

# BLIND TURBO MULTIUSER DETECTION FOR LONG-CODE UPLINK CDMA WITH UNKNOWN INTERFERENCES

Zigang Yang and Xiaodong Wang

Texas A&M University, Department of Electrical Engineering,  
College Station, TX 77843.

## ABSTRACT

We consider the problem of demodulating the multiuser symbols in an uplink long-code CDMA system in the presence of unknown out-cell multiple-access interference and narrowband interference. A novel blind Bayesian multiuser detector is derived based on the Bayesian inference of all unknown quantities. The Gibbs sampler, a Markov chain Monte Carlo (MCMC) procedure, is then used for Bayesian computation. Being soft-input and soft-output, the blind Bayesian multiuser detector is designed to be a part of Turbo multiuser receiver, which refines its processing based on the information from the decoding stage.

## 1. INTRODUCTION

Existing CDMA standards (such as IS-95 [1]) employ long spreading codes on the reverse link, i.e., PN sequences with very long periods. However, to date, most research on blind multiuser detection has focused on the more tractable short code case. In [2], we have proposed a blind multiuser detector for an asynchronous CDMA system employing long spreading sequences in unknown multipath channel with white Gaussian noise. In this paper, we consider the blind multiuser detection problem when such a system are exposing to the unknown interference, such as out-cell multiple-access interference (OMAI) and narrowband interference (NBI).

The proposed blind Bayesian multiuser detector for interfered CDMA system computes the MAP estimation for the multiuser symbols, based on the assumption that the total effect of white Gaussian noise, OMAI and NBI can be modeled as colored Gaussian noise with some unknown covariance matrix. Such a detector is based on the Bayesian inference of all unknown parameters, and Gibbs sampler, a Markov chain Monte Carlo (MCMC) technique is employed for Bayesian computation. Being soft-input and soft-output in nature, this blind multiuser detector is easy to fit into the Turbo receiver framework and exchange the extrinsic information with the MAP decoder to successively refine the performance in a coded CDMA system.

## 2. SYSTEM DESCRIPTION

### 2.1. Signal Model

Consider a  $K$ -user uplink asynchronous CDMA system, employing normalized long pseudo-random spreading sequences, and signaling through multipath channels with white Gaussian noise and

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other unknown interference. The transmitted signal due to the  $k$ th user is given by

$$x_k(t) = \sum_{i=0}^{M-1} \sum_{j=0}^{P-1} b_k(i) c_{k,i}(j) \phi(t - iT - jT_c - d_k), \quad (1)$$

where  $M$  denotes the length of the data frame;  $P$  is the processing gain;  $T$  denotes the symbol interval;  $\{c_{k,i}(j)\}_{j=0}^{P-1}$  is a signature sequence assigned to the  $k$ th user for the  $i$ th symbol;  $\{b_k(i)\}$ , and  $d_k \in [0, T)$  denote respectively the symbol stream, and the initial delay of the  $k$ th user's signal;  $\phi$  is a normalized chip waveform. The  $k$ th user's signal  $x_k(t)$  propagates through a multipath channel whose impulse response is given by

$$g_k(t) = \sum_{l=1}^{\tilde{L}} \beta_{k,l} \delta(t - \tau_{k,l}), \quad (2)$$

where  $\tilde{L}$  is the total number of physical paths in the channel;  $\beta_{k,l}$  and  $\tau_{k,l}$  are, respectively, the complex path gain and the delay of the  $k$ th user's  $l$ -th path.

At the receiver, the received signal  $r(t)$  is filtered by a chip-matched filter and sampled at the chip-rate. The received signal at the matched filter output at time  $t = iT + qT_c$  is given by [2]

$$r_q(i) = \sum_{k=1}^K \sum_{m=0}^{\iota} \sum_{j=0}^P \{b_k(i-m) c_{k,i-m}(q-j) h_k(mP+j)\} + v_q(i), \quad (3)$$

where  $\iota$  is the maximum delay spread among all users in terms of symbol intervals;  $h_k(l)$  is the discrete-time channel response for the  $k$ th user at delay  $lT_c$ ;  $v_q(i)$  is the sampled noise at the matched filter output. For convenience, define  $\iota_k$  as the initial delay for the  $k$ th user; define  $L$  as the maximum channel delay among all users. Based on the assumption that both  $\iota_k$  and  $L$  are less than one symbol interval, the received signal model in vector form can be written as

