INTERPOLATED 3-D DIGITAL WAVEGUIDE MESH WITH FREQUENCY WARPING

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ABSTRACT

An interpolated 3-D digital waveguide mesh algorithm is elaborated. We introduce an optimized technique that improves a formerly proposed interpolated 3-D mesh and renders the 3-D mesh more homogeneous in different directions. Frequency-warping techniques are used to shift the frequencies of the output signal of the mesh in order to cancel the effect of dispersion error. The extensions improve the accuracy of 3-D digital waveguide mesh simulations enough so that in the future it can be used for acoustical simulations needed in the design of listening rooms, for example.

1. INTRODUCTION

The 3-D digital waveguide mesh (WGM) algorithm was introduced in 1994 [1] as an extension to the formerly developed 2-D WGM algorithm [2]. The WGM approach is suitable for modeling acoustic wave propagation in restricted media, such as in musical instruments or in a room. As the 3-D WGM can be used for simulating wave propagation in a space, it turns out more important for practical applications than the 2-D WGM, which is mostly useful for physical modeling of drum membranes or other flexible vibrating surfaces. The 3-D WGM could be used as an alternative to the popular ray-tracing, image-source, FEM, and BEM methods in numerous practical tasks, which include the acoustical design of concert halls, churches, auditoria, listening rooms, movie theaters, cabins of various vehicles, or loudspeaker enclosures.

The basic version of the 3-D WGM suffers from error in wave travel speed, which depends on both direction and frequency [3]. This is called the direction-dependent dispersion error. It is the main reason why the WGM method could not have been used in many design tasks until now. To reduce the dispersion, an interpolation technique was incorporated in the 3-D WGM [4]. While the mesh was made more homogeneous in different directions, the frequency-dependence was not cured. A similar effect has been formerly observed in the interpolated 2-D WGM [5]. As a solution, a frequency-warping method is used in the 2-D case to cut down the remaining error [6]. Alternative 3-D mesh structures, such as a tetrahedral network [3, 7], have been shown to be successful in suppressing the dispersion problem, but at the expense of a complicated tessellation of space. We believe that the usefulness of the method relies on an effortless filling of space, and thus we prefer the rectangular mesh and aim at making it an accurate and reliable method for acoustic simulations.

The contributions of this paper are a new optimally interpolated 3-D mesh structure which is preferable to the former one, and frequency-warping methods that are optimized for the new interpolated 3-D WGM. This paper is organized as follows. In Section 2, we give a formulation of the 3-D WGM update rule as a finite difference scheme and present an error analysis. Section 3 discusses the interpolated 3-D WGM, and Section 4 focuses on the optimization of the interpolation coefficients. In Section 5, we apply the frequency-warping techniques to the optimally interpolated mesh and demonstrate how the error characteristics are improved. Section 6 presents results from a simulation of a rectangular space, which shows that a sufficient level of accuracy has been finally reached and that the method is ready for practical use.

2. 3-D DIGITAL WAVEGUIDE MESH

The digital waveguide mesh is based on digital waveguides [8]. In the original three-dimensional mesh there are digital waveguides in three orthogonal directions, and they are interconnected to each other. The final structure is a rectangular grid, in which each node has a neighbor at unit distance in six directions, namely up, down, left, right, front, and back. The wave propagation in such a structure is governed by the following difference equation

$$\begin{aligned} p(n+1,x,y,z) & \\ = \quad & \tfrac{1}{3}[p(n,x+1,y,z)+p(n,x-1,y,z) \\ & +p(n,x,y+1,z)+p(n,x,y-1,z) \\ & +p(n,x,y,z+1)+p(n,x,y,z-1)] \\ & -p(n-1,x,y,z) \end{aligned} \tag{1}$$

