PARAMETER SELECTION FOR OPTIMISING TIME-FREQUENCY DISTRIBUTIONS AND MEASUREMENTS OF TIME-FREQUENCY CHARACTERISTICS OF NON-STATIONARY SIGNALS

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ABSTRACT

Selecting a time-frequency distribution (TFD) which represents a signal in an optimal way is commonly done by visually comparing plots of different TFDs. This paper presents a procedure that allows the analyst to perform the same task in an automatic way. Using the resolution performance measure for TFDs, the procedure optimises all distributions considered and selects the one which results into best concentration of signal components around their instantaneous frequency laws, as well as best suppression of the interference terms in the time-frequency plane. To do this requires to define a methodology to measure the time-frequency characteristics of a signal from its optimal TFD. An algorithm which implements this methodology is described and results are presented.

1. INTRODUCTION

It is known that the majority of real-life signals are non-stationary and multicomponent [1]. Their representation in either time or frequency can not reveal these signal characteristics, at least not in an easy to understand way.

To illustrate this point, let us consider a whale signal in time (Figure 1) and frequency (Figure 2). The time representation of the whale signal shows how the amplitude of the signal varies with time, while the frequency representation indicates what frequencies are present in the signal and what their relative strengths are. However, none of the two representations is capable of providing us with any sort of information about the nature of the signal components nor their behaviour with respect to time or frequency. These are considered to be important features of a signal since, for example, they allow a deeper insight into the origin of the signal and characteristics of the propagation medium.

The limitations of the classical signal analysis tools inspired researchers to look for new tools which would allow both the representation and analysis of non-stationary signals (signals whose spectrum is a function of time) to be carried out in a more sophisticated manner. One of the most popular and used of these tools is the time-frequency signal analysis (TFSA). TFSA is capable of preserving all information about the signal and displaying them, using time-frequency distributions (TFDs), in a relatively simple way.

Currently in the literature, one can find dozens of TFDs to choose from when searching for a TFD to represent a signal in the joint time-frequency domain. The large number of time-frequency distributions gives a lot of flexibility to the analyst in his(her) choice, but it also creates a problem of its own: Which TFD is the absolute best for the given signal?

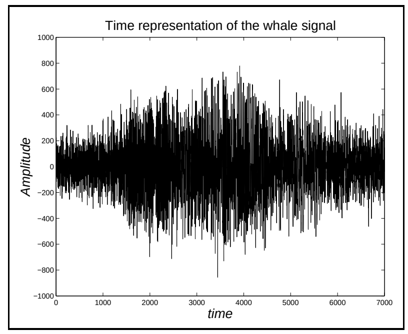


Figure 1: *Time representation of the whale signal*

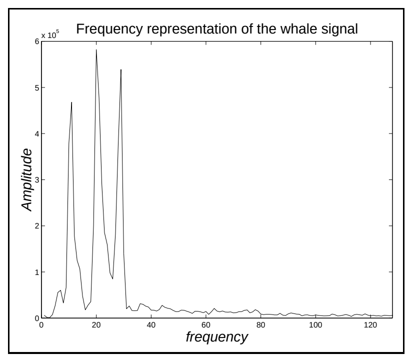
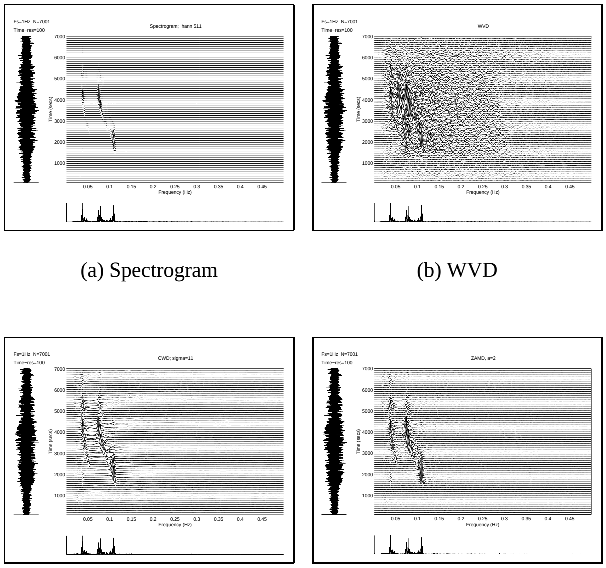


Figure 2: *Magnitude spectrum of the whale signal*

In Figure 3 we plotted the whale signal, whose time and frequency representations are given in Figures 1 and 2 respectively, using several popular TFDs [2, 3, 4, 5]: the spectrogram, the Wigner-Ville distribution (WVD), the Choi-Williams distribution (CWD), and the Zhao-Atlas-Marks distribution (ZAMD).

