IMPLEMENTATION OF PARALLEL COSINE AND SINE MODULATED FILTER BANKS FOR EQUALIZED TRANSMULTIPLEXER SYSTEMS

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ABSTRACT

Filter bank based transmultiplexer systems have certain advantages compared with existing DFT-based multicarrier systems and they are promising candidates for data transmission in frequency selective channels. We have recently proposed a novel and efficient channel equalization idea to be used with critically decimated perfect reconstruction cosine modulated transmultiplexer systems. The equalizer utilizes parallel cosine and sine modulated filter banks in the receiver end. This paper explores efficient realization structures for the needed parallel filter bank system, which finds applications also in other areas.

1. INTRODUCTION

Multicarrier modulation is an excellent method for efficient channel utilization in applications like very high-speed digital subscriber line (VDSL) modems and powerline communication systems. A filter bank based transmultiplexer (TMUX) system can be considered as a multicarrier system. The TMUX configuration changes the order of the analysis and synthesis filters, if compared with a typical subband signal coding system [1].

Cosine modulated filter bank (CMFB) based TMUX should be considered as a serious candidate when choosing future line coding methods. The filter bank design is flexible; we can adjust the selectivity and thus tolerate stronger narrowband interferences than existing DFT-based systems [2], [3]. The basic idea of CMFB is that subchannel filters with real coefficients can be derived form a single lowpass prototype filter by using cosine modulation [4]–[7]. Critically decimated perfect reconstruction (PR) CMFBs have efficient VLSI implementations based on the structure of the fast extended lapped transform (ELT) [4], [5] or lattice structure [6], [7].

In our previous papers, we have considered PR cosine modulated TMUXs for VDSL modems [8] and we have studied equalization issues for filter bank based multicarrier systems [9]. In [10], we have introduce a new equalization idea to be used with critically decimated PR cosine modulated TMUX systems. At the receiver, we need a new element: sine modulated filter bank (SMFB). Therefore, we need an efficient implementation for CMFB and SMFB to be used in parallel in the equalizer. Now, our aim is to find a structure where we have two separate block transforms (DCT-IV/DST-IV), whereas the prototype filter section should be combined as in Fig. 1. Similar ideas for modulated complex lapped transform, which can be used in audio processing applications, are considered in [11].

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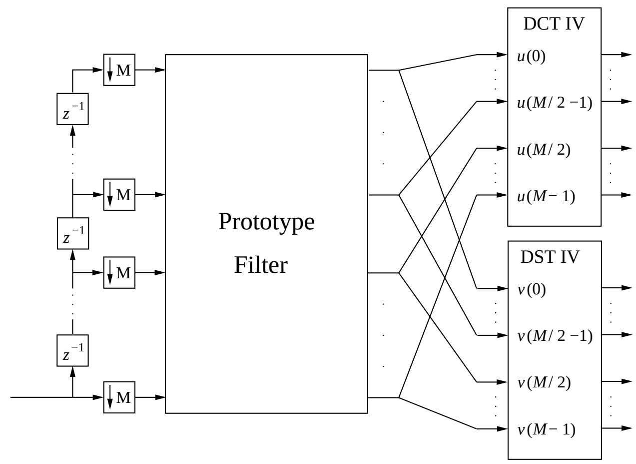

Fig. 1. General analysis filter bank structure with parallel CMFB and SMFB.

The rest of the paper is organized as follows: Section 2 introduces our equalizer for cosine modulated TMUXs. The relationship between CMFBs and SMFBs is presented in Section 3. An efficient implementation based on ELT type of structure is described in Section 4. In Section 5, lattice and direct form polyphase implementation structures are briefly studied.

2. A NEW EQUALIZATION IDEA FOR THE COSINE MODULATED TMUX SYSTEMS

Figure 2 shows the proposed cosine modulated TMUX system referred to as adaptive SMFB/CMFB equalizer for transmultiplexers (ASCET) [10], [12]. Because the channel is unknown or time-varying, we need an adaptive system to equalize the received signal. Here, we have included a SMFB at the receiver to make the adaptive structure simpler. In channels with flat frequency response within the subchannel bandwidth even two coefficients per subchannel are sufficient to equalize the system. Few independent coefficients for subchannel equalization leads also to fast converging equalizers. These are clear advantages over the alternative approach, multiple-input multiple-output (MIMO) combiners [3], [13].

