SPACE-SCALE ADAPTIVE NOISE REDUCTION IN IMAGES BASED ON THRESHOLDING NEURAL NETWORK

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ABSTRACT

Noise reduction has been a traditional problem in image processing. Recent wavelet thresholding based denoising methods proved promising, since they are capable of suppressing noise while maintaining the high frequency signal details. However, the local space-scale information of the image is not adaptively considered by standard wavelet thresholding methods. In this paper, a new type of thresholding neural networks (TNN) is presented with a new class of smooth nonlinear thresholding functions being the activation function. Unlike the standard soft-thresholding function, these new nonlinear thresholding functions are infinitely differentiable. Then a new nonlinear 2-D space-scale adaptive filtering method based on the wavelet TNN is presented for noise reduction in images. The numerical results indicate that the new method outperforms the Wiener filter and the standard wavelet thresholding denoising method in both peak-signal-to-noise-ratio (PSNR) and visual effect.

1. INTRODUCTION

Noise reduction has been a traditional problem in image processing. Recent wavelet thresholding based denoising methods proved promising [1-4], since they are capable of suppressing noise while maintaining the high frequency signal details. However, the local space-scale information of the image is not adaptively considered by standard wavelet thresholding methods. In standard wavelet thresholding based noise reduction methods [3,4], the threshold at certain scale is a constant for all wavelet coefficients at this scale. A major difficulty in achieving adaptive algorithm using wavelet thresholding methods is that the soft-thresholding function is a piece-wise function and does not have any high order derivatives. In this paper, first a new type of thresholding neural networks is presented and a new class of smooth nonlinear thresholding functions is developed as the activation function. Unlike the standard soft-thresholding function, these new nonlinear thresholding functions are differentiable. Then a new nonlinear 2-D adaptive filtering method based on wavelet thresholding neural network is presented for space-scale adaptive noise reduction in images. The numerical results indicate that the new method outperforms the Wiener filter and the standard wavelet thresholding denoising method in both peak-signal-to-noise-ratio (PSNR) and visual effect.

2. THRESHOLDING NEURAL NETWORK

2.1 Thresholding Neural Network Structure

We construct a type of thresholding neural network (TNN) to perform the thresholding in transform domain to achieve noise reduction. The structure of the TNN is shown in Fig. 1. The input of the TNN is noise corrupted signal samples $y_i=x_i+n_i$, $i=0,\dots,N-1$, where x is the true signal and n is additive noise. The transform in TNNs can be any linear orthogonal transform. For a specific class of signal, the appropriate linear transform may be selected to concentrate signal energy versus noise. By thresholding, the signal energy may be kept while the noise is suppressed. Here the thresholding function $\eta(x,t)$ is employed as nonlinear activation functions of the neural network. The inverse transform is employed to recover the signal from the noise-reduced coefficients $\hat{v}_i$ in transform domain.

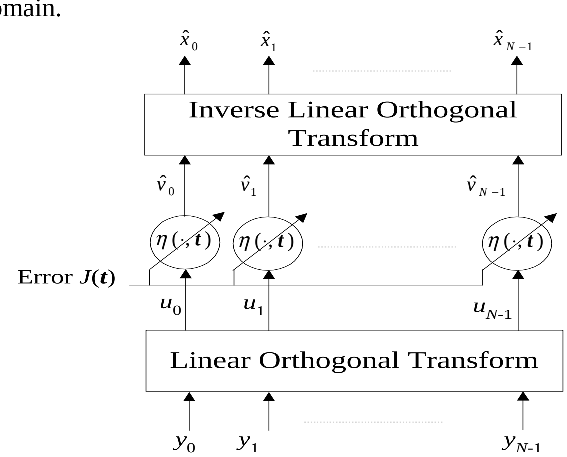


Fig. 1. Thresholding neural networks

Since many signals have some regularities and wavelet transform is a very good tool to efficiently represent such characteristics of the signal, discrete wavelet transform (DWT) [5] is often a suitable linear orthogonal transform in TNNs. Note that the TNN is different from the conventional multilayer neural network. In TNNs, a fixed linear transform is used and the nonlinear activation function is adaptive. It is possible to change the fixed linear transform to an adaptive linear transform and then the conventional multilayer neural network techniques can be incorporated. This will be a meaningful exploration we are going to pursue in the future.

