FREQUENCY DOMAIN ACTIVE NOISE CONTROL SYSTEM USING OPTIMAL STEP-SIZE PARAMETERS

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ABSTRACT

In this paper, we propose a frequency domain active noise control system using optimal step-size parameters at each frequency. The proposed ANC system can converge faster than the conventional ANC system using the Filtered-x LMS algorithm with the optimal step-size parameter. Moreover, the proposed system can converge by setting the step-size parameters at unstable frequencies to 0 in the case where the phase error of the secondary path model does not satisfy the stable condition, whereas the conventional ANC system cannot converge in this case. In this paper, the theoretical equation of the optimal step-size parameters is derived by using available information during system operation. Next, we present the structure of the ANC system using the optimal step-size parameters obtained from the theoretical equation. Moreover, a control technique determining unstable frequencies is introduced. Finally, simulation results demonstrate the efficiency of the proposed ANC system.

1. INTRODUCTION

Active noise control (ANC) [1] has recently been applied to a wide variety of acoustic noise problems. In the ANC system, the filtered-x algorithm [2] is usually used as an algorithm to update the adaptive filter coefficients. The filtered-x algorithm requires an estimation of the secondary path (from the secondary source generating anti-noise to the error sensor detecting residuals) prior to the operation of the ANC system. The estimated model is often called a secondary path model. However, the secondary path model generally differs from the physical one. In this case, the phase errors between the secondary path and its model would lead to system instability or suboptimal performance [3, 4]. The upper bound for the step-size parameter in this case is also smaller than that in the case where the phase error is 0. We could know whether the system operates stable and the upper bound for the step-size parameter if the modeling error is known. However, we cannot obtain such information.

To solve this problem, we have already derived a theoretical equation of the upper bound for the step-size parameter using available information during system operation [5]. The upper bound for the step-size parameter is different at each frequency. Therefore, we propose an ANC system using the optimal step-size parameters at each frequency. In the proposed system, the filtering of the adaptive filter operates in the time domain and the updating operates in the frequency domain. This system is the same structure as the delayless frequency domain adaptive filter [6]. As the proposed system uses the optimal step-size parameters at each frequency, the proposed system can converge faster than the conventional one. Moreover, the proposed system can operate stable by setting the step-size parameters at unstable frequencies to 0, whereas the conventional system cannot operate stable in this case. In the proposed system, the unstable frequencies are determined by the theoretical equation and the measurement of error spectrum.

2. THEORETICAL EQUATION FOR THE OPTIMAL STEP-SIZE PARAMETERS

In this section, we derive a theoretical equation of the optimal step-size parameter by using the results in [5]. Figure 1 shows a single-channel ANC system in an air-duct.

Defining the step-size parameter at frequency l as μ_l, the input signal power as $|X_l|^2$, and the transfer functions of the secondary path and its model as C_l and $\hat{C}_l$, respectively, the coefficient $\left(1 - \mu_l \, |X_l|^2 \, C_l \hat{C}_l^*\right)$ has to exist in unit circle on the complex plane in order to decrease the acoustic noise. Figure 2 shows the

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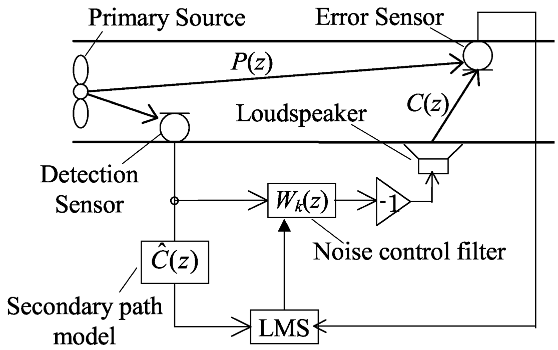

