LIMITS ON LINEAR DETECTOR PERFORMANCE FOR CLOSE USERS IN A WIRELESS MULTIPATH ENVIRONMENT

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ABSTRACT

In multiuser wireless systems using multiple antennas in a multipath environment it has been asserted that whenever users are at least a distance of $\lambda/2$ apart, it is possible to recover their respective signals with appropriate signal processing [1]. This paper significantly strengthens this remarkable assertion by: 1) showing explicitly that separability of the users' signals is possible using the simplest class of linear detectors; 2) showing that the $\lambda/2$ rule of thumb can be quite conservative with meaningful signal separability achievable at separations around $\lambda/10$; and 3) identifying critical geometrical properties, such as position of users and multipath geometry that permit the best separability.

1. INTRODUCTION

It is generally acknowledged that if an interfering user is very close to a desired user in a mobile wireless communications environment then the difficulty in accurately reproducing the desired user's signal at the receiver is increased. In mobile wireless communications research it is often assumed that the users are a large enough distance apart that the interfering users' signals may be nulled using beamforming techniques with an antenna array receiver [2, 3]. It is sometimes also simply assumed that in a mobile environment the users are moving (mobile) and so any problems in signal detection associated with the users being too close is only momentary. In fact, it is pointed out in [1] that, particularly in wireless multipath environments, there will always be pathological geometries, corresponding to any particular location of the users, for which sufficiently accurate signal detection is impossible. Rather, receiver performance should be taken as an average over various locations and geometries. In this paper, we present results which give credence and clarification to these assumptions.

We introduce the concept of *separability* and define it as a measure of the degree to which a set of desired signals can be extracted from interference and noise. We simulate receiver performance in a wireless multipath environment in which a desired user and an interfering user are positioned very close together. We apply our definition of separability to the receiver performance to get an indication of how close an interfering user can be to a desired user in such an environment while maintaining acceptable signal reproduction. We show the surprising result that, given a rich multipath environment, users as close as $\lambda/10$ can retain sufficient separability. At a carrier wavelength of 1GHz this corresponds to 3cm. Users, in a practical situation, are unlikely to find themselves this close.

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2. SEPARABILITY

We develop an abstract concept of separability. It is applicable to any system in which the extraction of desired signals from a received signal corrupted by interference and noise is desired. We then develop a stronger version by imposing the constraint of linearity on the receiver structure. Finally, we apply separability to the problem of detection of close users in a multiuser wireless multipath communication system.

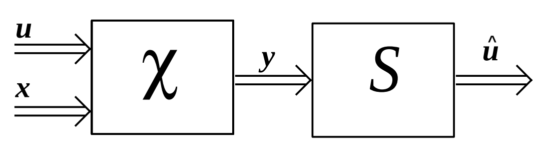

Fig. 1. Generic system model for defining separability. The desired signals are $\mathbf{u}$, extraneous signals, $\mathbf{x}$, interference function, $\chi(\cdot,\cdot)$, with multiple outputs, $\mathbf{y}$, separation function, $\mathcal{S}$ and outputs, $\hat{\mathbf{u}}$, estimates of the desired signals.

We base our definition of separability on the system in Fig. 1. All inputs and outputs are generically assumed to be vectors. Let the input, $\mathbf{u}$, represent the desired signals and $\mathbf{x}$ represent extraneous inputs such as noise and signals from interfering users. Let $\chi(\cdot,\cdot)$ represent an interference function that maps the desired signals and any extraneous inputs to multiple outputs, $\mathbf{y} = [y_1, \ldots, y_L]^T$ where $\{\cdot\}^T$ denotes transpose. Parameter $L > 1$ models the situation, for example, when we have multiple sensors. Let $\mathcal{S}(\cdot)$ represent a signal processing function which operates on $\mathbf{y}$ to produce an estimate, $\hat{\mathbf{u}}$, of the desired signals.

In the context of this paper, $\chi(\cdot,\cdot)$ represents the communication channel and $\mathcal{S}(\cdot)$ represents the receiver operating in a oneshot mode. We define separability as follows.