where $p(n,x,y,z)$ represents the sound pressure at time step n at position (z,y,z) [1]. This structure can be analyzed by Von Neuman analysis (see, e.g., [9]), in which a spatial Fourier transform is performed to the difference scheme. Formerly this same technique has been used for 2-D meshes [2]. This leads us to the dispersion factor, which is a function of three spatial frequencies ξ_x, ξ_y, and ξ_z. The dispersion factor for the 3-D WGM is

$$k(\xi_x,\xi_y,\xi_z) = \frac{c'(\xi_x,\xi_y,\xi_z)}{c} = \frac{\sqrt{3}}{2\pi\xi}\arctan\frac{\sqrt{4-b(\xi_x,\xi_y,\xi_z)^2}}{b(\xi_x,\xi_y,\xi_z)} \tag{2}$$

in which $b(\xi_x,\xi_y,\xi_z)$ is

$$b(\xi_x,\xi_y,\xi_z) = \tfrac{2}{3}(\cos\omega_1 cT + \cos\omega_2 cT + \cos\omega_3 cT) \tag{3}$$

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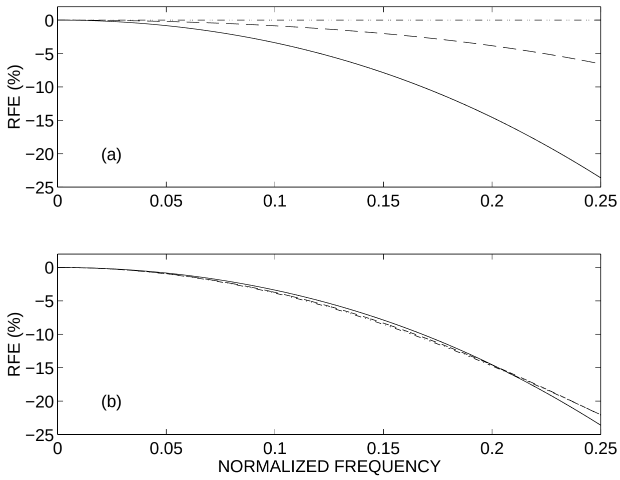

Fig. 1. The relative frequency error (RFE) in (a) the original 3-D WGM and (b) in the optimally interpolated WGM. The curves show the RFE in axial (solid line), 2-D diagonal (dashed line), and 3-D diagonal (dash-dotted line) directions up to the normalized frequency 0.25 which is the upper limit in the original mesh.

where $\omega_1 = 2\pi\xi_x$, $\omega_2 = 2\pi\xi_y$, and $\omega_3 = 2\pi\xi_z$.

Figure 1 shows the relative frequency error (RFE) in two 3-D WGM structures. The RFE is related to the dispersion factor by the following equation:

$$E(\xi_x, \xi_y, \xi_z) = \frac{k(\xi_x, \xi_y, \xi_z) - k_{dc}}{k_{dc}} \cdot 100\% \quad (4)$$

where $k_{dc} = \lim_{\xi_x,\xi_y,\xi_z \to 0} k(\xi_x, \xi_y, \xi_z)$ in this case equals to 1. In the original rectangular 3-D mesh the maximal RFE is 23.6% as can be seen from Fig. 1(a).

3. INTERPOLATED 3-D DIGITAL WAVEGUIDE MESH

In earlier studies, it was shown that by using interpolation it is possible to achieve wave propagation characteristics which are nearly independent of the wave propagation direction in the 2-D case [5, 6]. The same technique works also in 3-D WGM systems as shown in [4]. Similar results may be obtained with some other mesh geometries such as a tetrahedral one [7], but they are more laborious to construct than the simple rectangular configuration which is applicable to the interpolated mesh.

The basic structure for the interpolated 3-D WGM is illustrated in Fig. 2. In the original rectangular mesh each node has a connection to six neighbors (Fig. 2(a)), and that causes the direction dependent dispersion. In the interpolated mesh the number of neighbors is increased by adding delay-lines from a node to its diagonal neighbors. Finally, a node has connections of three separate type, 6 axial neighbors (Fig. 2(a)), 12 2-D diagonal neighbors (Fig. 2(b)), and 8 3-D diagonal neighbors (Fig. 2(c)), or altogether 26 neighbors as illustrated in Fig. 2(d).

