OPTIMAL HISTOGRAM MODIFICATION WITH MSE METRIC

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ABSTRACT

In this paper we propose a method to modify the histogram of a signal to a desired specific histogram. Traditionally, points having the same value in the input signal are all mapped to same value in the output signal. Hence, the desired histogram can only be approximated. Here we formulate our problem as finding a transformation such that the error between the input and output signal is minimized and the output signal has the desired histogram. It turns out that this problem is equivalent to an integer linear programming problem. This method might be specifically useful for histogram based watermarking and compression.[1]

1. INTRODUCTION

Histogram modification is an old problem[1]. Histogram equalization is applied to images so that details which cannot be seen in the original image can be rendered. Recently, histogram modification has been used to embed watermarks to the images by Coltuc et.al.[2].[2] In this work histogram of an image has been modified to a specific watermark histogram. This is achieved by defining an ordering of the pixels in the image. In this method, there is no direct control of how the image quality is affected due to histogram modification.

Here, we propose a histogram modification method where the mean square error (MSE) is minimized between the modified and the original image. Again the histogram can be modified any way we want.

A second application of histogram modification can be in compression. We may want to modify the histogram of an image such that only some gray levels will be allowed in the modified image. By doing so, we will reduce the number of gray levels in an image so that we need less bits/pixel to store the image. Notice that, further compression can be achieved on the modified image. In eliminating some of the gray levels we will again try to find a modified image such that the MSE between the original image and the modified image is minimum and the histogram of the modified image contains only predefined gray levels. We will assume that these predefined gray levels are given.

We will define our histogram modification problem in Sec. 2. The equivalent linear programming problem will be shown in Sec. 3. This will be followed by defining histogram modification for compression in Sec. 4. Then we will show our experimental results in Sec. 5. This will be followed by conclusion.

[1]This work was supported by Office of Naval Research Grant N00014-99-1-1002.

[2]A recent state of the art review of watermarking methods can be found in [3].

2. HISTOGRAM MODIFICATION PROBLEM

Let us assume that the signal has L levels: $g_1, \ldots, g_L$. The original signal will have an histogram $h_1, \ldots, h_L$ where $h_i \geq 0$ for $i = 1, \ldots, L$. Let us denote the desired histogram as $h'_1, \ldots, h'_L$. Now we will allow a fraction of points having value h_i to become h'_j and we will denote the number of these points as $\gamma_{i,j}$ (For simplicity let $\gamma_{i,i} = 0$). Apparently, $\gamma_{i,j}$ is a nonnegative integer. Also, from the preservation of points, for each grey level the following holds:

$$h_i - \Sigma_{j=1}^{L} \gamma_{i,j} + \Sigma_{j=1}^{L} \gamma_{j,i} = h'_i \quad \text{for} \quad i = 1, \ldots, L. \quad (1)$$

In order to find feasible $\gamma_{i,j}$ we have to ensure that

$$\Sigma_{i=1}^{L} h_i = \Sigma_{i=1}^{L} h'_i \quad (2)$$

i.e., the equations in (1) are consistent. Another constraint comes from the fact that the total number of points which have signal level h_i and which become some other signal level should not be greater than h_i:

$$\Sigma_{j=1}^{L} \gamma_{i,j} \leq h_i \quad \text{for} \quad i = 1, \ldots, L. \quad (3)$$

Now let us calculate the error when this type of histogram modification is applied on the signal. Let $f(g_i, g_j)$ denote the error due to a point which has signal level g_i and becomes g_j. Thus the sum of errors due to histogram modification will be:

$$E = \Sigma_{i=1}^{L} \Sigma_{j=1}^{L} \gamma_{i,j} f(g_i, g_j) \tag{4}$$

The common error functions used are square error function and absolute error function:

$$\begin{array}{rl} \text{Square error function:} & f(x,y) = (x-y)^2 \\ \text{Absolute error function:} & f(x,y) = |x-y| \end{array}$$

The method described in Sec. 3 works for more general, in fact arbitrary, nonnegative error functions.