Definition 1 (MSE-ε Separability) *Let desired signals,* $\mathbf{u}$*, and extraneous input signals,* $\mathbf{x}$*, be acted upon by an interference function,* $\chi(\cdot,\cdot)$*, with vector output,*

$$\mathbf{y} = \chi(\mathbf{u}, \mathbf{x}). \quad (1)$$

Then $\mathbf{u}$ *is said to be* MSE-ε separable *from* $\mathbf{x}$ *under* $\chi(\cdot,\cdot)$ *if* $\exists$ *a function,* $\mathcal{S}(\cdot)$*, acting upon* $\mathbf{y}$*, which produces an estimate,* $\hat{\mathbf{u}}$*, such that* $\mathrm{E}\{|\mathbf{u}-\hat{\mathbf{u}}|^2\} \leq \varepsilon, \varepsilon > 0$.

Remarks

1. This definition only requires existence of the separability function, $\mathcal{S}(\cdot)$ - no indication is given nor required of how to go about finding such a function for a given $\chi(\cdot,\cdot)$.
2. Whenever the threshold, ε, is small enough, e.g., with $\mathbf{u} = [u_1, \cdots, u_k, \cdots, u_K]^T, u_k \in \{\pm 1\}$, $\varepsilon = 0.01$, the receiver $\mathcal{S}(\cdot)$ can be said to have sufficiently accurately recovered the desired signals, $\mathbf{u}$, despite distortions induced by $\chi(\cdot,\cdot)$.
3. The degree of separability can be gleaned from the closeness of ε to 0.
4. For the remainder of this paper we use the term "separable" to imply MSE-ε separable.

Having established a general definition of separability, we can now develop stronger definitions by introducing constraints. For example, we can constrain $\mathcal{S}(\cdot)$ to be linear in the sense defined below.

Definition 2 (MSE-ε Linear Separability) *Let desired signals,* $\mathbf{u}$*, and extraneous input signals,* $\mathbf{x}$*, be acted upon by an interference function* $\chi(\cdot,\cdot)$*, with vector output,* $\mathbf{y} = \chi(\mathbf{u}, \mathbf{x})$*. Then* $\mathbf{u}$ *is* MSE-ε linearly separable *from* $\mathbf{x}$ *under* $\chi(\cdot,\cdot)$ *if*

1) it is MSE-ε separable

2) $\exists$ *a matrix* $\mathbf{S}$ *and a scalar memoryless nonlinearity* $f(\cdot)$ *such that* $\hat{\mathbf{u}} = \mathcal{S}(\mathbf{y}) = f(\mathbf{S}^{\mathbf{H}}\mathbf{y})$ *where* $\{\cdot\}^{\mathbf{H}}$ *denotes conjugate transpose.*

Remarks

1. The function $f(\cdot)$ is dependent upon the nature of the signals, $\mathbf{u}$ and $\chi(\cdot,\cdot)$. For example, if the elements of $\mathbf{u}$ are binary, $u_k \in \{-1, 1\}$, $\chi(\cdot,\cdot)$ is complex and $\Re(\cdot)$ indicates real part, then,

$$f(\cdot) = \mathrm{sgn}(\Re(\cdot)). \quad (2)$$

2. Such a linear detector is feasible whenever the interference function is linear, such as that used to model multipath.
3. For the remainder of this paper we use the term "linearly separable" to imply MSE-ε linearly separable.

3. APPLICATION OF SEPARABILITY TO CLOSE USERS

3.1. Problem Description

We now consider an application of linear separability. We consider close users in a wireless multipath environment with an antenna array receiver incorporating a linear detector. It is often considered that the usefulness of antenna arrays in multiuser wireless communications is limited to the utilization of beamforming techniques

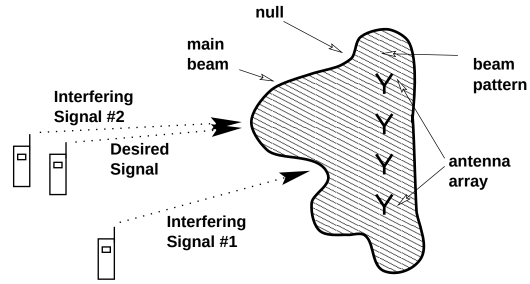

Fig. 2. Beamforming in a multiuser environment, with the main beam directed at the desired signal and, unavoidably, also at interfering signal # 2. A null is directed at interfering signal # 1.

such as nulling the interfering signal as shown for interfering user #1 in Fig. 2. If the interfering and desired users have an angular separation of less than the beamwidth, as with interfering user #2, in Fig. 2, then beamforming is usually ineffective.

