shows that when block size is small, zero-th order interpolation is sufficient for the reconstruction. In this case, constant weights associated with a dominant edge direction are interpolated as in (13) and used for the entire damaged block.

$$w^{1,2} = \operatorname{argmin} \sum_{(i,j) \in \Omega_N} g(\|\nabla f_{i,j}\|)(w^{1,2} - w_{i,j}^{1,2})^2 \tag{13}$$

Subsequently, the expression in (10) can be further simplified with  $W_{1,2} = w^{1,2} \cdot I$ . Furthermore, if we quantize the

**Table 1.** PSNR(dB) of reconstruction using zeros substitution (ZS), smoothness measure(SM), directional smoothness measure(DSM) and its fast implementation (FDSM).

Type 1	None	DC	First 5AC	All AC	All
ZS	32.91	19.82	26.61	25.66	18.98
SM	32.91	32.00	31.23	28.76	27.20
DSM	32.91	32.26	31.74	29.78	28.71
FDSM	32.91	32.00	31.42	29.47	28.34

  

Type 2	None	DC	First 5AC	All AC	All
ZS	32.91	19.75	26.84	25.61	18.91
SM	32.91	31.45	30.41	27.73	26.62
DSM	32.91	32.22	31.53	29.06	28.12
FDSM	32.91	31.92	31.11	28.83	27.82

constant weights  $w^{1,2}$  to a set of pre-determined values and use the quantized weights to evaluate  $\hat{\mathbf{a}}_l$  (11), then matrix computation in (11) can be converted off-line by computing and storing a set of matrices for different quantized weights. On-line concealment only needs to estimate the weights and choose the corresponding reconstruction matrices.

When some of the boundary values are not available, we set them to zero for the initial estimates of the lost coefficients. Damaged blocks are iteratively reconstructed using previously recovered boundary values until the reconstructed values converge.

#### 4. EXPERIMENTAL RESULTS

We tested the proposed algorithm on the  $256 \times 256$  grayscale Lena image. Each  $8 \times 8$  block undergoes a DCT and the DCT coefficients are quantized. Figure 1 shows the corrupted image with all coefficients lost in the damaged blocks and two reconstruction results, one based on the smoothness measure in [4] and the other based on the proposed directional smoothness measure. Two loss patterns are tested. The first type of loss, shown in column (a), simulates the situation where spatial interleaving scheme is adopted for packetization, while the second type, shown in column (b), simulates the situation with no spatial interleaving. The damaged blocks spreading over the entire image contain a variety of local structures. Four outer layers of surrounding pixels are used in a zero-th order polynomial interpolation to obtain the weighting terms for a damaged block. When concealing consecutive block loss, the proposed method converges much faster than [4]. Encouraging improvement in visual quality for both types of loss is observed in figure 1. Local structures are smoothly and faithfully recovered by the proposed scheme with much reduced artifacts.

The directional smoothness measure-based concealment also improves the peak signal-to-noise ratio (PSNR) quantitatively. Table 1 lists PSNR of the reconstruction results



**Fig. 1.** Reconstruction results. (a) Type 1 loss. (b) Type 2 loss. First row: damaged image; second row: reconstruction with maximum smoothness measure; third row: reconstruction with directional smoothness measure.

using zero substitution, the smoothness measure (SM) in [4], the proposed directional smoothness measure (DSM) as well as its fast implementation (FDSM) with weights quantized into 16 levels. For both loss patterns, we simulate the following situations, no coefficient is lost, only DC coefficients are lost, the first 5 AC coefficients are lost, all AC coefficients are lost and the entire set of DCT coefficients are lost. In the most severe situation where all DCT coefficients are lost, the proposed method has 1.5dB PSNR improvement. The comparison between SM-based concealment and other concealment schemes is discussed in [4].

## 5. CONCLUSIONS

We demonstrate the importance of keeping structural consistency for faithful error concealment through spatial cor-

relation. A directional smoothness measure is derived to recover lost and damaged DCT coefficients and ensures consistent structure reconstruction. Various edge profiles are consistently extended to damaged blocks from uncorrupted surrounding areas guided by adaptive weighting functions. The proposed non-iterative concealment scheme takes advantage of any correctly received coefficients and is suitable for various packetization designs with different spatial and frequency interleaving schemes [2]. A fast implementation is also proposed. Compared to the conventional smoothness-based schemes, the proposed concealment method demonstrates prominent improvement in the subjective image quality as well as in the reconstruction PSNR.

## 6. REFERENCES

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