$$\begin{aligned} r(i) = & \sum_{k=1}^K \{b_k(i) \mathbf{C}_{k,i}^{(0)} \mathbf{h}_k + b_k(i-1) \mathbf{C}_{k,i-1}^{(1)} \mathbf{h}_k \\ & + b_k(i-2) \mathbf{C}_{k,i-2}^{(2)} \mathbf{h}_k\} + \mathbf{v}(i), \end{aligned} \quad (4)$$

where  $i = 0, 1, \dots, M-1$ ;  $r(i) \triangleq [r_1(i), r_2(i), \dots, r_P(i)]^T$ ;  $\mathbf{v}(i) \triangleq [v_1(i), \dots, v_P(i)]^T$ ;  $\mathbf{h}_k \triangleq [h_k(\iota_k+1), \dots, h_k(\iota_k+L)]^T$ .

It is easy to verify that  $\mathbf{C}_{k,i}^{(n)}$  can be expressed as

$$\begin{bmatrix} \left[ \mathbf{C}_{k,i}^{(0)} \right]_{P \times L} \\ \left[ \mathbf{C}_{k,i}^{(1)} \right]_{P \times L} \\ \left[ \mathbf{C}_{k,i}^{(2)} \right]_{P \times L} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{t_k \times 1} & \mathbf{0} & \dots & \mathbf{0} \\ c_{k,i}(1) & c_{k,i}(1) & & \\ c_{k,i}(2) & c_{k,i}(1) & & \\ \vdots & c_{k,i}(2) & \ddots & \\ c_{k,i}(P) & \vdots & c_{k,i}(1) & \\ & c_{k,i}(P) & c_{k,i}(2) & \\ & & \ddots & \vdots \\ & & & c_{k,i}(P) \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}_{3P \times L} \quad (5)$$

which is determined by the initial delay and the spreading sequence, and is assumed to be known throughout the paper.

## 2.2. Noise Model

In cellular DS-CDMA, the same uplink/downlink pair of frequency bands are reused for each cell. Therefore, a signal transmitted in one cell may cause interference in neighboring cells, resulting in out-cell multiple-access interference (OMAI). In addition, narrowband communication systems sometimes can overlay with CDMA systems, and thus cause narrowband interference (NBI) to the latter. Hence, the noise component  $\mathbf{v}(i)$  in (4) may consist of white Gaussian noise (WGN), OMAI and NBI.

The OMAI has the same structure as the in-cell CDMA signals. When the total number of out-cell users is large, by the central limit theorem, OMAI signal vector  $\mathbf{v}_{\text{OMAI}}(i)$  approaches a Gaussian vector with zero mean and a covariance matrix. The NBI signal is typically modeled as a correlated Gaussian process, hence the NBI signal vector  $\mathbf{v}_{\text{NBI}}(i)$  is Gaussian with zero mean and a covariance matrix. Combining these three components, the noise vectors  $\{\mathbf{v}(i)\}$  can be modeled as colored Gaussian vectors with zero mean and a covariance matrix, denoted by  $\Sigma$ .

## 2.3. System Model

Now, we consider the problem of blind multiuser detection for the above asynchronous uplink CDMA system. To resolve the phase ambiguity, which is inherent in any blind receiver, we differentially encoding the BPSK symbols  $\{a_k(i)\}_{i=1}^{M-1}$  to yield the symbol stream  $\{b_k(i)\}_{i=0}^{M-1}$ . Each symbol  $b_k(i)$  is then modulated and transmitted through a multipath channel  $h_k(m)$ . The received signals are given by (4).

Define  $\mathbf{Y} \triangleq \{r(0), r(1), \dots, r(M-1)\}$ , define the *a priori* LLR of the data bit  $a_k(i)$  as  $\rho_k(i)$ . In Section 3, we consider the estimation the *a posteriori* probabilities of the multiuser bits

$$P[a_k(i) = +1 | \mathbf{Y}], \quad (6)$$

based on the received signals  $\mathbf{Y}$ , the signal structure (4) and the prior information  $\{\rho_k(i)\}_{k=1, i=1}^{K, M-1}$ , without knowing the channel response  $\{h_k\}_{k=1}^K$  and the noise covariance matrix  $\Sigma$ . The prior information  $\{\rho_k(i)\}$  can be set to zero when there is no prior information available, when such a detector is fit into the Turbo receiver, this prior information is the extrinsic information delivered by the channel decoder from last iteration.

