# IMPULSE NOISE CANCELLATION IN MULTICARRIER TRANSMISSION

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**ABSTRACT**

A parallel between Reed Solomon codes in the complex field and multicarrier transmission using OFDM is first presented. This shows that when the signal is sent over some channel composed of Gaussian plus impulse noise, the impulse noise can be removed by a procedure similar to channel decoding, using information carried by the "syndrome". These results are first derived in a simple situation (oversampled DMT, additive channel), which is merely of theoretical interest. Several extensions are then provided in order to increase the practical usefulness of the method. Simulations combining classical convolutive codes with the above mentioned approach are provided.

## 1. INTRODUCTION

The main idea behind OFDM is to split the transmitted data sequence into $N$ parallel symbol sequences. This structure allows the use of a very simple equalization scheme when the signal is sent over multipath propagation channels. In fact, intersymbol interference (ISI) can be avoided when a guard interval (IG) is implemented between each block of time domain samples to be transmitted. However, some carriers can be strongly attenuated, then it is necessary to incorporate a powerful channel coder combined with frequency and time interleaving. In this way, close coded bits are not likely to fall simultaneously in a spectral null. Therefore, the coded orthogonal frequency division multiplex (COFDM) technique has become extremely popular in many applications, such as broadcasting, ADSL modems, Local area Networks (HiperLAN2). However, in some of these applications, it is well known that channel noise is not only made from measurement (Gaussian) noise, but also encompasses some large bursts of errors.

In this case, we propose to use the OFDM modulator as some specific impulse noise canceller, the structure of which is well suited to the nature of the problem (i.e. a single impulse shows up as a single error), rather than counting on the classical channel coder to solve the problem. Practically, of course, both type of codes will have to cooperate, in order to process both Gaussian and impulse noise.

Note that the proposed approach makes use of techniques that are similar to previous papers by Wolf [1] and Redinbo [2] . The contributions of this paper are : (i) RS decoding in the complex field is easily applied in OFDM system, (ii) they can be extended in the sense that the pilot tones can be seen as additional syndromes, (iii) the method still holds when ISI is present, (iv) a combination of classical and complex codes is efficient under the presence of Gaussian plus impulse noise.

## 2. TRANSMISSION SCHEME AND CONNECTION WITH SPECTRAL CODES

### 2.1. Discrete model of OFDM system

A binary message is coded and mapped to a sequence of complex data stream $\{I_k(n)\}$ which belong to a given constellation. The OFDM system splits the initial data stream (to be transmitted at rate $T_s$) into $N$ substreams, each one being transmitted over its own carrier. All symbols emitted during the same duration $NT_s$ constitute an OFDM symbol $I(n) = (I_0(n) \dots I_N(n))^T$[3]. The orthogonality property between carriers ensures the perfect reconstruction of the emitted symbols at the receiver.

A discrete model of the OFDM system is easily obtained by computing $M$ samples of the signal to be sent onto the channel during one OFDM symbol. i.e. $MT_e = NT_s$, $T_e \leq T_s$ ($T_e$ the sampling period). Moreover, if one considers the simple multicarrier system where the prototype filter is a rectangular pulse of duration $NT_s$, modulated with spacing between carriers equal to $1/NT_s$, these samples are computed as :

$$c_k(n) = \sum_{m=0}^{N-1} I_m(n-1) e^{\frac{2j\pi km}{M}}$$

which is exactly the inverse discrete Fourier Transform (IDFT) of the $\{I_m(n-1)\}$ sequence enlarged by $(M-N)$ zeroes. In the following, we assume that $M - N = 2t$, $t$ a positive integer.

At the receiver the Analog to Digital Converter (ADC) samples the signal $r(n)$, at rate $T_e$ and a DFT is performed. Therefore, the received signal is converted into the frequency domain $\{Y_k\}$, where $Y_k$ is given by the following equation:

$$Y_k = I_k + N_k \ , 0 \leq k \leq N-1$$

where $N_k$ is the length $M$ Fourier transform of the noise sequence $\{n_k\}$ (see Fig.1)

### 2.2. Channel model and capacity

First assuming a memoryless channel, each emitted sample is modified by the channel according to

$$r_k = c_k + b_k + w_k, \ k \in \{0 \dots M-1\}$$

where $w_k$ is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_w^2$ and $b_k$ is the impulse noise.