So, how do we say which of these distributions is best? Common practice is to visually compare the above plots, and based on ones own impressions of them choose the best one. Clearly this is



(a) Spectrogram (b) WVD

(c) Choi-Williams dist. (d) Zhao-Atlas-Marks dist.

Figure 3: *TFDs of the whale signal*

a very subjective procedure since what looks good to one person, may not be acceptable to another. The answer also largely depends on what features of the signal we are interested in obtaining from its time-frequency representations.

To avoid this confusion when selecting an optimal TFD for the given application, we define in this paper a generalised and objective procedure to evaluate the performance of TFDs and select the one which allows the best extraction of the signal time-frequency characteristics.

2. SELECTING AN OPTIMAL TFD FOR EXTRACTING TIME-FREQUENCY FEATURES OF A SIGNAL

2.1. The Methodology

In order to define a methodology for extraction of signal properties from its optimal time-frequency representation, we need to:

1. *Define a set of criteria of what information we expect to gain from a TFD*: What an analyst is interested in getting from the signal time-frequency representations is information about the number of components, their relative amplitudes (hence allowing extraction of the energy information), and components (frequency) modulation laws.[1] This requires a TFD to have signal components well concentrated around their respective IFs and well separated from each other (small components sidelobes and small cross-terms).

2. *Define a way to objectively measure the performance of time-frequency distributions based on these criteria*: One such measure was defined in [6] and it was further refined (normalised) in [7]. This measure, P, favours TFDs which result into components of good concentration and good resolution. So, for a slice of TFD taken at the time instant $t = t_0$, the distribution resolution performance measure is defined as [7]:

$$P = 1 - \frac{1}{3}\left\{\frac{A_S}{A_M} + \frac{A_X}{2A_M} + D\right\} \qquad (1)$$

where A_M, A_S and A_X are the average amplitudes of the mainlobes, sidelobes and cross-terms, respectively, of any two consecutive components of the multicomponent signal, and $D = (V_1 + V_2)/[2(f_2 - f_1)]$ is the components relative frequency separation (V_k and f_k, $k = 1, 2$, are the instantaneous bandwidth and the instantaneous frequency of the k^{th} component). How we actually measure each of these parameters is explained in Section 2.2.

Large values of P, close to 1, correspond to a good performing TFD, while small P (close to 0) indicates poor performance.

3. *Before the comparison, optimise each TFD to match the chosen criteria as close as possible*: The optimisation procedure for any TFD with parameter α, say, (e.g. CWD with parameter σ) consists of the following steps. First, we have to choose an initial value for α, usually the smallest value the parameter can take, and calculate the TFD for the given signal. For each time instant, we take a slice of the TFD (instantaneous spectrum) and find A_M, A_S, A_X and D, which are then combined using equation (1) to give the value of the performance measure P for the particular slice of the TFD. Note that in order to obtain the frequency separation between the signal components (D), we have to find the frequencies corresponding to the components peaks and the peaks instantaneous bandwidths. This will allow us to reconstruct the IF law of each component by combining the centres of the peaks for a range of distribution slices taken.

Once all instantaneous measures P are obtained, they are combined into the overall measure, $P_{overall}$, of the TFD for the given value of α. Since in real life we deal with noisy signals, in order to minimise the effect of the outliers on $P_{overall}$, the overall measure is taken to be the median of the instantaneous measures.

The above procedure is repeated for the next value of the parameter α. An increment in α should be neither too small (long computation time), nor too large (may skip the optimal value of α). A more refined search can be done at the later stage, after the initial optimal value of the smoothing parameter is found; the value of α resulting into largest $P_{overall}$. For TFDs whose parameters have no upper bounds, once $P_{overall}$ starts to decrease continuously, the optimisation is stopped.

4. *Quantitatively compare different TFDs and select the optimal one*: When all TFDs used to represent a given signal are optimised, they can be easily compared using their $P_{overall}$s. A TFD which results into the largest value of $P_{overall}$ should be selected as optimal for representing the signal characteristics in the joint time-frequency domain.

[1] TFSA is known to be especially useful when analysing frequency modulated asymptotic signals, with amplitude either slowly varying or being constant [1].