## 3. BLIND BAYESIAN MULTIUSER DETECTION

In this section, we develop the blind Bayesian multiuser detector for colored Gaussian noise. It is assumed that  $\mathbf{v}(i)$  in (4) have a complex joint Gaussian distribution, i.e.,

$$p[\mathbf{v}(i)] = \left( \frac{\det(\Sigma^{-1})}{\pi} \right)^P \cdot \exp(-\mathbf{v}^H(i) \Sigma^{-1} \mathbf{v}(i)). \quad (7)$$

Denote  $\mathbf{b}_k \triangleq \{b_k(i)\}_{i=0}^{M-1}$ ;  $\mathbf{B} \triangleq \{\mathbf{b}_k\}_{k=1}^K$ ;  $\mathbf{A} \triangleq \{a_k(i)\}_{k=1, i=1}^{K, M-1}$ ;  $\mathbf{H} \triangleq \{h_k\}_{k=1}^K$ ;  $\mathbf{S}_{k,i}(\mathbf{b}_k) \triangleq b_k(i) \mathbf{C}_{k,i}^{(0)} + b_k(i-1) \mathbf{C}_{k,i-1}^{(1)} + b_k(i-2) \mathbf{C}_{k,i-2}^{(2)}$ . Then (4) can be written as

$$\mathbf{r}(i) = \sum_{k=1}^K \mathbf{S}_{k,i}(\mathbf{b}_k) \mathbf{h}_k + \mathbf{v}(i). \quad (8)$$

### 3.1. Gibbs Sampler

To the problem of joint sequence detection and channel estimation, recent paper [3] has shown that *Gibbs sampler*, a blind Bayesian approach, is a very powerful Bayesian solution. Let  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_d]^T$  be a vector of unknown parameters,  $\mathbf{Y}$  be the observed data. Algorithmically, we can describe the Gibbs sampler as follows:

- For  $i = 1, \dots, d$ , we draw  $\theta_i^{(t+1)}$  from the conditional distribution

$$p(\theta_i^{(n+1)} | \theta_1^{(n+1)}, \dots, \theta_{i-1}^{(n+1)}, \theta_{i+1}^{(n)}, \dots, \theta_d^{(n)}, \mathbf{Y})$$

It is known that under regularity conditions, [4],

- The distribution of  $\boldsymbol{\theta}^n$  converges geometrically to  $p[\boldsymbol{\theta} | \mathbf{Y}]$ , as  $n \rightarrow \infty$ .
- $\frac{1}{N} \sum_{n=1}^N f(\boldsymbol{\theta}^{(n)}) \xrightarrow{a.s.} \int f(\boldsymbol{\theta}) p[\boldsymbol{\theta} | \mathbf{Y}] d\boldsymbol{\theta}$ , as  $n \rightarrow \infty$ , for any integrable function  $f$ .

### 3.2. Prior Distributions

The unknown quantities in this case are  $(\mathbf{H}, \Sigma^{-1}, \mathbf{B})$ , which are assumed to be independent with each other. Next, we specify the prior distributions.

1. For the unknown channel  $h_k$ , a complex Gaussian prior distribution is assumed,

$$p[h_k] \sim \mathcal{N}_c(h_{k0}, \Sigma_{k0}). \quad (9)$$

Note that large value of  $\Sigma_{k0}$  corresponds to less informative prior.

2. For the inverse of noise covariance matrix  $\Sigma^{-1}$ , an complex Wishart distribution [5] is assumed, i.e.,

$$p[\Sigma^{-1}] \sim \mathcal{W}_c(\Psi^{-1}, m), \quad (10)$$

Small values of  $m$  and  $\Psi$  correspond to less informative prior. According to [5], a random matrix with a Wishart distribution with  $m$  degrees of freedom (10) can be generated by  $\sum_{i=1}^{m+1} \mathbf{u}_i \mathbf{u}_i^H$ , where  $\{\mathbf{u}_i\}$  are i.i.d. Gaussian random vectors with zero mean and covariance  $\Psi^{-1}$ .

3. The data bit sequence  $b_k$  is a Markov chain, encoded from  $\{a_k(i)\}_{i=1}^{M-1}$ . Its prior distribution can be expressed easily from the prior distribution of  $a_k(i)$ .