2.2 A New Class of Differentiable Thresholding Functions

Most learning algorithms of neural network employ the gradients and higher derivatives of the network activation function [6]. It is desirable that the activation function has high-order derivatives so that the neural network has better numerical properties and various gradient-based learning algorithms can be developed. However, the standard soft-thresholding function is only weakly differentiated and does not have any high-order derivative. The author's previous work presented a new type of soft-thresholding functions which has second order weak derivatives and proved to be useful [2]. Here we present a new type of smooth soft-thresholding, which is infinitely differentiable and can keep the good properties of the standard soft-thresholding, as follows.

$$\eta_\lambda(x,t) = x + \frac{1}{2}\left(\sqrt{(x-t)^2+\lambda} - \sqrt{(x+t)^2+\lambda}\right) \qquad (1)$$

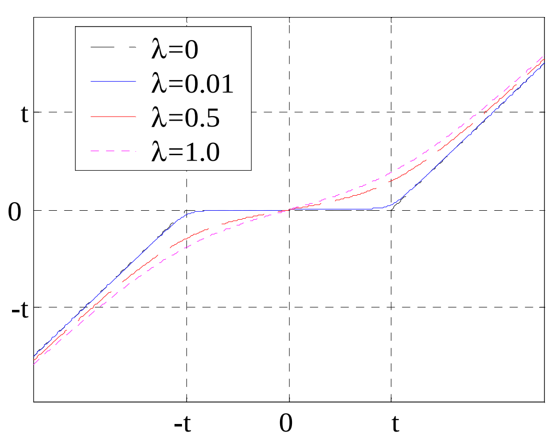


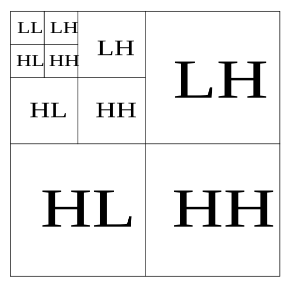
Fig. 2. Thresholding functions $\eta_\lambda(x,t)$

Obviously, $\eta_\lambda(x,t)$ has all higher order derivatives for $\lambda>0$. Note that when $\lambda=0$, $\eta_\lambda(x,t)$ is just the standard soft thresholding function [3] $\eta_s(x,t)=\mathrm{sgn}(x)(|x|-t)_+$. The standard soft-thresholding function ($\lambda=0$) and the new functions are shown in Fig. 2. The parameter λ determines the degree of the thresholding effect and the adjustability of the adaptive algorithm based on the function (amplitude of derivatives).

3. SPACE-SCALE ADAPTIVE 2-D NOISE REDUCTION BASED ON TNN

3.1 Space-scale Data Stream Preparation

In the new 2-D adaptive noise reduction method, the 2-D DWT is adopted as the linear transform in TNN and the noise corrupted image y is the input of the TNN. To achieve space-scale adaptive noise reduction, we need to prepare the 1-D coefficient data stream which contains the space-scale information of 2-D images. This is somewhat similar to the "zigzag" arrangement of the DCT (Discrete Cosine Transform) coefficients in image coding applications. In this data preparation step, the 2-D DWT coefficients are rearranged as a 1-D coefficient series in spatial order so that the adjacent samples represent the same local areas in the original image. An example of the rearrangement of an 8×8 transformed image is shown in Fig. 3, which will be referred to as a *1-D space-scale data stream*. Note that [5] the DWT of an image consists of four frequency channels: HH, HL, LH and LL. "H" represents high frequency channel and "L" represents low frequency channel. The first letter represents the horizontal direction and the second letter represents the vertical direction. The LL part at each scale is decomposed recursively, as illustrated in Fig. 3 (a). Each number in Fig. 3(b) represents the spatial order of the 2-D coefficient at that position corresponding to Fig. 3(a).