The impulse noise is an additive disturbance that arises primarly from the switching electric equipment [4]. In the following, the impulse noise is modeled as in [5] as:

$$b_k = e_k g_k \ \forall k \in \{0 \dots M-1\}$$

where $e_k$ stands for a Bernoulli process, an i.i.d. sequence of zeroes and ones with $pr(e_k = 1) = p$, and $g_k$ is a complex white Gaussian noise with zero mean and variance $\sigma_b^2$ such as $\sigma_b^2 \gg \sigma_w^2$. Note that this model assumes the presence of a large interleaver, so that bursts of errors can be scattered along time, resulting in independent noise sequences.

Under this model, the probability density of the channel noise $n_k = b_k + w_k$ can be expressed as

$$p(n) = (1-p)G(n, 0, \sigma_w^2) + pG(n, 0, (\sigma_w^2 + \sigma_b^2))$$

where $G(n, m_x, \sigma_x)$ is the Gaussian density with mean $m_x$ and variance $\sigma_x^2$.

This expression allows to compute the capacity of this channel, in order to estimate the impact of a given impulse noise on the capacity of a Gaussian channel. Practically, this capacity has been computed by an iterative procedure proposed by Blahut and Arimoto [6] applicable to arbitrary discrete memoryless channels.

Fig.2 depicts the capacity of the "Gaussian plus Bernoulli Gaussian" channel in bits per second normalized by the bandwidth of the channel (W), as a function of P for several values of $p$, $\sigma_b = 1$, $\sigma_w = 6.10^{-2}$. We note that, even for somewhat large values of $p$, the capacity of the channel is approximately similar to that of the AWGN channel. For example, if $p = 10^{-2}$, and P=1, then the capacity of the "Gaussian plus Bernoulli plus Bernoulli Gaussian" channel is $4bit/s/Hz$, which is approximately the same value as for the AWGN channel. If $p = 5.10^{-2}$ then we transmit at most 3.3 bits/s/Hz that means that we lost only 0.7 bit per second/Hz, this decrease of capacity being due to the impulse error. However, if no specific procedure is used in an OFDM system, it is unlikely that such similar performances can be obtained : consider the case of a 64 QAM constellation emitted over 64 subbands. Each impulse drastically impairs 384 bits at a time, and it can be stated that the OFDM demodulator acts as an impulsive noise amplifier... This is clearly in favor of a processing taking into account the specific nature of the impulsive noise and the OFDM system.

### 2.3. Spectral codes

We have seen above that implementing an OFDM modulation is similar to adding consecutive null symbols at the input of the block to be modulated. Since the zeroes emitted through a "Gaussian plus Bernoulli Gaussian" channel are not recovered after demodulation, a question arises: to have performance similar to the ones of AWGN channel, is it possible to remove the impulse error with the sole knowledge that some of the demodulator input should be null?

The similarity between OFDM modulator and RS codes can be used at that point, following the work by Blahut. It has been shown in [7], that the ideas of spectral coding theory can be translated in the frequency domain, i.e. over the complex field $C$. Reed Solomon codes can be defined [7] as follow:

**Definition 1** *Let F contain an element of order M. The (M,M-2t) Reed Solomon block length M with symbols in F is the set of all vectors c whose spectrum (in F) satisfies:* $C_k = 0 \; \forall k \in \mathcal{A}$ *where* $\mathcal{A} = \{k_0+1 \ldots k_0+2t\}$. *This is described briefly as an (M,M-2t) Reed Solomon code over F.*

The spectrum of a Reed Solomon codeword lives in the same field as the code word. Then, to form a Reed Solomon code, a block of $(2t)$ consecutive spectral components are chosen as parity frequencies, (to be set to zero) and the remaining are information symbols. Marshall [8] has shown that conventional decoding algorithm for finite field cyclic codes could be employed for real and complex numbers.

The basic remark that we have used in this work is that a discrete sequence of complex numbers containing $(2t)$ consecutive zeroes are transmitted over the OFDM system, therefore, the output of the OFDM modulator can be considered as a Reed Solomon codeword (their spectrum contains consecutive zeroes). After transmission over "Gaussian plus Bernoulli Gaussian" channel, the DFT of the received discrete time sequence no longer has $(2t)$ zeroes, and this is due only to the channel. Hence, the OFDM modulator can be seen as a complex-valued RS code, the correction capacity is given by :

**BCH Bound 1** *if (2t consecutive frequencies belong to* $\mathcal{A}$ *) then (*$d_{min} > 2t + 1$ *).*

where $\mathcal{A}$ is the set of the $(2t)$ zeroes.

However, strictly speaking, there are more than $(\frac{2t+1}{2})$ errors if one uses our channel model : all samples are polluted by noise. Therefore, we concentrate on the removal of the sole impulse noise, considering the Gaussian component as background noise. The classical decoding techniques have to be adapted to the presence of this background noise.