2.2. The Signal Parameters Measurement Algorithm

The algorithm used to measure the parameters of two consecutive components (with equal amplitudes) of a multicomponent signal TFD slice to obtain the instantaneous value of P is described below.

1. For a give signal $s(t)$ calculate its TFD, $\rho(t, f)$, with parameter α
2. Take a slice, $\rho_{t_0}(t, f)$, of $\rho(t, f)$ at time $t = t_0$
3. Normalise $\rho_{t_0}(t, f)$ such that it has the absolute maximum equal to 1
4. Determine the three largest maxima (peaks) of $\rho_{t_0}(t, f)$
5. The cross-term is located between the auto-terms, so initially set the middle peak to be the cross-term
6. By making sure that the ratio between the amplitudes of remaining two peaks is close to 1, and that the peak chosen as the cross-terms is close to the middle point between the centres of the other two peaks, we check whether our assumption in the previous step is correct. If not, select the two largest peaks of $\rho_{t_0}(t, f)$ as the auto-terms, and the extremum (maximum or minimum) located half-way between these auto-terms IFs (centres of their peaks) as the cross-term
7. Once the auto-terms and the cross-term locations are determined, we can measure their amplitudes. The average of the amplitudes of auto-terms is A_M in equation (1), while the amplitude of the cross-term is A_X
8. We then measure the rms bandwidths (at ≈ 0.71 of the peak amplitude) of the auto-terms, and check whether the components are resolved: If the value corresponding to the sum of the centre of the first component peak (the component IF) and the half of its bandwidth is less than or equal to the difference between the IF of the second component and the half of its bandwidth, the two components are resolved. For the resolved components we take the average of their bandwidths, and divide this average by the difference between the auto-terms IFs. The obtained quantity is the parameter D in equation (1)
9. Identify the outer extrema of the auto-terms (to the left from the first component and to the right from the second one) with largest amplitudes as the components "outer" sidelobes; these are the components sidelobes if there are no extrema between the auto-terms and the cross-term. Otherwise, find the largest "inner" sidelobes for each component (now searching from the components towards the cross-term). The larger of the "inner" and "outer" sidelobes is chosen as the component sidelobe, and the average of the components sidelobes is A_S in equation (1)
10. Once A_M, A_S, A_X and D have been measured, they are combined, using equation (1), into the instantaneous value of the performance measure P

    10 a) By repeating the above steps for a range of time instants, we obtain a set of instantaneous values of the measure P, whose average (or median for noisy signals) is the overall performance measure $P_{overall}$

    10 b) We then repeat the whole procedure for different values of the distribution smoothing parameter α. The value of α which results into the largest $P_{overall}$ gives the optimal $\rho(t, f)$ for the signal $s(t)$

    10 c) Finally, by first optimising different TFDs, and selecting the one which has largest $P_{overall}$ among them, we can find the optimal TFD to represent $s(t)$

3. EXAMPLE

To illustrate how the above defined methodology, and the accompanying measurement algorithm, for selecting an optimal TFD to extract the time-frequency characteristics of a given signal are used in practice, we consider the following example.

Let us define a two-component signal in noise:

$$\begin{aligned} s(t) &= s_1(t) + s_2(t) + n(t) \\ &= \cos(2\pi(0.1t + \frac{\alpha}{2}t^2)) + \cos(2\pi(0.2t + \frac{\alpha}{2}t^2)) + n(t) \end{aligned}$$

where $\alpha = 0.0016$ is the bandwidth-duration ration of the signal components, and $n(t)$ is additive Gaussian noise. In this example we chose the signal-to-noise ration of 10 dB.

Signal $s(t)$, whose length is $N = 128$, is analysed in the time-frequency domain using the spectrogram, the Wigner-Ville distribution, the Choi-Williams distribution, the Born-Jordan distribution (BJD), the Zhao-Atlas-Marks distribution, and the recently introduced Modified B-distribution (MBD) [8].

The TFDs are first optimised (the WVD and BJD have no smoothing parameters, hence they can not be optimised) and then compared: $P_{overall}$ is calculated for each of the optimised TFDs, and the one with the largest value of $P_{overall}$ is selected as optimal in representing $s(t)$.

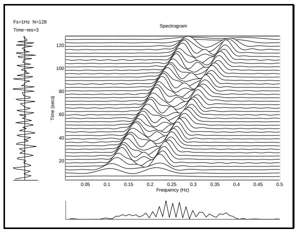
Table 1 contains the results of the "optimisation" process, and, as it can be seen, for signal $s(t)$ the "optimal" TFD is the Modified B-distribution with parameter $\alpha = 0.04$ since it results into the largest value of $P_{overall}$. The time-frequency plots of the optimised TFDs are shown in Figure 4.