64	63	15	45	3	9	33	39
62	61	30	60	6	12	36	42
14	44	13	43	18	24	48	54
29	59	28	58	21	27	51	57
2	8	32	38	1	7	31	37
5	11	35	41	4	10	34	40
17	23	47	53	16	22	46	52
20	26	50	56	19	25	49	55

(a) (b)

Fig. 3. Data preparation of the image

3.2 Learning Algorithm for Thresholding Neural Network

Let u_i in Fig. 1 denote the space-scale data stream of the 2-D DWT coefficients of the input noisy image y and $u_i=v_i+z_i$, $i=0,\ldots,N-1$, in which v_i is the space-scale data stream of the 2-D DWT coefficients of the true image and z_i is additive white Gaussian noise in the transform domain. Our objective is to obtain the estimate $\hat{v}_i$ of the true image DWT coefficients v_i, which minimize the MSE (Mean Square Error) risk

$$J(\mathbf{t}) = E\{(\hat{v}_i - v_i)^2\} = \frac{1}{N}\sum_{i=1}^{N}[\eta_\lambda(u_i,\boldsymbol{t}(i)) - v_i]^2 \qquad (2)$$

In the new adaptive noise reduction scheme, the parameter $\boldsymbol{t}(i)$ is adaptively adjusted for the nonlinear thresholding function $\eta_\lambda(x,t)$ to minimize the MSE, where $\boldsymbol{t}$ denotes vector $[t_1, t_2,\ldots,t_M]^T$ and t_m is the threshold at wavelet channel m. In practice, since the original image x is usually unknown, its DWT coefficients v_i cannot be used as reference to estimate the risk $J(\boldsymbol{t})$. Therefore, a practical reference is adopted: two noise corrupted signals y and y' are obtained from the same image x plus uncorrelated noise n and n', and y' is used as the reference. This is reasonable since in some applications, we may have an array of sensors and obtain more than one corrupted version of the signal. For example, in adaptive echo cancellation applications, two measurements for the same source signal are commonly used [7]. It can be proved that using such noisy reference signal leads to the same optimum threshold as using the true signal [1].

We use a gradient-based LMS (Least Mean Square) stochastic adaptive learning algorithm [6,7] for the TNN to track local changes within the image and take advantage of the time-varying local estimation error, i.e., the threshold parameter $\boldsymbol{t}$ at position i is adjusted by $\Delta\boldsymbol{t}(i)=\boldsymbol{\alpha}(i)\cdot\partial\hat{v}_i/\partial\boldsymbol{t}\cdot\varepsilon_i$, $i=1,\ldots,N$, where the instantaneous error $\varepsilon_i=\hat{v}_i-v'_i$, $\boldsymbol{\alpha}(i)$ is a learning parameter and v'_i is the space-scale data stream of the 2-D DWT coefficients of the reference image y'. The threshold parameter $\boldsymbol{t}$ is dependent on not only different channels in transform domain but also spatial position, i.e., it is fully space-scale adaptive.

4. EXAMPLES

The 256×256 "cameraman" image is used as a test image to illustrate the new method based on the TNN. The original clean image is shown in Fig. 4(a). Two noisy images are generated with additive i.i.d. Gaussian noise with same noise variance. One of them is used as a reference image y'. The Daubechies 8-tap least asymmetrical wavelet filters are used. The largest scale of the two-dimensional DWT is set to be 3 in the experiments. The new soft-thresholding function $\eta_\lambda(x,t)$ with λ=0.01 is used. The algorithm is different noise variances. The peak-signal-to-noise-ratio (PSNR) results are shown in Table 1. The first column is the original PSNRs of noisy images. The new space-scale adaptive image denoising method is denoted as "TNN" in the table. For comparison, Table 1 also shows the results of the non-adaptive conventional wavelet denoising schemes. They are calculated using functions provided in Matlab Wavelet Toolbox. The "VisuShrink" is the universal soft-thresholding denoising technique [3]. The column "Wiener" represents the denoising results by Wiener filtering, which is the optimal solution of the linear filtering technique. As can be seen, the TNN based space-scale adaptive image denoising has the best performance in terms of PSNR improvement, especially when the PSNR of the original noisy image is high. This can be expected since the amplitudes of the few coefficients representing the signal in transform domain are much higher than those coefficients representing the noise, and then more signal energy can be preserved when cutting off all the coefficients with a threshold.