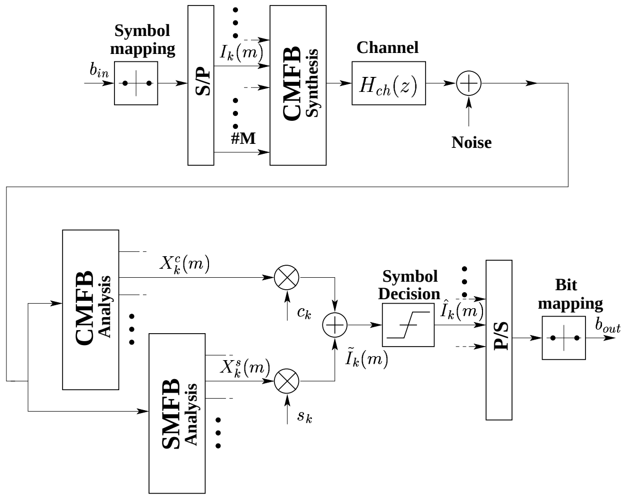

Fig. 2. Zero-order ASCET structure. Interference terms are reduced in the combined subchannel signal.

Interested reader can find more detailed information about ASCET in [10], [12], where we also consider methods to improve the performance if two coefficients per subchannel are not enough. In addition, we have demonstrated how the ASCET equalizer can be used in passband channel communications, and still take the advantage of simple equalization.

3. COSINE MODULATED AND SINE MODULATED FILTER BANKS

The prototype filter, from which we can derive the cosine modulated subchannel filters, is a symmetric lowpass FIR filter. In this paper, the order of the of the prototype filter $h_p(n)$ is fixed to be $N = 2KM - 1$, where K is an integer (called as overlapping factor) and M is the number of channels. The analysis filters $h_k^c(n)$ and synthesis filters $f_k^c(n)$ are formed in the following way [7]:

$$h_k^c(n) = 2h_p(n)\cos\left((2k+1)\frac{\pi}{2M}(n - \frac{N}{2}) + (-1)^k\frac{\pi}{4}\right) \quad (1)$$

and

$$f_k^c(n) = 2h_p(n)\cos\left((2k+1)\frac{\pi}{2M}(n - \frac{N}{2}) - (-1)^k\frac{\pi}{4}\right), \quad (2)$$

where $k = 0, \ldots, M-1$ and $n = 0, \ldots, N$. It follows from the above equations that the synthesis filters are time-reversed versions of the analysis filters, i.e., $f_k^c(n) = h_k^c(N-n)$.

The subchannel filters for the SMFB are obtained when we replace the cosine term in Eqs. (1) and (2) with sine term. Furthermore, the subchannel filters in CMFBs and SMFBs are related as follows

$$h_k^s(k,n) = (-1)^k h_k^c(k, N-n) = (-1)^k f_k^c(k,n). \quad (3)$$

The modulation blocks used in the CMFB and SMFB implementations are DCT-IV and DST-IV transforms, respectively [11]. DCT-IV and DST-IV are $M \times M$ block transforms, which are defined as follows [14]:

$$[\boldsymbol{C}]_{kn} = \sqrt{\frac{2}{M}}\cos\left((n+\frac{1}{2})(k+\frac{1}{2})\frac{\pi}{M}\right) \quad (4)$$

and

$$[\boldsymbol{S}]_{kn} = \sqrt{\frac{2}{M}}\sin\left((n+\frac{1}{2})(k+\frac{1}{2})\frac{\pi}{M}\right). \quad (5)$$

They are related as

$$\boldsymbol{S} = \boldsymbol{I}_{\pm}\boldsymbol{C}\boldsymbol{J}, \quad (6)$$

where $\boldsymbol{I}_{\pm} = \mathrm{diag}(1, -1, 1, -1, \ldots, -1)$ and $\boldsymbol{J}$ is reversal (anti-diagonal) matrix.