TFD	*Optimal parameter*	$P_{overall}$
Spectrogram	Bartlett window, length 31	0.8611
Wigner-Ville	N/A	0.6223
Choi-Williams	$\sigma = 1$	0.8152
Born-Jordan	N/A	0.8122
Zhao-Atlas-Marks	$a = 2$	0.6707
Modified B	$\alpha = 0.04$	0.8756

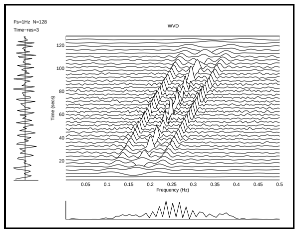
Table 1: *Optimisation results for the TFDs of signal* $s(t)$

It was reported in [6] that for a practical time-frequency analysis one of the properties a TFD should satisfy is to reveal the IF laws of the signal components by its peaks. In Figure 5 we compare the true IF laws of the two signal components with those measured from the peaks of the optimised MBD (the procedure is explained in Section 2.1). Even though $s(t)$ is embedded in 10 dB Gaussian noise, the distribution peaks still provide very good approximations of the components true IFs.

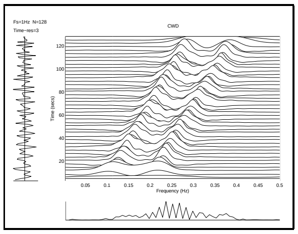
From the optimal TFD of the signal $s(t)$ we can also measure other important signal characteristics (the components bandwidths, the components mainlobe and sidelobe amplitudes, and



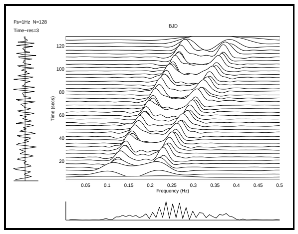
(a) Spectrogram



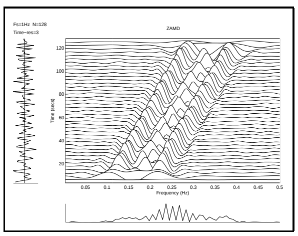
(b) WVD



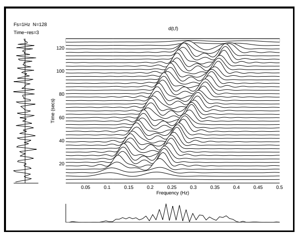
(c) CWD



(d) BJD



(e) ZAMD



(f) MBD

Figure 4: *Optimised TFDs of signal* $s(t)$

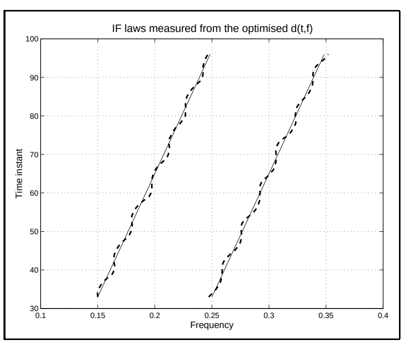


Figure 5: *Comparison of the measured (dashed) and true (solid) IF laws of the component* $s_1(t)$ *(left), and* $s_2(t)$ *(right) of* $s(t)$

the cross-term amplitude) by using the algorithm described in Section 2.2. These signal parameters are recorded in Table 2.[2]

[2]Note that in this example all amplitudes are normalised, and the sampling frequency is 1 Hz.

Parameter	*Component* $s_1(t)$	*Component* $s_2(t)$
Bandwidth	0.0194	0.0195
Mainlobe amplitude	1.0002	0.9574
Sidelobe amplitude	0.0900	0.0858
Cross-term amplitude	0.1503	

Table 2: *Measurements of the signal* $s(t)$ *components parameters*

4. CONCLUSION

One of the major obstacles to a wider use of time-frequency distributions is the nonexistence of methodologies for choosing a distribution that suits best a given signal. In this paper we have used a resolution performance measure for time-frequency distributions to optimise, compare and select a TFD which is the most appropriate for extracting important signal features (e.g. the number of components, components amplitudes, IF laws and bandwidths) in the joint time-frequency plane. The methodology was applied to closely spaced multicomponent FM signals, and it was shown that for these signals a recently introduced TFD, the Modified B-distribution, provided the optimal time-frequency representation.

5. REFERENCES

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