We will use square error function from now on to illustrate the power of our method. Also note that we will check whether Eqn (2) holds before starting our minimization. In Eqn. (1) there are L equations and only $L-1$ of them are independent (If we sum all L equations we end up with Eqn. (2).).

3. EQUIVALENT LINEAR PROGRAMMING PROBLEM

Now, let γ be defined as follows:

$$\gamma = [\gamma_{1,2}, \ldots, \gamma_{1,L}, \gamma_{2,1}, \gamma_{2,3}, \ldots, \gamma_{2,L}, \ldots, \gamma_{L,L-1}]^T \tag{5}$$

Simply, γ will contain all $\gamma_{i,j}$'s except when $i = j$. Let us also define $\mathbf{c}$:

$$\mathbf{c} = [c_{1,1}, \ldots, c_{1,L-1}, c_{2,1}, c_{2,L-1}, \ldots, c_{L,1}, \ldots, c_{L,L-1}]^T \tag{6}$$

where

$$c_{i,j} = \begin{cases} (g_i - g_j)^2 & \text{if } i < j \\ (g_i - g_{j+1})^2 & \text{if } i \geq j \end{cases} \tag{7}$$

Now, it can be shown that the first $L-1$ equations of Eqn. (1) can be written in matrix form as

$$\mathbf{A}\gamma = \mathbf{d} \tag{8}$$

where $\mathbf{d} = [h_1 - h'_1, \ldots, h_{L-1} - h'_{L-1}]^T$, and $\mathbf{A} = [a_{i,j}]$ is defined to be:

$$a_{i,j} = \begin{cases} 1 & \text{if } (i-1)(L-1) < j \leq i(L-1) \\ -1 & \text{if } j > i(L-1) \text{ and } \frac{j-i-i(L-1)}{L-1} \in \mathcal{Z} \\ -1 & \text{if } j \leq (i-1)(L-1) \text{ and } \\ & \quad \frac{j-i+1-(i-2)(L-1)}{L-1} \in \mathcal{Z} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Similarly Eqn. (3) can be written as

$$\mathbf{B}\gamma \leq \mathbf{e} \tag{10}$$

where $\mathbf{e} = [h_1, \ldots, h_L]^T$, and $\mathbf{B} = [b_{i,j}]$ is defined to be:

$$b_{i,j} = \begin{cases} 1 & \text{if } (i-1)(L-1) < j \leq i(L-1) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Now, our problem can be formulated as follows:

$$\begin{array}{rl} & \text{minimize} \quad \mathbf{c}^T\gamma \\ \text{subject to} & \mathbf{A}\gamma = \mathbf{d}, \quad \mathbf{B}\gamma \leq \mathbf{e} \quad \text{and} \quad \gamma \geq 0 \end{array} \tag{12}$$

Exactly we also want the elements of γ to be integers, then the optimal solution is an integer linear programming problem of Eqn. (12)[4]. Since integer programming takes more time than linear programming, if we assume that the number points in the signal is huge, then the above problem can be approximated by solving the linear programming of Eqn (12).

Example: When $L = 3$ and $g_i = i$, then the required matrices for the integer linear programming problem will be as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 & 0 & -1 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$\gamma = [\ \gamma_{1,2} \quad \gamma_{1,3} \quad \gamma_{2,1} \quad \gamma_{2,3} \quad \gamma_{3,1} \quad \gamma_{3,2}\ ]^T,$$

$$\mathbf{c} = [\ 1 \quad 4 \quad 1 \quad 1 \quad 4 \quad 1\ ]^T,$$

$$\mathbf{d} = [\ h_1 - h'_1 \quad h_2 - h'_2\ ]^T, \ \mathbf{e} = [\ h_1 \quad h_2 \quad h_3\ ]^T.$$