## 3. DECODING ALGORITHM

The procedure is as follows : choose a classical decoding algorithm, adapt it to the presence of the background noise, and correct the estimated errors. Redinbo [2] recently presented a decoding procedure for real number constructed in the discrete Fourier transform (DFT) domain. In our work, performed simultaneously in the context of joint source and channel coding [9], the basic algorithm was different, since we used a modified Peterson-Gorenstein-Zierler algorithm to locate and correct "impulse errors", based only on a syndrome evaluation (the $(2t)$ consecutive zeroes that one should observe at the output of the OFDM modulator in the absence of noise).

After transmission, the corresponding received components of $\{Y_k\}$ will no longer be null (Fig.1)

$$Y_k = B_k + \Gamma_k, \; \forall k \in \{N+1 \ldots M-1\}$$

where $B_k$ if the DFT of the impulse noise $b_n$, and $\Gamma_k$ that of the background noise $w_n$.

At the receiver, the correction of impulse noise must operate on the $(2t)$ syndromes $S_k$ which are given by: $S_k = Y_{N+k-1}, \; k = 1 \ldots 2t$.

There are two contributions in these terms : one is the Fourier transform of the Gaussian background noise, hence is still Gaussian, and the other one is a sum of Fourier transforms of impulses, hence is a sum of complex sinusoids, the frequencies of which correspond to the localization of the errors. The decoding problem is thus the estimation of the number of sinusoids, together with their frequencies and amplitudes, polluted by Gaussian noise. The two main differences with classical signal processing situations are (i) that the number of samples is orders of magnitude smaller than usual, (ii) that one has the knowledge that the frequencies take integer values.

The decoding algorithm works in three steps: (i) estimate the number $\nu$ of impulse errors (ii) seek the error locations and (iii) correct the errors. Classically, the procedure is finished at this step. We have added a control step, which is able to carefully estimate whether the decoding procedure has worked correctly. In this way, we are able to begin a truncated enumeration of all possible error localizations (the most sensitive part of the algorithm) among the most likely ones. . . This truncated enumeration is necessary because of the presence of the background noise which introduces some fuzziness in the computations.

## 4. EXTENSIONS

The procedure just described cannot be applied as such in OFDM system, since the $2t$ zeroes do not correspond to a part of the spectrum which is actually available (analog shaping filters are here to limit the bandwidth). Only a small number of these zeroes can be practically used. However, in many cases, pilot tones are emitted, for synchronization or channel estimation purposes. These pilot tones consist in known symbols that are emitted, scattered among the information ones.

We outline below that a procedure similar to that of the RS decoding can be used in this situation. This is easily understood by combining situations in which: (i) $(2t)$ consecutive symbols are known (and not null), (ii) the pilot symbols are uniformly distributed, and (iii) when the pilot symbols are uniformly distributed and a channel $C$ is considered in addition to the "Gaussian plus Bernoulli Gaussian" channel .

Two extensions are trivial , and will not be detailed due to lack of space :

- if the emitted symbols are known (rather than zero), the extension consists in subtracting the known value. The rest of the algorithm remains unchanged.
- if the OFDM system goes through a channel with ISI, one uses the classical cyclic prefix procedure, which transforms the ISI channel to a set of parallel multiplicative constants. If this channel is known (which is assumed), divide by the correct constant, and the algorithm explained above applies with minimal modifications.

The only point which is more tricky is the extension when the pilot tones are scattered among the symbols. A special case when the pilot tones are regularly spaced can be deduced from the Hartmann-Tzeng theorem [10] :

**Theorem 1** *Suppose that the field F contains an element of order* $M$ *and locate the syndromes in* $K$ *blocks of size* $d-K$*. Then the error correction capacity of the code is upper bounded by* $\frac{d-1}{2}$ *if* $pgcd(M,K)=1$.

This theorem is easily used in a special case, when $gcd(M,K)=1$ and blocks have size 1. So no loss in error correction capacity occurs because $d=2t+1$ then correction capacity is $t$. Therefore, decoding can be performed in the same way as already explained.

## 5. SIMULATIONS

Due to short space, the simulations concentrate on the efficiency of the impulse noise cancellation. The BER curves, and the combination with classical coders will be presented in greater details in a forthcoming paper.

A first simulation is concerned with the plain, initial algorithm using only the consecutive null carriers, and the straightforward analogy between OFDM systems and BCH codes. The total number of carriers is 65, the number of zeroes is 12, the probability of impulsive noise $p=5.10^{-4}$, $\sigma_b=4$ and 4QAM symbols are emitted. One can observe on Fig.3, where we plot 1/EQM (dB), that the RS code in the complex domain has met the expectations, since after decoding, the EQM between the emitted and received symbols closely follows the curve containing the Gaussian noise only.