Noisy	VisuShrink	Wiener	TNN
20.0159	20.4768	26.2899	26.6423
25.0290	22.4583	29.1181	29.9728
30.0057	24.6496	32.4476	33.7289
34.9942	26.7662	36.3116	37.8609
39.9867	28.8212	40.6440	41.9908

Table 1. The PSNRs (dB) of different denoising methods.

Fig. 4(b) shows the noisy image with PSNR=20dB (the first row in Table 1). The denoised images using different methods are shown in Fig. 4(c)-(e). Apparently, the TNN based space-scale adaptive denoising method gives the best visual result as well as PSNR improvement.

5. CONCLUSION

In this paper, we presented a new type of thresholding neural network (TNN) structure for adaptive noise reduction, which combines the linear filtering and thresholding methods. We created a new type of soft and hard thresholding functions to serve as the activation function of TNNs. Unlike the standard thresholding functions, the new thresholding functions are infinitely differentiable. A new practical 2-D space-scale adaptive noise reduction method based on TNN was presented. Using the instantaneous error of the TNN, a gradient-based LMS (Least Mean Square) stochastic adaptive learning algorithm is employed in TNN. The learning algorithm proved to be efficient and effective. Numerical examples are given for different noise reduction algorithms including the conventional wavelet thresholding algorithms and linear filtering method. It is shown that the TNN based space-scale adaptive noise reduction algorithm exhibits much superior performance in both PSNR and visual effect.

Further investigations proved that under ideal conditions, the stochastic learning algorithm converges to the optimal solution of the TNN in certain statistical sense. It is also shown that by using the TNN and the new thresholding functions, many effective learning algorithms can be developed for various applications [8].

6. REFERENCES

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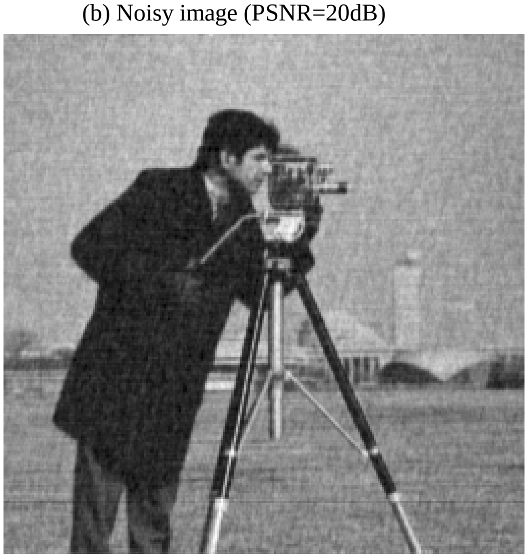
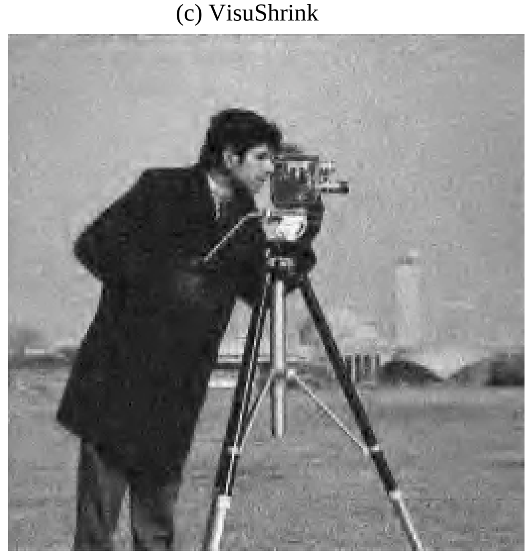
(a) Original image

(b) Noisy image (PSNR=20dB)

(c) VisuShrink



(d) Wiener filtering



(e) TNN based space-scale adaptive denoising

Fig. 4. Test images