4. COMPRESION PROBLEM

In some cases we may want to reduce the number of levels in a signal for compression. In this case, some of the signal levels are not allowed in the modified histogram: $g_{m_1}, \ldots, g_{m_M}$ where $0 < m_i \leq L$ for $i = 1, \ldots M$. Here M determines how many gray levels will be in the modified image. Then, the constraints for this problem can be written as follows:

$$\begin{array}{ll} \Sigma_{j=1}^{L} \gamma_{m_i,j} = h_{m_i} & \text{for} \quad i = 1, \ldots, M. \\ \Sigma_{j=1}^{L} \gamma_{i,j} \leq h_i & \text{for} \quad i = 1, \ldots, L. \\ \gamma_{i,j} \geq 0 & \text{for} \quad i,j = 1, \ldots, L. \end{array} \tag{13}$$

Thus, this problem is again an integer programming problem.

5. EXPERIMENTAL RESULTS & DISCUSSION

In this section, we will illustrate our method of histogram modification on Lena image. The original histogram of Lena image is shown in Fig. 1. We have chosen a raised cosine as our desired histogram as our first example. The modified histogram is shown in Fig.

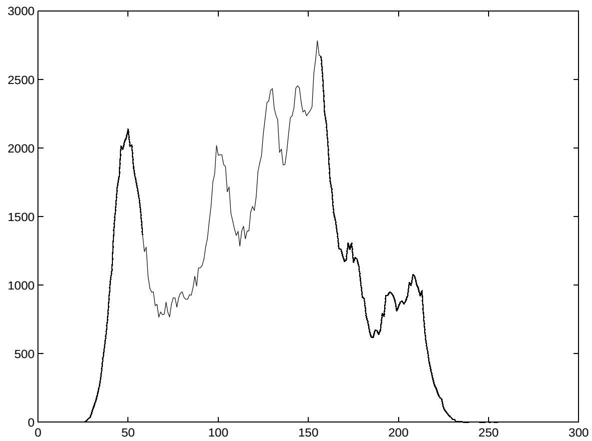

Figure 1: Original histogram of Lena Image.

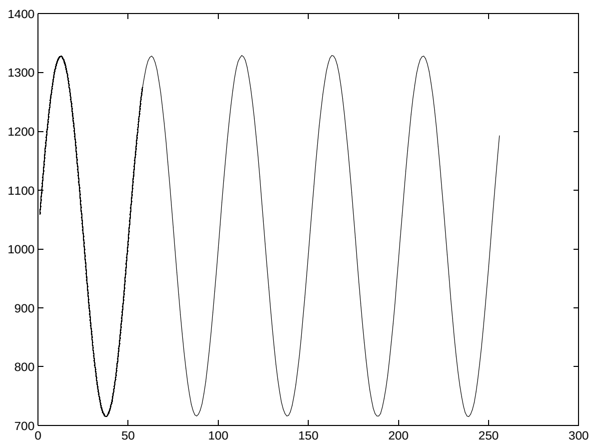

Figure 2: Histogram of modified Lena Image (raised cosine)(PSNR=19.42dB).

2 (PSNR=19.42dB).[3] We can as well modify the histogram to have a constant value (histogram equalization).

Notice that, we degrade the image quality (in terms of MSE) very much if we want to modify all portions of the histogram into a very different one. In histogram based watermarking, a watermark (a particular signal) is embedded in the histogram of an image such that the watermarked and the original images are visually the same. As an example, we have made the first half of the histogram to be a ramp. The modified histogram is shown in Fig. 3 (PSNR=35.09dB). We observed that the histogram-modified image and the original image cannot be distinguished visually.[4] In another example, we have made a portion of histogram to look like a sine wave. This histogram is shown in Fig. 4 (PSNR=36.05dB). For visual comparison we show the original image in Fig. 5 and the histogram modified image in Fig. 6. The images are visually indistinguishable.

[3]PSNR values in this paper refer to the PSNR values between the original and the modified images.

[4]All histogram-modified images can be found at [5].