The second simulation is reminiscent of the HiperLan2 standard, although we do not claim at present any practical usefulness in this context. The number of carriers is $M=64$ and the guard interval has length $\mathcal{D}=16$ samples.This second curve also plots the 1/EQM (dB), but in a situation containing a mixture of all extensions we have developed : Among the $M$ carriers, $12$ carriers are null-carriers. Among the remaining, $52$, $4$ are fixed pilots carrying known 4QAM symbols while $48$ subcarriers, convey the information. The zeroes and the pilot symbols are uniformly distributed. Low-level Gaussian noise samples with variance $\sigma_w^2$ are added to each position independently, modeling the background noise. We also included a channel C, which is a realization of the typical channel Model A specified by Hiperlan2. For this simulation, the parameter of the Bernoulli sequence is $p=10^{-3}$ and the variance of the impulse noise $\sigma_b=2$.

The algorithm also shows good behavior under these circumstances, since curve after correction of the impulse noise is only marginally different from the curve obtained with Gaussian noise only (see Fig.4).

Fig.5, shows the performances in terms of BER, under the same conditions as those explained for Fig. 4. The improvement in terms of EQM clearly also shows in terms of BER. Note that this simulation was not containing any classical channel coder. The question which remains to be answered concerns the amount of redundancy which has to be assigned to the RS code in the complex field (if the inherent one using the pilot tones is not sufficient) compared to that which is devoted to the classical convolutive code.

## 6. CONCLUSION

In this paper we have described a procedure for removing impulse noise in OFDM system. Implementing a digital OFDM modulator often requires working with an oversampled version of the emitted analog signal, this is functionally similar to add null consecutive symbols to the block to be transmitted. The impulse error-correcting procedure is based on the relationship between Fourier transform and Reed Solomon codes defined over the field of complex numbers. A suitably modified Peterson-Gorenstein-Zierler was examined as an alternative for determining impulse error location. This procedure can also be applied when pilot symbol are uniformly distributed in the output of the OFDM modulator. Many extensions are under consideration, in order to increase the practical usefulness of this approach.


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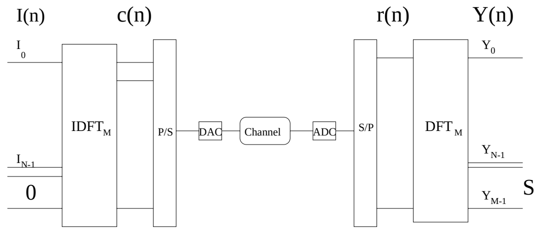


**Fig. 1**. OFDM system

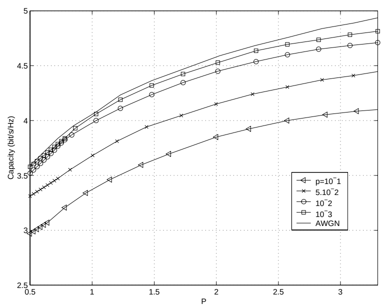


**Fig. 2**. The "Gaussian plus Bernoulli Gaussian" channel capacity

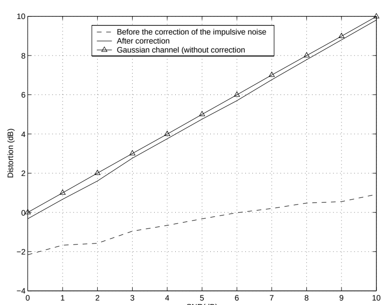


**Fig. 3**. Distortion performance when we consider a "Gaussian plus Bernoulli Gaussian" channel, and consecutive syndrome locations

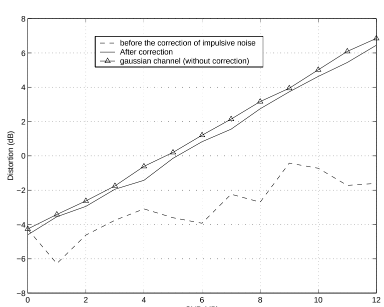


**Fig. 4**. Distortion performance when we consider a channel C, scattered null carriers and pilots tones

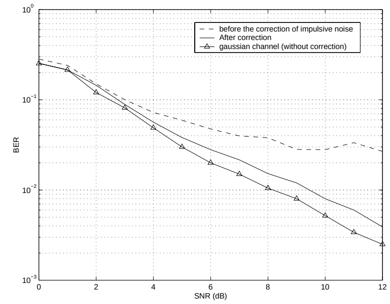


**Fig. 5**. BER performance when we consider a channel C, scattered null carriers and pilots tones

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