Fig. 1. Single-channel ANC system in an air-duct

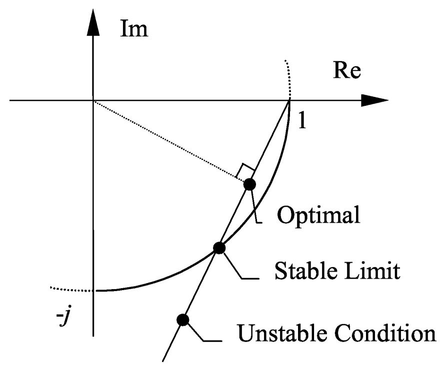

Fig. 2. Change of the coefficient $\left(1-\mu_l\left|X_l\right|^2 C_l \hat{C}_l^*\right)$ for variation of step-size parameter μ_l

variation of the coefficient $\left(1-\mu_l\left|X_l\right|^2 C_l \hat{C}_l^*\right)$ when μ_l varies. In this case, the coefficient moves on a straight line as μ_l varies. In order for the error to decrease the fastest, μ_l must be determined as the position of the coefficient becomes the nearest to the origin. Hence, the optimal step-size parameter is half of the upper bound derived in [5]. Defining the tap length of the adaptive filter as N and assuming the Gaussian input whose variance is σ_x^2, the optimal step-size parameter μ_l^{opt} is represented by

$$\mu_l^{opt}=\frac{C_l^r \hat{C}_l^r+C_l^i \hat{C}_l^i}{2N\sigma_x^2\left[\left(C_l^r \hat{C}_l^r+C_l^i \hat{C}_l^i\right)^2+\left(C_l^i \hat{C}_l^r-C_l^r \hat{C}_l^i\right)^2\right]}, \tag{1}$$

where C_l^r and C_l^i are the real and imaginary parts of C_l, and $\hat{C}_l^r$ and $\hat{C}_l^i$ are the real and imaginary parts of $\hat{C}_l$, respectively.

Next, using the available information during system operation, equation (1) is rewritten as

$$\mu_l^{opt}=\frac{2\left|\hat{C}_l\right|^2-\varepsilon G}{4N\sigma_x^2\left|\hat{C}_l\right|^4}, \tag{2}$$

where G is the gain of the secondary path and its model, and ε is the modeling error. The modeling error ε can be derived by

$$10\log_{10}\frac{1}{\varepsilon}=S/N+10\log_{10}\left(\frac{2}{\alpha}-1\right) \tag{3}$$

in [7]. If the NLMS algorithm is used in the adaptive algorithm for prior identification, the estimation accuracy (modeling error) can be computed by eq. (3) from the S/N of the desired signal and the ambient noise at the time of prior identification and the magnitude of the step-size parameter α in the NLMS algorithm.

If the step-size parameter obtained by eq. (2) is the negative number, the step-size parameter is set to 0. This case occurs when the phase error does not satisfy the stable condition. Hence, the above procedure can prevent unstable system operation.

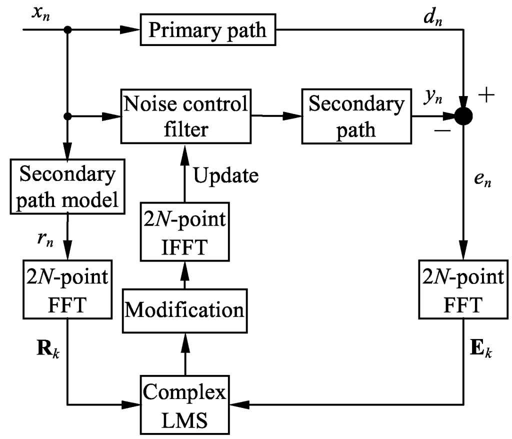

Fig. 3. Block diagram of the ANC system updated on frequency domain

3. FREQUENCY DOMAIN ANC SYSTEM USING OPTIMAL STEP-SIZE PARAMETERS

In this section, we present the structure and the update equation of the proposed ANC system. Figure 3 shows the block diagram. If n, k, and N are a sample time, a block time, and the tap length of the noise control filter, respectively,

$$n=kN+i, i=0,1,\cdots,N-1. \tag{4}$$

In Fig. 3, x_n is the input signal, e_n is the error signal, and r_n is the filtered reference signal at a sample time n, respectively.