The difference scheme for the interpolated 3-D WGM is [4]

$$\begin{gathered} p(n+1, x, y, z) = \\ \sum_{l=-1}^{1} \sum_{m=-1}^{1} \sum_{p=-1}^{1} h(l,m,p) p(n, x+l, y+m, z+p) \\ -p(n-1, x, y, z) \end{gathered} \quad (5)$$

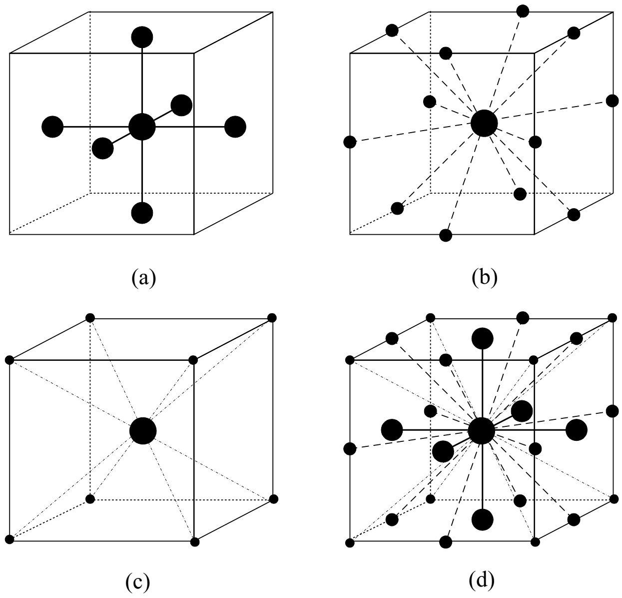

Fig. 2. (a) Six nearest neighbors in the original WGM, (b) 12 2-D diagonal neighbors, (c) 8 3-D diagonal neighbors, and (d) all neighbors in the interpolated WGM. The center node is indicated with the large dot in all cases.

where $h(l, m, p)$ are the weighting coefficients for different neighbor types. In the following the coefficients are denoted as follows

$$h(l,m,p) = \begin{cases} h_a, & \text{if } |l|+|m|+|p| = 1 \\ h_{2D}, & \text{if } |l|+|m|+|p| = 2 \\ h_{3D}, & \text{if } |l|+|m|+|p| = 3 \\ h_c, & \text{if } |l|+|m|+|p| = 0 \end{cases} \quad (6)$$

For the original rectangular mesh $h_a = \frac{1}{3}$ and $h_{2D} = h_{3D} = h_c = 0$.

For the dispersion analysis (2) is still valid, but $b(\xi_x, \xi_y, \xi_z)$ gets a new formulation.

$$\begin{gathered} b(\xi_x, \xi_y, \xi_z) = 2[h_a \sum_{i=1}^{i=3} \cos \omega_i cT \\ + h_{2D} \sum_{i=1}^{i=6} \cos \delta_i cT + h_{3D} \sum_{i=1}^{i=4} \cos \gamma_i cT + \tfrac{h_c}{2}] \end{gathered} \quad (7)$$

where δ_i correspond to the centers of all the edges of a unit cube, and γ_i are all the corners of the spatial frequency unit cube. The values for δ_i and γ_i are shown in Table 1.

Table 1. The values for spatial frequency coordinates δ_i and γ_i representing the 2-D diagonal and 3-D diagonal neighbors of a node.