4. IMPLEMENTATION OF CMFB AND SMFB BY USING ELT TYPE OF STRUCTURES

We have been mainly interested in the ELT type of realizations, because they have very regular structure which leads to the fast implementation for any K [4], [5]. Moreover, the coefficient quantization can be done in a way that this kind of structure maintains its PR property [15]. As shown in Fig. 3, the prototype filter is implemented with cascaded orthogonal butterflies $\boldsymbol{D}_j$ and pure delays. These butterfly matrices have nonzero values only on their diagonals and anti-diagonals:

$$\boldsymbol{D}_j = \begin{bmatrix} -c_{0j} & & & & & s_{0j} \\ & \ddots & & & \iddots & \\ & & -c_{\frac{M}{2}-1,j} & s_{\frac{M}{2}-1,j} & & \\ & & s_{\frac{M}{2}-1,j} & c_{\frac{M}{2}-1,j} & & \\ & \iddots & & & \ddots & \\ s_{0j} & & & & & c_{0j} \end{bmatrix}, \quad (7)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for $i = 0, 1, \ldots, M/2 - 1$ and $j = 0, 1, \ldots, K-1$. The cosine modulation is implemented with DCT-IV transform.

Let us consider a case in which $K = 2$ and we have moved the decimation by M blocks after the DCT-IV. After that, we have to write z^{-sM} instead of z^{-s} for the delays between the butterfly stages. Now we can easily determine the relation between the M-sample sequence $u(i)$, which comes to DCT-IV, and the M-sample input sequence $x(i)$ for $i = 0, 1, \ldots, M-1$. When we feed one impulse to the ELT structure, we can write $u(i)$ in the following way [4]:

$$\begin{aligned} u(M/2+i) \; = \; & \Delta_{3M}\Big(c_{i0}c_{i1}x(i) - c_{i0}s_{i1}x(M-1-i)\Big) \\ & + \Delta_M\Big(s_{i0}s_{i1}x(i) + s_{i0}c_{i1}x(M-1-i)\Big) \end{aligned}$$

and

$$\begin{aligned} u(M/2-1-i) \; = \; & \Delta_{2M}\Big(-s_{i0}c_{i1}x(i) + s_{i0}s_{i1}x(M-1-i)\Big) \\ & + \Big(c_{i0}s_{i1}x(i) + c_{i0}c_{i1}x(M-1-i)\Big), \end{aligned}$$

where Δ_M is the M-sample delay operator. The impulse response of the prototype filter can be found from the $u(i)$ sequence. Now

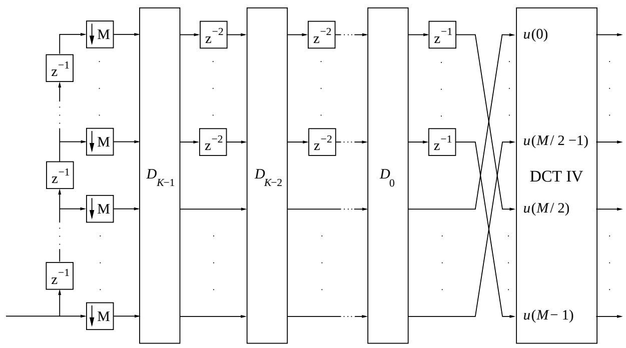

Fig. 3. Fast ELT structure (analysis filter bank).