Let us explain the update equation. The weight vector $\mathbf{w}_k$ is updated every N sample time. The modification is obtained by the IFFT of the frequency domain modification. That is,

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \Delta\mathbf{w}_k \quad (5)$$
$$\Delta\mathbf{w}_k = IFFT\left[\Delta\mathbf{W}_k\right] \quad (6)$$
$$\Delta\mathbf{W}_k = \Lambda\left[\mathbf{E}_k diag\left[\mathbf{R}_k^*\right]\right], \quad (7)$$

where Λ, which is the diagonal matrix whose diagonal elements are $2N$ optimal step-size parameters, is defined as

$$\Lambda = diag\left[\mu_0\ \mu_1\ \cdots\ \mu_{2N-1}\right]. \quad (8)$$

$\mathbf{R}_k$ is obtained by combining two filtered reference signal vectors and transforming by $2N$-point FFT.

$$\mathbf{R}_k = FFT\begin{bmatrix}\mathbf{r}_{k-1}\\ \mathbf{r}_k\end{bmatrix} \quad (9)$$

$\mathbf{E}_k$ is obtained by inserting N zeros into the error signal vector and transforming by $2N$-point FFT.

$$\mathbf{E}_k = FFT\begin{bmatrix}\mathbf{0}\\ \mathbf{e}_k\end{bmatrix} \quad (10)$$

where $\mathbf{r}_k$ and $\mathbf{e}_k$ are the filtered reference signal and error signal vectors, and represented as

$$\mathbf{r}_k = \left[r_{kN}\ r_{kN+1}\ \cdots\ r_{kN+N-1}\right]^T \quad (11)$$
$$\mathbf{e}_k = \left[e_{kN}\ e_{kN+1}\ \cdots\ e_{kN+N-1}\right]^T \quad (12)$$

4. CONTROL PREVENTING UNSTABLE OPERATION

In the conventional filtered-x LMS algorithm, if the phase error of the secondary path model exceeds the region $-\pi/2 \sim \pi/2$, the system operation becomes unstable regardless the value of the step-size parameter. In contrast, the proposed system could operate stable because the step-size parameters at all unstable frequencies become the negative number by eq.(2). However, there is no guarantee that the step-size parameter becomes the negative number at all frequencies where the phase error of the secondary path model exceeds the region $-\pi/2 \sim \pi/2$ because eq.(2) is an approximation. Therefore, we incorporate a control procedure, which can determine the unstable frequencies during system operation, into the proposed ANC system.

First, we define the absolute value of each element in $\mathbf{E}_0$ as the threshold to decide the stability. Next, the weight is set to 0 at the frequency where the absolute value of the error spectrum exceeds the threshold so that the system can operate stable. As the instant value of error spectra is not uniform, the error spectra are averaged by the low pass filter as

$$A_k^{'} = \beta A_{k-1}^{'} + (1-\beta)A_k, \quad (13)$$

where A_k and $A_k^{'}$ are the input and output signals of the low pass filter at a block time k, respectively.

Table 1. Condition of simulation

Primary path	256
Noise control filter	256
Secondary path	256
Secondary path model	256
S/N	19[dB]
Sampling frequency	2000[Hz]
β	0.95
Input signal	White noise (Power 1.0)

5. SIMULATION RESULTS

Table 1 shows the simulation condition. In order to verify the efficiency of the proposed system, two secondary path are chosen as the follows: (A) the phase error satisfies the stable condition at all frequencies; (B) the phase error does not satisfy the stable condition at some frequencies. The evaluation criterion in the following convergence property is defined as

$$Reduction[dB] = 10\log_{10}\frac{\sum_{i=0}^{N-1} d_i^2}{\sum_{i=0}^{N-1} e_i^2}. \quad (14)$$

The below convergence propeties represent ensemble averages over 50 independent runs.