$\delta_1 = \omega_1 + \omega_2$	$\gamma_1 = \omega_1 + \omega_2 + \omega_3$
$\delta_2 = \omega_1 + \omega_3$	$\gamma_2 = \omega_1 - \omega_2 + \omega_3$
$\delta_3 = \omega_2 + \omega_3$	$\gamma_3 = \omega_1 + \omega_2 - \omega_3$
$\delta_4 = \omega_1 - \omega_2$	$\gamma_4 = \omega_1 - \omega_2 - \omega_3$
$\delta_5 = \omega_1 - \omega_3$	
$\delta_6 = \omega_2 - \omega_3$	

Table 2. The optimized values for interpolation coefficients in the interpolated 3-D WGM.

h_a	h_{2D}	h_{3D}	h_c
0.124867	0.0387600	0.0133567	0.678827

4. OPTIMIZATION OF INTERPOLATION

Previously, trilinear interpolation was applied in the 3-D WGM [4], but here we show how to optimize the interpolation coefficients. In the interpolated three-dimensional WGM there are two constraints that the coefficient values must satisfy. First of all the stability criterion states that $b_{max} = 2$, that is

$$b_{max} = 2[3h_a + 6h_{2D} + 4h_{3D} + \frac{h_c}{2}] = 2 \tag{8}$$

Therefore the coefficient for the center node is

$$h_c = 2 - 6h_a - 12h_{2D} - 8h_{3D} \tag{9}$$

The second constraint comes from the dispersion factor, which should equal to 1 at the zero frequency, that is

$$k_{dc} = \lim_{\xi_x, \xi_y, \xi_z \to 0} k(\xi_x, \xi_y, \xi_z) = \sqrt{12h_{2D} + 12h_{3D} + 3h_a} \tag{10}$$

From that we can solve another coefficient. Let us choose h_{3D}.

$$h_{3D} = \frac{1}{12}(1 - 12h_{2D} - 3h_a) \tag{11}$$

The optimization of coefficients was performed such that the maximal and minimal error curves are as close to each other as possible by minimizing the area between the two curves. The resulting coefficients are presented in Table 2.

In the interpolated mesh there still remains dispersion which increases steadily as a function of frequency, as shown in Fig. 1(b). In this case the dispersion is nearly independent of the propagation direction which can be seen by comparing the RFE curves in three different directions in Fig. 1(b).

5. APPLYING FREQUENCY WARPING TO REDUCE THE DISPERSION ERROR

There are two different principles to apply the frequency warping. In our previous studies we have utilized warping in the time domain, and the results have been good in the 2-D case [10, 6]. In the case of the 3-D WGM more accurate results are obtained by warping in the frequency domain. In the following we show results for both techniques.

5.1. Frequency warping in the time domain

Since the error curves are smooth and nearly the same in all the directions, it is possible to apply a frequency warping to reduce the dispersion [6]. The warping is performed to the input signal of the mesh using a warped FIR filter [11]. A warped FIR filter is an FIR filter, in which each unit delay element has been replaced with a first-order allpass filter having the transfer function

$$A(z) = (z^{-1} + \lambda)/(1 + \lambda z^{-1}) \tag{12}$$

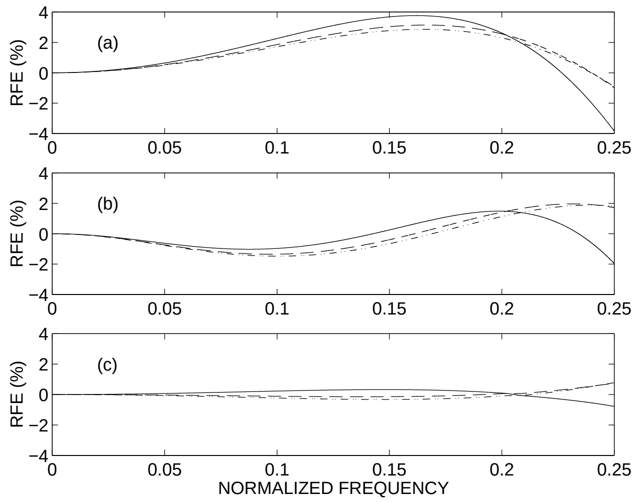

Fig. 3. Obtained RFE in three different directions after frequency warping in the optimally interpolated 3-D WGM in three different cases: (a) a single warping with $\lambda = -0.252902$, (b) a two-stage multiwarping ($\lambda_1 = 0.275389$, $\lambda_2 = -0.577291$, $D_1 = 2.48579$, $D_2 = 0.852843$), and (c) a frequency-domain warping.