We use the concept of separability to show that under certain channel conditions a desired user and an interfering user can be as close as $\lambda/10$ in a wireless multipath environment while still obtaining acceptable performance from an antenna array receiver incorporating a linear detector. That is, the desired user's signal is linearly separable from interference and noise under the given channel conditions.

3.2. System Model

We consider a system with one desired signal, one interfering signal and noise. All signals are specified at a given time instant. Let the desired user be user 1, with signal b_1 and let the interfering user be user 2 with signal b_2. We form the user signal vector, $\mathbf{b} = [b_1 \ b_2]^T, b_k \in \{-1, 1\}, k \in 1, 2$.

Let each user's signal have P multipath components. The p^{th} multipath component of the k^{th} user's signal has an angle of arrival (AOA) of θ_{pk} at the array. Let L be the number of array sensors and $\mathcal{G}(l, \theta_{pk})$ be the additional phase imposed upon a signal with an AOA of θ_{pk}, by sensor l, due to the sensor's physical position in the array. We form an $L \times P$ sensor phase matrix, $\mathbf{G_k}$, for user k, with elements $\mathcal{G}(l, \theta_{pk})$.

Let the vector of multipath component complex gains for user k be $\mathbf{a_k} = [a_{1k}, \cdots, a_{Pk}]^T$. We assume that the multipath profile for each user is time-invariant (i.e., the users and scatterers in the environment are stationary). The signal propagation channel for user k is represented by the channel coefficient vector $\mathbf{c_k} = [c_{1k}, \cdots, c_{Lk}]^T$ and is given by,

$$\mathbf{c_k} = \mathbf{G_k}\mathbf{a_k}. \quad (3)$$

Allowing for one output for each sensor, the array output vector is $\mathbf{y} = [y_1, \cdots, y_L]^T$. Let the channel coefficient matrix for both users be $\mathbf{C} = [\mathbf{c_1} \ \mathbf{c_2}]^T$. Let the vector of noise components from each sensor be $\mathbf{n} = [n_1, \cdots, n_L]^T$, where the components are statistically independent, zero mean, additive, white, gaussian noise (AWGN) each with variance σ_n^2. Then,

$$\begin{aligned} \mathbf{y} &= \mathbf{G_1}\mathbf{a_1}b_1 + \mathbf{G_2}\mathbf{a_2}b_2 + \mathbf{n} \\ &= \mathbf{c_1}b_1 + \mathbf{c_2}b_2 + \mathbf{n} \\ &= \mathbf{C}\mathbf{b} + \mathbf{n}. \end{aligned} \quad (4)$$

Remarks

1. It is assumed that the delay spread of the multipath is small compared with the symbol duration.
2. Equation (4) is in the form of (1) where b_1 is the desired signal and $\chi(\cdot)$ adds noise, multi-user interference and multipath.

4. LINEAR DETECTORS

A memoryless linear detector generates an estimate of the desired signal from a linear combination of the current received signals. The received signals used to generate the estimate in our simulations are the L sensor outputs. We compare the separability performances of three well-known linear detectors, the matched filter (MF) [4], decorrelating (DEC) [5] and minimum mean square error (MMSE) [6] detectors.

Let $\mathbf{h_1}$ be the estimate coefficient vector for the desired signal for a given linear detector. In accordance with Definition 2, the estimate, $\hat{b}_1$, is a function of $\mathbf{h_1^H y}$, where $\mathbf{h_1}$ is a special case of the separability matrix $\mathbf{S}$. From (2) the estimate for binary b_1 is,

$$\hat{b}_1 = \mathrm{sgn}[\Re(\mathbf{h_1^H y})]. \quad (5)$$

The value of $\mathbf{h_1}$ chosen for a given detector can depend on the particular performance criterion being used. We choose

$$\mathbf{h_1}_{\mathrm{MF}} = \frac{\mathbf{c_1}}{\mathbf{c_1^H c_1}} \quad (6)$$

$$\mathbf{h_1}_{\mathrm{DEC}} = \frac{\mathbf{c_1 c_2^H c_2 - c_2 c_2^H c_1}}{\mathbf{c_1^H c_1 c_2^H c_2 - c_1^H c_2 c_2^H c_1}} \quad (7)$$

$$\mathbf{h_1}_{\mathrm{MMSE}} = (\mathbf{C R_{bb} C^H + R_{nn}})^{-1} \mathbf{C r_{b1}}. \quad (8)$$

where $\mathbf{R_{nn}} = \sigma_n^2 \mathbf{I}$ is the noise covariance matrix and $\mathbf{R_{bb}}$ is the user signal cross-correlation matrix with $\mathbf{r_{b}}_k$ its k^{th} column.