### 3.3. Conditional Posterior Distributions

The following conditional posterior distributions are required by the blind Bayesian multiuser detector.

1. The conditional distribution of the  $k$ th user's channel response  $h_k$  given  $\Sigma^{-1}, \mathbf{B}, \mathbf{H}_k$ , and  $\mathbf{Y}$  is [where  $\mathbf{H}_k \triangleq \mathbf{H} \setminus h_k$ .]

$$p[h_k | \mathbf{B}, \Sigma^{-1}, \mathbf{H}_k, \mathbf{Y}] \sim \mathcal{N}_c(h_{k*}, \Sigma_{k*}), \quad (11)$$

$$\text{with } \Sigma_{k*}^{-1} \triangleq \Sigma_{k0}^{-1} + \sum_{i=0}^{M-1} \mathbf{S}_{k,i}^H(b_k) \Sigma^{-1} \mathbf{S}_{k,i}(b_k),$$

$$\text{and } h_{k*} \triangleq \Sigma_{k*} [\Sigma_{k0}^{-1} h_{k0} + \sum_{i=0}^{M-1} \mathbf{S}_{k,i}(b_k) \Sigma^{-1} (r(i) - \sum_{j \neq k} \mathbf{S}_{j,i}(b_j) h_j)].$$

2. The conditional distribution of the inverse of noise covariance matrix  $\Sigma^{-1}$  given  $\mathbf{H}$ ,  $\mathbf{B}$ , and  $\mathbf{Y}$  is

$$p[\Sigma^{-1} | \mathbf{H}, \mathbf{B}, \mathbf{Y}] \sim \mathcal{W}_c((\Psi + \mathbf{Q})^{-1}, m + M), \quad (12)$$

$$\text{with } \mathbf{Q} \triangleq \sum_{i=0}^{M-1} (r(i) - \sum_{k=1}^K \mathbf{S}_{k,i}(b_k) h_k) (r(i) - \sum_{k=1}^K \mathbf{S}_{k,i}(b_k) h_k)^H.$$

3. The conditional distribution of the data bit  $b_k(i)$  given  $\mathbf{H}$ ,  $\Sigma^{-1}$ ,  $\mathbf{B}_{ki}$ , and  $\mathbf{Y}$  can be obtained from [where  $\mathbf{B}_{ki} \triangleq \mathbf{B} \setminus b_k(i)$ .]

$$\begin{aligned} & \frac{P[b_k(i) = +1 | \mathbf{H}, \Sigma^{-1}, \mathbf{B}_{ki}, \mathbf{Y}]}{P[b_k(i) = -1 | \mathbf{H}, \Sigma^{-1}, \mathbf{B}_{ki}, \mathbf{Y}]} \\ &= \exp[b_k(i+1) \rho_k(i+1) + b_k(i-1) \rho_k(i) - \text{tr}(\Delta \mathbf{Q} \cdot \Sigma^{-1})], \end{aligned} \quad (13)$$

with

$$\begin{aligned} \Delta \mathbf{Q} &\triangleq -4\mathcal{R}\{\mathbf{r}_k(i) \mathbf{h}_k^H \mathbf{C}_{k,i}^{(0)T} + \mathbf{r}_k(i+1) \mathbf{h}_k^H \mathbf{C}_{k,i+1}^{(1)T} + \mathbf{r}_k(i+2) \mathbf{h}_k^H \mathbf{C}_{k,i+2}^{(2)T}\} \\ \mathbf{r}_k(l) &\triangleq \mathbf{r}(l) - \sum_{j \neq k} \mathbf{S}_{j,l}(b_j) h_j - \mathbf{S}_{k,l}(b_{k,l}^0) h_k, \\ b_{k,l}^0 &\triangleq \{b_k \text{ with } b_k(l) = 0\}. \end{aligned}$$

### 3.4. Gibbs Procedure

Using the above conditional posterior distributions, the Gibbs sampling implementation of the blind Bayesian multiuser detector proceeds iteratively as follows. Given the initial values of the unknown quantities  $\{\mathbf{H}^{(0)}, \Sigma^{-1(0)}, \mathbf{B}^{(0)}\}$  drawn from their prior distributions, and for  $n = 1, 2, \dots$