There are various techniques to find an optimal value for the warping coefficent λ [6]. We decided to minimize the maximal error. By this method we obtained the value $\lambda = -0.252902$, and the resulting RFEs are presented in Fig. 3(a). The maximal error with this technique is 3.8% in the frequency range [0,0.25]. Please note that the warping requires a resampling operation by factor $D = (1 - \lambda)/(1 + \lambda)$ to compensate for the warping at low frequencies which is undesirable. In earlier studies [10, 6] the resampling has been performed after warping, but in this case a more accurate result was achieved when the resampling was done before warping.

The error can be still reduced by applying the multiwarping technique in which multiple signal resampling and frequency warping operations are cascaded [12, 13]. By using multiwarping which contains two warpings and two signal resamplings, a maximal error of 2.0% is achieved. The corresponding curves are given in Fig. 3(b).

5.2. Frequency warping in the frequency domain

Warping in the frequency domain is conducted by non-uniform resampling of the Fourier transformed signal [14], [15](see page 13). In this case the resampling intervals are determined by the relative wave propagation speed curves. The applied warping function corresponds to the average of RFEs shown in Fig. 1(b) thus minimizing the maximal error. This resulting RFE is illustrated in Fig. 3(c). Using this technique the maximal error is reduced to 0.78%.

6. SIMULATION EXAMPLE OF A CUBE

As an example an ideal cube was simulated. The mesh consisted of $8 \times 8 \times 8 = 512$ nodes and the walls had a reflection coefficient -1. An impulse excitation was located near one corner, and the receiver was at the opposite one. In the simulation, 3298 time

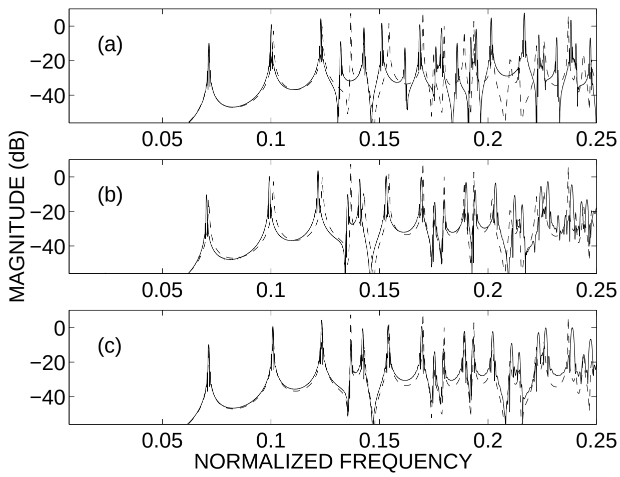

Fig. 4. A cubic space is simulated and a transfer function is calculated (a) with the original rectangular mesh, (b) with the optimally interpolated mesh applying multiwarping, and (c) with the optimally interpolated mesh using warping in the frequency domain. The solid line represents the simulation result and the dashed line is the analytical solution.

steps were calculated and the magnitude response was computed by Fourier transforming the obtained impulse response. Figure 4 illustrates the result in three different cases. In all the figures the dashed line stands for the analytically solved magnitude response. In the original rectangular mesh, represented in Fig. 4(a), some of the modes are at correct locations and some others are too low. Both the optimally interpolated mesh with multiwarping (Fig. 4(b)) and the optimally interpolated mesh with warping in the frequency domain (Fig. 4(c)) enhance the situation remarkably. It is easy to see that the most accurate result is obtained when the warping is performed in the frequency domain as already shown in Section 5.2. In all the simulations the obtained RFEs are in good agreement with the curves shown in Figs. 1(a) and 3(b,c), respectively.