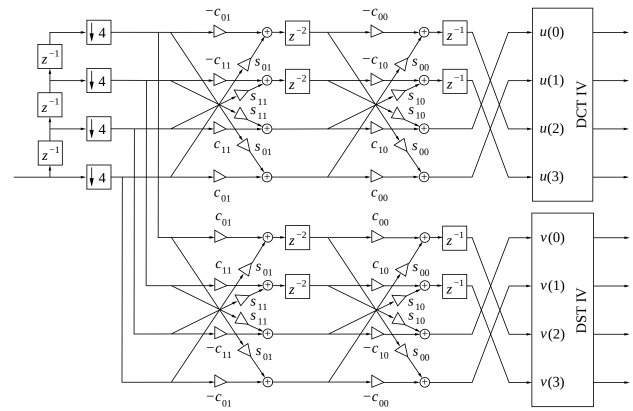

Fig. 4. ELT type of structure with parallel CMFB and SMFB ($M = 4$ and $K = 2$).

we notice that the prototype filter which can be obtained from the ELT structure is not perfectly symmetric. All the coefficients are correct but some signs do not match. This is because the ELT structure is so far optimized to make use of $M \times M$ DCT-IV.

At first glance, one could expect that the butterflies in the CMFB could be directly adopted for SMFB butterflies. Unfortunately, this does not work directly. We know by observing Eq. (3) that the impulse response of the prototype filter and modulation function has to be reversed. Moreover, every other subchannel filter have to be multiplied by -1. The reversed prototype filter can be obtained by using inverse ELT structure and other requirements are fulfilled by using DST-IV in place of DCT-IV. Now the M-sample sequence $v(i)$ which comes to DST-IV can be written as:

$$v(M/2+i) \;=\; \Delta_{3M}\Big(c_{i0}c_{i1}x(i) + c_{i0}s_{i1}x(M-1-i)\Big) \\ +\Delta_M\Big(s_{i0}s_{i1}x(i) - s_{i0}c_{i1}x(M-1-i)\Big)$$

and

$$v(M/2-1-i) \;=\; \Delta_{2M}\Big(+s_{i0}c_{i1}x(i) + s_{i0}s_{i1}x(M-1-i)\Big) \\ +\Big(-c_{i0}s_{i1}x(i) + c_{i0}c_{i1}x(M-1-i)\Big).$$

So, the fast ELT structure can be converted to SMFB structure by changing the DCT-IV transform to the DST-IV transform. Additionally, it is necessary to change the position of the minus signs in the butterflies. The analysis structure with parallel CMFB and SMFB is presented in Fig. 4. In order to simplify the figures, we have considered only 4-channel structures. The number of multiplications is roughly twice as high as in the fast ELT structure. Only the multiplications of the first butterfly stage can be shared. However, the complexity can be further reduced by scaling the butterflies [4], [5]. After that, the butterfly matrices $\boldsymbol{D}_1$ to $\boldsymbol{D}_{K-1}$ have only 1 and -1 elements in the diagonal. In order to compensate the modifications, the inverse scaling has to be applied to $\boldsymbol{D}_0$. In order to obtain CMFB and SMFB for $K > 2$, we can add more butterfly stages and z^{-2} delays to the structure [4], [5].

Is this really the best structure, if we consider implementation complexity, e.g., in terms of the number of multiplication, additions and delay elements? In the next Section, we study lattice and direct form polyphase structures for comparison. However, we want to keep the transform part fixed, so the same sequences $u(i)$ and $v(i)$ are used. Moreover, we use the same butterfly angles also in these structures.

5. LATTICE AND POLYPHASE STRUCTURES

When we write just the signs of the prototype filter coefficients obtained from the ELT structure in the case of $M = 4$ and $K = 2$, we verify that the prototype filter is not perfectly symmetric:

$h(0)$	$h(1)$	$h(2)$	$h(3)$	$h(4)$	$h(5)$	$h(6)$	$h(7)$
+	+	+	+	+	+	+	+
+	+	–	–	–	–	+	+
$h(15)$	$h(14)$	$h(13)$	$h(12)$	$h(11)$	$h(10)$	$h(9)$	$h(8)$

If we can just change the signs of the elements from $h(2)$ to $h(5)$ and from $h(10)$ to $h(13)$, we have obtained the reversed version of the prototype filter. The solution is to use polyphase representation with lattice structure as in Fig. 5. To one branch we place terms that have to be multiplied by -1 and to another branch terms that proceed without multiplication. Of course, we obtain the original impulse response when we take both branches without multiplication by -1.