1. Draw  $h_k^{(n)}$  from  $p[h_k | \mathbf{H}_k^{(n-1)}, \Sigma^{-1(n-1)}, \mathbf{B}^{(n-1)}, \mathbf{Y}]$  given by (11);
2. Draw  $\Sigma^{-1(n)}$  from  $p[\Sigma^{-1} | \mathbf{H}^{(n)}, \mathbf{B}^{(n-1)}, \mathbf{Y}]$  given by (12);

3. For  $i = 0, 1, \dots, M-1$

For  $k = 1, 2, \dots, K$

Draw  $b_k^{(n)}(i)$  from  $P[b_k(i) | \mathbf{H}^{(n)}, \Sigma^{-1(n)}, \mathbf{B}_{ki}^{(n-1)}, \mathbf{Y}]$  given by (13).

To ensure convergence, the above procedure is usually carried out for  $(n_0 + N)$  iterations and samples from the last  $N$  iterations are used to calculate the Bayesian estimates for unknown parameters. By the second convergence property mentioned before, we get

$$E[a_k(i) | \mathbf{Y}] \cong \frac{1}{N} \sum_{n=n_0+1}^{n_0+N} b_k^{(n)}(i) b_k^{(n)}(i-1). \quad (14)$$

The posterior distribution  $p[a_k(i) = +1 | \mathbf{Y}]$  can be easily computed from (14).

## 4. BLIND TURBO MULTIUSER DETECTOR

We consider employing iterative joint multiuser detection and decoding to improve the performance of the blind Bayesian multiuser detector in a coded CDMA system. Because it utilizes the *a priori* symbol probabilities, and it produces symbol *a posteriori* probabilities, the blind Bayesian multiuser detector developed in this paper is well suited for iterative (Turbo) processing.

The Turbo receiver consists of two stages: the blind Bayesian multiuser detector followed by a soft-input soft-output channel decoder [6]. The two stages are separated by deinterleavers and interleavers. A receiver structure is shown in Figure 1. The extrinsic information provided by multiuser detector is reverse interleaved and fed into the channel decoder as prior information. The extrinsic information delivered by decoders is interleaved and fed back to the multiuser detector as prior information for the next iteration.

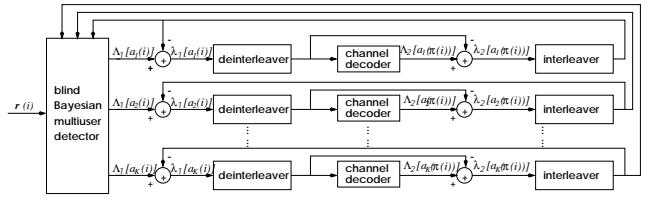
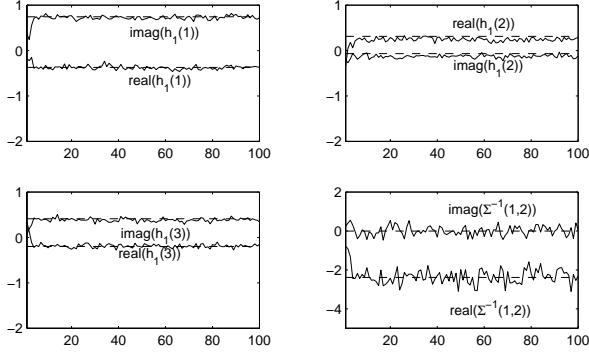


Fig. 1. Turbo multiuser receiver.

## 5. SIMULATION RESULTS

We consider a CDMA system with processing gain  $P = 10$ . The long spreading sequences of all users are generated randomly. The data block size for each user is  $M = 101$ ; the number of path for each user is  $L = 3$ ; the transmitter delay  $\iota_k$  is generated randomly with the restriction  $\iota_k < P$ . For each data block, the Gibbs sampling is performed for 100 iterations, with the first 50 iterations as the “burning-in” period, i.e.,  $n_0 = N = 50$  in (14). The out-cell multiple-access interference (OMAI) and narrowband interference (NBI) is simulated as interference. For convenience, we denote SNR as the in-cell user signal to WGN ratio; denote SIR as the in-cell user signal to NBI ratio; denote  $K$  and  $K'$  as the number of in-cell and out-cell users respectively. The NBI is modeled as a 2nd order AR model with coefficients  $(1.8, -0.81)$ . The OMAI is generated with energy 12dB below the in-cell user.