7. CONCLUSIONS

An optimally interpolated 3-D digital waveguide mesh with rectangular structure was presented. By applying the interpolation, nearly direction independent wave propagation characteristics are obtained. The remaining dispersion can be reduced by frequency warping, which can be performed either in the time domain or in the frequency domain. The least relative frequency error of 0.78% is obtained when the warping is done in the frequency domain. The new method improves the frequency accuracy of the original 3-D mesh remarkably.

8. REFERENCES

[1] L. Savioja, T. Rinne, and T. Takala, "Simulation of room acoustics with a 3-D finite difference mesh," in *Proc. Int. Computer Music Conf.*, Aarhus, Denmark, Sept. 1994, pp. 463–466.

[2] S. Van Duyne and J. O. Smith, "The 2-D digital waveguide mesh," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, New Paltz, NY, Oct. 1993.

[3] S. Van Duyne and J. O. Smith, "The tetrahedral digital waveguide mesh," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, Mohonk, New Paltz, NY, Oct. 1995.

[4] L. Savioja, "Improving the three-dimensional digital waveguide mesh by interpolation," in *Proc. Nordic Acoustical Meeting*, Stockholm, Sweden, Sept. 1998, pp. 265–268.

[5] L. Savioja and V. Välimäki, "Improved discrete-time modeling of multi-dimensional wave propagation using the interpolated digital waveguide mesh," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Munich, Germany, 19-24 April 1997, vol. 1, pp. 459–462.

[6] L. Savioja and V. Välimäki, "Reducing the dispersion error in the digital waveguide mesh using interpolation and frequency-warping techniques," *IEEE Trans. Speech and Audio Process.*, vol. 8, no. 2, pp. 184–194, March 2000.

[7] S. Van Duyne and J. O. Smith, "The 3D tetrahedral digital waveguide mesh with musical applications," in *Proc. Int. Computer Music Conf.*, Hong Kong, 19-24 Aug. 1996, pp. 9–16.

[8] J. O. Smith, "Principles of digital waveguide models of musical instruments," in *Applications of Digital Signal Processing to Audio and Acoustics*, M. Kahrs and K. Brandenburg, Eds., chapter 10, pp. 417–466. Kluwer Academic, Boston, MA, 1997.

[9] J. Strikwerda, *Finite Difference Schemes and Partial Differential Equations*, Chapman & Hall, New York, NY, 1989.

[10] L. Savioja and V. Välimäki, "Reduction of the dispersion error in the triangular digital waveguide mesh using frequency warping," *IEEE Signal Processing Letters*, vol. 6, no. 3, pp. 58–60, March 1999.

[11] A. Oppenheim, D. Johnson, and K. Steiglitz, "Computation of spectra with unequal resolution using the Fast Fourier Transform," *Proc. IEEE*, vol. 59, no. 2, pp. 299–301, Feb. 1971.

[12] V. Välimäki and L. Savioja, "Interpolated and warped 2-D digital waveguide mesh algorithms," in *COST G-6 Conference on Digital Audio Effects*, Verona, Italy, 7-9 Dec. 2000, Invited paper.

[13] L. Savioja and V. Välimäki, "Multiwarping for enhancing the frequency accuracy of digital waveguide mesh simulations," *IEEE Signal Processing Letters*, 2001, in press.

[14] J. O. Smith, *Techniques for Digital Filter Design and System Identification with Application to the Violin*, Ph.D. thesis, Elec. Eng. Dept., Stanford University, June 1983.

[15] J.-M. Jot, V. Larcher, and O. Warusfel, "Digital signal processing issues in the context of binaural and transaural stereophony," in *the 98th Audio Engineering Society (AES) Convention, preprint no. 3980*, Paris, France, Feb. 1995.