5. MSE PERFORMANCE MEASURE

We have based our definition of separability on the MSE performance measure. The MSE for the desired signal is,

$$\mathrm{MSE} = \mathrm{E}\{|b_1 - \hat{b}_1|^2\}. \quad (9)$$

Let R_{11} be the 0 delay autocorrelation coefficient of the desired signal. Then, the general equation for the MSE between the desired and estimated signals, for a linear detector, using the received signal model in (4), is,

$$\mathrm{MSE_{LIN}} = R_{11} - \mathbf{h_1^H C r_{b1}} - \mathbf{r_{b1}^H C^H h_1} + \mathbf{h_1^H (C R_{bb} C^H + R_{nn}) h_1}. \quad (10)$$

6. RESULTS

6.1. Linear Separability Performance

For the first set of simulations the scatterers are distributed on an arc, centred on the users, subtending an angle of $\Delta\theta_u$, with corresponding multipath spread at the array, $\Delta\theta_a$. The radius, r, of the scatterer distribution arc is a ratio, R_D, to the distance, D, between the users and the array. Figures 3, 4 and 5 show the linear separability performance of the MF, DEC and MMSE detectors with increasing distance, $d \ll D$, between the positions of the desired and interfering users, for different values of R_D. Results for each d are averaged over several interfering user positions and scatterer distribution centre angles.

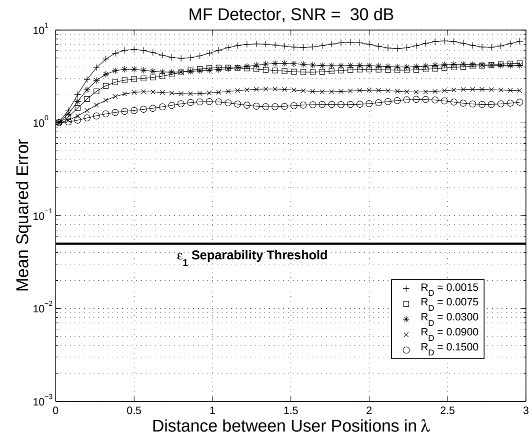

Fig. 3. MF linear separability performance with increasing distance, d, between the user positions for various ratios, R_D, of scatterer distribution radius to the distance between the users and the array, with the scatterer distribution centred on the users.

As the bit error rate (BER) has more practical significance in communications than the MSE, we select an acceptable BER and use this as a guide in selecting an appropriate MSE threshold. We select a BER of 0.001. As a lower limit, it can be shown that the best possible MSE for a given BER is twice the BER value, giving 0.002. As an upper limit, it an be shown that for a Gaussian distribution with a BER of 0.001, the variance (≡ MSE) is 0.1. We compromise with the MSE threshold, $\varepsilon_1 = 0.05$.

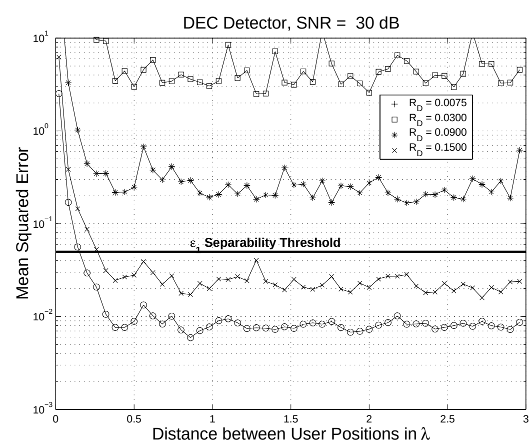

Fig. 4. DEC linear separability performance with increasing distance, d, between the user positions for various ratios, R_D, of scatterer distribution radius to the distance between the users and the array, with the scatterer distribution centred on the users.