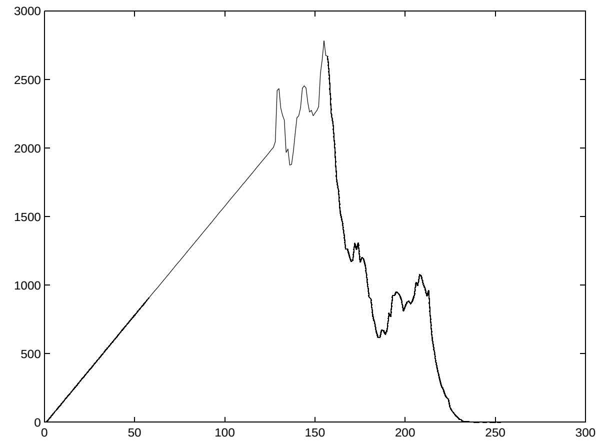

Figure 3: Histogram of modified Lena Image (Partial ramp)(PSNR=35.09dB).

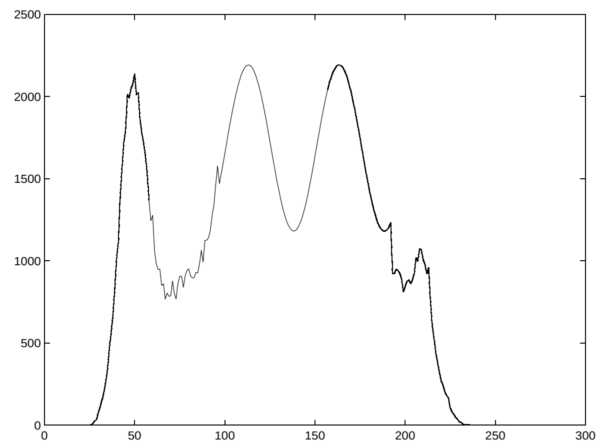

Figure 4: Histogram of modified Lena Image (Partial sine wave)(PSNR=36.05dB).

Our last example is about reducing the number of gray levels in a given image. We have chosen 13 out of 256 gray levels which can appear in the histogram modified image. The modified histogram is shown in Fig. 7 (PSNR=34.97dB). Thus, we have reduced the number of gray levels in an image without degrading the image very much. Choosing the gray levels which can appear in the histogram modified image is another problem, and this problem is not addressed here.

Figure 5: Original Lena Image.

Figure 6: Histogram modified Lena Image (Partial sine wave)(PSNR=36.05dB).

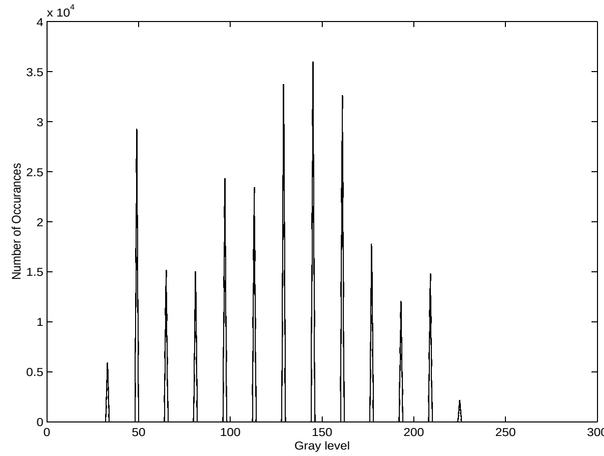

Figure 7: Histogram of modified Lena Image (Only 13 of 256 gray levels exist)(PSNR=34.97dB).

6. CONCLUSION

In this paper we have proposed an optimal algorithm to modify the histogram of an image to any desired histogram. The optimality criterion is chosen to be the minimization of MSE between the modified and the original image. However, other types of error criterion can also be used in the algorithm. We have shown that histogram modification problem is equivalent to integer linear programming problem. We have also shown examples of histogram modification which can be useful for watermarking and compression applications.

7. REFERENCES

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[5] http://www.systems.caltech.edu/mese/histogram/