**Fig. 2.** Samples drawn by the Gibbs sampler with  $K = 3$ ,  $K' = 18$ ; for each in-cell user, SNR = 20dB and SIR = -15dB.

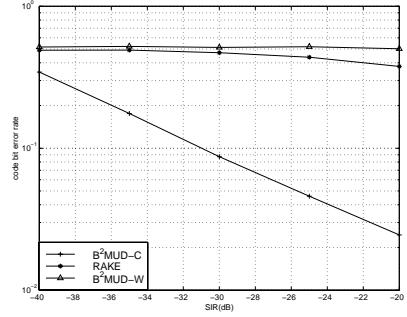
1) *Convergence behavior of the Gibbs sampler:* In Figure 2, we plot the first 100 samples drawn by the Gibbs sampler of  $h_1$  and  $\Sigma^{-1}(1, 2)$ . The corresponding true values of  $-h_1$  and  $\Sigma^{-1}(1, 2)$  are also shown in the same figure with dashed lines. The Gibbs sampler reaches convergence within the first several iterations. The channel response samples converges to  $-h_k$  or  $h_k$  randomly due to the phase ambiguity. It is seen that  $\Sigma^{-1}(1, 2)$  is far from 0, which indicates that the noise covariance matrix is not diagonal any more with the existence of OMAI and NBI.

2) *Performance of the blind Bayesian multiuser detector:* Figure 3 compare the performance of the proposed the blind Bayesian multiuser detector assuming colored Gaussian noise ( $B^2$ MUD-C) with other two detection scheme: RAKE receiver with perfect channel knowledge; and the blind Bayesian multiuser detector assuming white Gaussian noise ( $B^2$ MUD-W) derived in [2]. The bit error rate of  $\{a_k(i)\}$  is plotted with the change of SIR for the three receiver scheme. It is seen that with highly correlated noise, both RAKE receiver and  $B^2$ MUD-W fail to work, but  $B^2$ MUD-C can achieve very good performance.

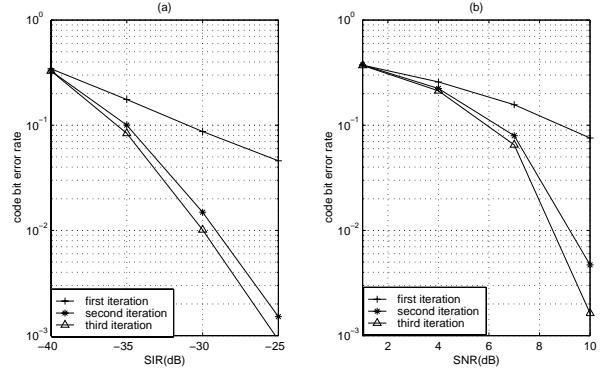
3) *Performance of Turbo multiuser receiver:* In a coded CDMA system, the channel code is set to a rate of  $\frac{1}{2}$ , constraint length-5 convolutional code (with generators 23, 35 in octal notation). The interleaver is generated randomly and fixed for all simulations. The bit error rate of the code bits  $\{a_k(i)\}$  at the output of the multiuser detector is averaged among all the users, and is plotted for the first three iterations in Figure 4. It is seen that by incorporating the extrinsic information provided by the channel decoders, the proposed blind Bayesian multiuser detector achieves significant performance improvement by the Turbo procedure.

## 6. CONCLUSION

We have proposed a blind Turbo multiuser receiver for asynchronous CDMA system employing long spreading sequences in the presence of unknown out-cell multiple-access interference and narrowband interference. A novel blind Bayesian multiuser detector is developed for joint multiuser detection and differential decoding, which is “soft-input soft-output” in nature to fit into the proposed Turbo multiuser receiver framework. Finally, we have provided simulation examples to demonstrate the effectiveness of the proposed techniques.



**Fig. 3.** Performance of blind Bayesian multiuser detector in colored Gaussian noise with  $K = 3$ ,  $K' = 18$  and fixed SNR = 15dB for all in-cell users.



**Fig. 4.** Performance of blind Turbo multiuser receiver with  $K = 3$ ,  $K' = 18$ , (a) with fixed SNR = 15dB for all in-cell users, (b) with fixed SIR = -20dB for all in-cell users.

## 7. REFERENCES

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