Linear separability is achieved at $d = 0.1\lambda$ for $R_D = 0.15$ and 0.09 for the MMSE detector and at $d = 0.15\lambda$ and 0.25λ

for $R_D = 0.15$ and 0.09, respectively, for the DEC detector. It is not achieved, for $d < 3\lambda$, for the MF detector. It is expected that the MMSE detector would perform better than the MF and DEC detectors because 1) it is better matched to the definition of separability and 2) it takes into account both the interfering signal and the noise. Note that once the desired and interfering users are at least the separable distance, d_s, apart, the performance tends not to improve for further increases in d. For example, the separability performances when the interfering user is distances, d_s and $10d_s$, from the desired user are effectively the same. This appears to be true for the DEC and MMSE detectors, at least. However, the MSE performance of the MF detector does not appear to conform to this assumption.

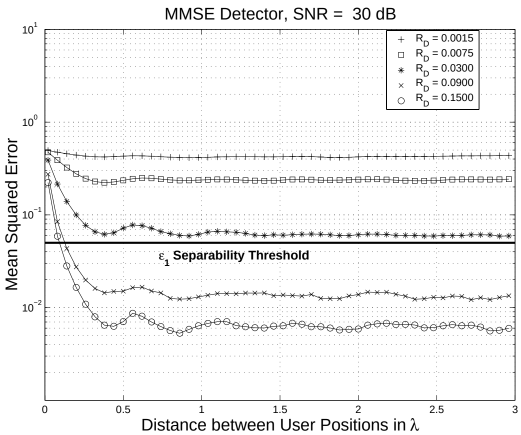

Fig. 5. MMSE linear separability performance with increasing distance, d, between the user positions for various ratios, R_D, of scatterer distribution radius to the distance between the users and the array, with the scatterer distribution centred on the users.

6.2. Angular Spread Effect on Linear Separability Performance

In general, the performance improves for increasing R_D. This is because the scatterers are placed further from the users with a given angular spread, $\Delta\theta_u$, the corresponding angular spread with respect to the antenna array, $\Delta\theta_a$, increases. In [7] it is shown that for increasing angular spread of multipath components arriving at the antenna array, receiver performance is improved.

Figure 6 shows the MMSE linear separability performance with increasing distance between the desired and interfering users for different values of $\Delta\theta_u$. The scatterer distribution is array-centric with $\Delta\theta_a$ kept constant at 20°. $\Delta\theta_u$ is varied by varying the radius, r, of the scatterer distribution about the array, keeping $\Delta\theta_a$ constant, with increasing r giving a corresponding increase in $\Delta\theta_u$. The performance improves with increasing $\Delta\theta_u$. Linear separability is achieved at $d = 0.1\lambda, 0.25\lambda$ and 0.7λ for $\Delta\theta_u = 74^\circ, 23^\circ$ and 10°, respectively. These angular spreads of scatterers local to the users are much less than the 360° usually assumed.

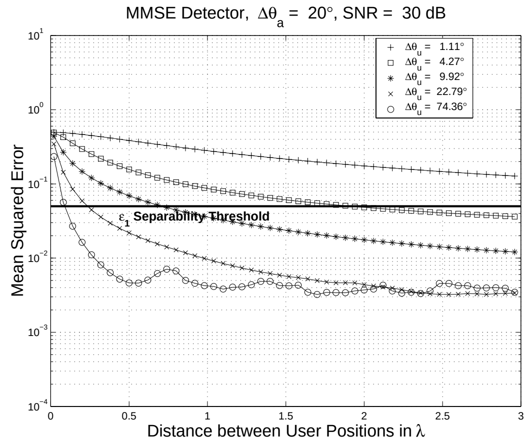

Fig. 6. MMSE linear separability performance with increasing distance between the user positions for various multipath spreads, $\Delta\theta_u$, with respect to the users. $\Delta\theta_a$ is kept constant at 20°.

7. CONCLUSIONS

The concept of separability of a desired signal from extraneous signals and noise under a given interference function has been introduced. The achievable separability level gives an indication of the fundamental performance limits of a system. A specific application of linear separability has been presented. The example used has produced the surprising result that the signal of a desired user in a multiuser multipath environment can be sufficiently accurately reproduced using a linear detection algorithm when the desired user is as close as $\lambda/10$ to an interfering user. We have also shown that for a given angular spread of multipath about the antenna array receiver, an increase in angular spread about the users has a marked effect on the separability of the desired signal.

8. REFERENCES

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