First, let us examine the convergence property in the case (A). Figure 4 shows the convergence properties of the conventional filtered-x LMS algorithm when the step-size parameter is changed. It can be seen from Fig. 4 that the property converges the fastest in case of $\mu = 0.0004$ and the system operation becomes unstable in case of $\mu = 0.0006$. Hence, $\mu = 0.0004$ is the optimal value in this case. Note that this value is unknown prior to system operation. In contrast, Figure 5 shows the convergence properties of the proposed system and the conventional one with $\mu = 0.0004$. From Fig. 5, it can be seen that the proposed system can converge faster than the conventional system using the optimal step-size parameter.

Next, let us examine the convergence property in the case (B). Figure 6 show the convergence properties

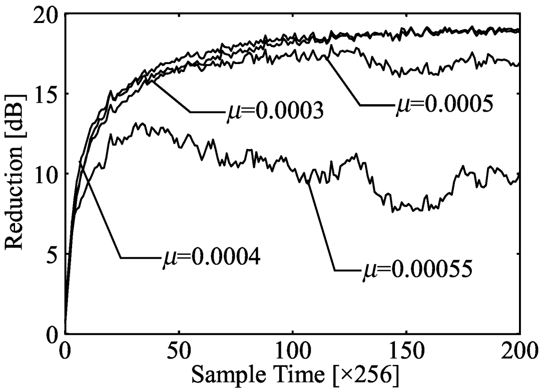

Fig. 4. Convergence properties of the conventional ANC system

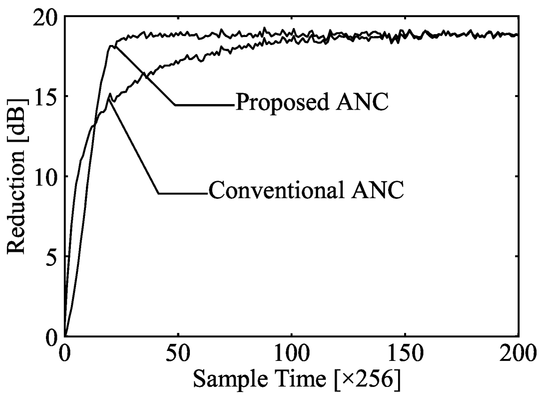

Fig. 5. Convergence property of the proposed ANC system

of the proposed system using the control procedure explained in the previous section and not using that. It can be seen from Fig. 6 that the system operation becomes unstable in case of not using the control procedure, whereas the system can operate stable by incorporating the control procedure. This fact means that the approximation equation (2) cannot determine the unstable frequencies completely. However, by incorporating the control procedure, the proposed ANC system can converge in poor environment as the conventional system cannot converge. Hence, the effectiveness of the proposed system has been demonstrated.

6. CONCLUSIONS

In this paper, we have proposed the frequency domain ANC system using the optimal step-size parameters at each frequency. The proposed system can converge faster and more stable than the conventional one using

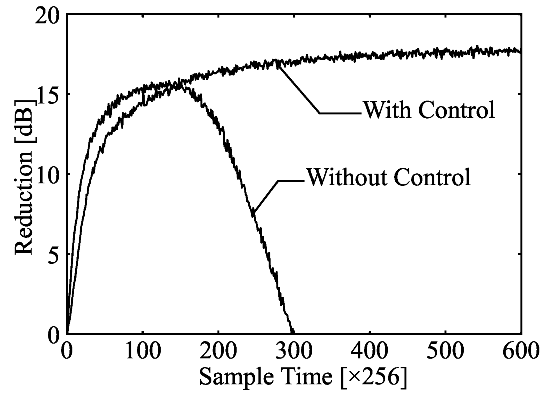

Fig. 6. Convergence property of the proposed ANC system

the optimal step-size parameter. Moreover, the proposed system can converge in poor environment as the conventional system cannot converge by incorporating the control procedure, which can determine the unstable frequency. Simulation results have demonstrated the effectiveness of the proposed system.

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