When we add more same kind of butterfly stages and z^{-2} delays before the existing butterflies, we obtain filter banks for $K > 2$. It is worth noting, that we have to change the signs of the branches in the case of odd K for SMFB. Moreover, we can reduce the complexity by scaling the butterflies.

Another approach is to use direct form polyphase structure as in Fig. 6. For this structure we can define the impulse response coefficients as follows:

$$h(0) = c_{00}c_{01} \quad h(1) = c_{11}c_{10} \quad h(2) = c_{11}s_{10} \quad h(3) = s_{01}c_{00}$$
$$h(4) = c_{01}s_{00} \quad h(5) = c_{11}s_{10} \quad h(6) = s_{11}s_{10} \quad h(7) = s_{01}s_{00}$$

For $K > 2$, we have to compute new impulse response values and check which branches have to be multiplied by -1. However, one advantage of using the direct form polyphase structure is that we can also use this kind of structure to implement nearly perfect reconstruction (NPR) filter banks. NPR designs could result in lower implementation complexity than PR designs [16].

All these three structures, after using the complexity reduction tricks we have mentioned, have the same number of multipliers, adders, and delay elements. Therefore, we can select appropriate structure depending on how we are going to implement the system. One structure can be easier and more efficient to implement using VLSI and another using signal processors.

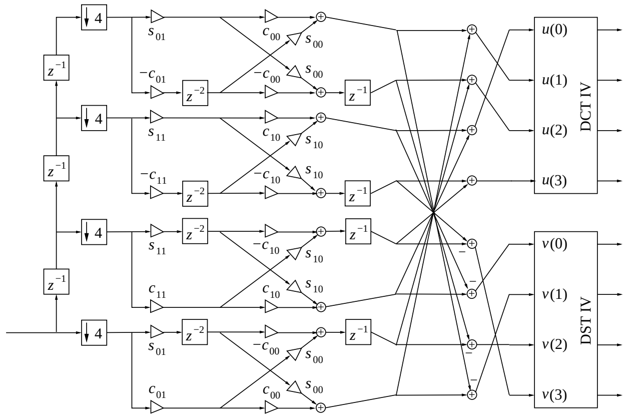

Fig. 5. Lattice structure ($M = 4$ and $K = 2$).

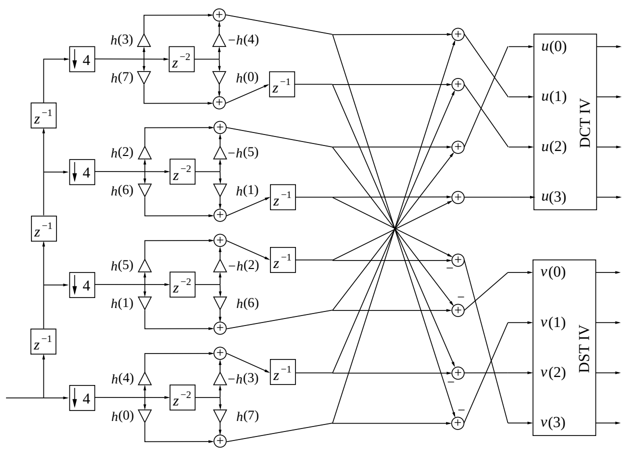

Fig. 6. Direct form polyphase structure ($M = 4$ and $K = 2$).

6. CONCLUSION

We have introduced a new equalization idea to be used with cosine modulated TMUX systems. In this structure, we need both the CMFB and SMFB in the receiver. In this paper, we have studied how to efficiently implement these filter banks. Our aim has been to find a structure where we have common prototype filter and separate transforms for cosine modulated and sine modulated parts. The computational complexity for all three presented structures appear to be quite the same. Therefore, the approach where we use ELT type of structures seems to be very suitable for our equalizer purposes. However, the final selection of the structure depends on the chosen implementation technology.

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