A SECOND-ORDER METHOD FOR BLIND SEPARATION OF NON-STATIONARY SOURCES

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ABSTRACT

The question addressed in this paper is whether and under what conditions blind source separation is possible using only second-order statistics. It is well known that for stationary, i.i.d. sources the answer is negative due to the inherent unitary matrix ambiguity of output second-order information. It is shown in this paper however, that if the sources' power is allowed to vary with time, unique identifiability can be achieved without resorting to higher order statistics. In many applications the sources' power does change with time (e.g., speech or fading communication signals), and therefore the result has practical relevance. A novel second-order source separation method is proposed based on a generalized eigen-decomposition of appropriate correlation matrices and the identifiability conditions are investigated. Asymptotic performance results for the output SIR are developed.

1. INTRODUCTION

The use of sensor array to separate a desired signal from unwanted interferers has received considerable attention. Recently, there has been a renewed interest in it driven by applications in wireless and cellular communications. The classical assumption to simplify the problem that the array manifold is parameterized by the direction of arrivals (DOAs) is less useful for communication applications due to multipath and angle spread effects. Instead, a more generic *mixing matrix* formulation is preferred.

When training data are available, the problem is simplified to standard MMSE filtering concepts. In the absence of training data however, the statistical characteristics of the received signal have to be exploited to achieve *blind* signal separation. For stationary sources, the common blind approaches are (explicitly or implicitly) based on the signal's higher (than second) order statistics (HOS) (e.g., [3]) and assumes non-Gaussian signals. The question of whether second-order based blind methods are applicable is still a valid one, considering the success of such methods in the case of SIMO systems (e.g., [8]). Second-order methods may have the added benefit of requiring shorter data records than HOS-based methods.

When the sources are stationary, i.i.d. (in space and time) it is well known that second-order methods suffer from a unitary matrix ambiguity and cannot successfully separate the signals [1]. If the i.i.d. assumption (in time) is relaxed, it was shown in [2] that second-order based solutions are possible, by exploiting information present in correlation matrices of non-zero lags.

In this paper we maintain the i.i.d assumption and exploit certain non-stationarities in the source signals to achieve a second-order separation method. A typical non-stationarity of signals may arise from possible variations in the sources' power. Examples include communication signals propagating through a fading channel. Our method is based on generalized eigenanalysis of two correlation matrices obtained at different time periods. Under certain conditions it directly yields the zero-forcing solution.

The exploitation of nonstationarities of the signals in order to separate them attracts a lot interest recently. Related work includes [10], which dealt with "gated signals", i.e., situations where the signal of interest is present in one observation interval but absent in another. [7] generalized that work to the case where the signal of interest changes its power from one level to another. Both approaches assume stationary interferers and propose subspace solutions. The method of [7] uses generalized eigenanalysis and in that respect is closer to our approach. Our method however, considers the more general case where the interferers as well as the signal of interest are nonstationary. In that case it turns out that the subspace solution has a different interpretation from that in [7]. The same signal model has also been studied in [5] where an algorithm based on maximum-likelihood criterion was proposed. Other related work include [9, 11] which dealt with the problem of detecting a CDMA signal using the data obtained before and after the onset of its transmission. The exploitation of non-stationarities in the source signals in order to separate them is not entirely new.

2. PROBLEM STATEMENT

We consider a narrowband mixing problem of the form

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) \ , \tag{1}$$

where the received vector $\mathbf{x}(n)$ consists of the output of L sensors, the source vector $\mathbf{s}(n) = [s_1(n), \ldots, s_K(n)]^T$ contains K sources, and $\mathbf{A}$ is the $L \times K$ mixing matrix. Due to the lack of space only results for the noiseless case will be presented here. We focus on the case where the sources are non-stationary. Two observation intervals are taken into account,

$$\begin{array}{rcll} \mathbf{x}_1(n) & = & \mathbf{A}\mathbf{s}_1(n), & n = M_1, \ldots, N_1 + M_1 - 1 \ , \\ \mathbf{x}_2(n) & = & \mathbf{A}\mathbf{s}_2(n), & n = M_2, \ldots, N_2 + M_2 - 1 \ , \end{array} \tag{2}$$

where $\mathbf{x}_1(n)$, $\mathbf{x}_2(n)$, $\mathbf{s}_1(n)$, and $\mathbf{s}_2(n)$ are segments of $\mathbf{x}(n)$ and $\mathbf{s}(n)$ in the two intervals, respectively. We assume that in each interval the sources are stationary but the sources' power may be different resulting in a second-order non-stationarity. Notice that the array response is assumed unchanged through the observation period. We further make the following assumptions with regard to (2):

(AS1) $L \geq K$ (no less sensors than sources);

(AS2) $\mathbf{s}_1(n)$ and $\mathbf{s}_2(n)$ are spatially independent, temporally i.i.d., zero-mean;

(AS3) The mixing matrix $\mathbf{A}$ is full column-rank.

Given the model (2) and the above assumptions, second-order information of the output signal is limited to the two correlation matrices,

$$\begin{array}{rclcl} \mathbf{R}_1 & = & \mathrm{E}[\mathbf{x}_1(n)\mathbf{x}_1^H(n)] & = & \mathbf{A}\mathbf{D}_1\mathbf{A}^H \ , \\ \mathbf{R}_2 & = & \mathrm{E}[\mathbf{x}_2(n)\mathbf{x}_2^H(n)] & = & \mathbf{A}\mathbf{D}_2\mathbf{A}^H \ , \end{array} \quad (3)$$

where $\mathbf{D}_1 = diag\{\sigma_{11}^2, \dots, \sigma_{K1}^2\}$, $\mathbf{D}_2 = diag\{\sigma_{12}^2, \dots, \sigma_{K2}^2\}$, and σ_{i1}^2 and σ_{i2}^2 are the power of $\mathbf{s}_i(n)$ in the two intervals, respectively. If furthermore, the sources are Gaussian, no further output statistical information can be obtained.

3. SEPARATION OF NON-STATIONARY SOURCES USING SECOND-ORDER STATISTICS

3.1. A Zero-forcing Solution

Let us consider the generalized eigendecomposition of $(\mathbf{R}_1, \mathbf{R}_2)$, i.e., to find $(\lambda, \mathbf{g})$ such that

$$(\mathbf{R}_1 - \lambda\mathbf{R}_2)\mathbf{g} = \mathbf{0} \ . \quad (4)$$

Notice that $\mathbf{R}_1$ and $\mathbf{R}_2$ share identical signal and noise subspace because of their structure (see (3)). The $L-K$ noise eigenvectors satisfy (4) as they are orthogonal to the columns of $\mathbf{A}$. Therefore, they are also the generalized eigenvectors of $(\mathbf{R}_1, \mathbf{R}_2)$ with arbitrary eigenvalues. Of course, if $L = K$ no noise eigenvectors exist.

Of more interest are the other K generalized eigenvectors that lie in the signal subspace. It will be instructive to first make clear the following notation. Let $\mathbf{a}_i$ be the ith column of $\mathbf{A}$ (i.e., $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_K]$) representing the signature of the ith source. Let further $\mathbf{A}_i = [\mathbf{a}_1, \dots, \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, \dots, \mathbf{a}_K]$ be the collection of the signatures of interferers to the ith source. The column space of $\mathbf{A}_i$ is the *interference subspace* to the ith source. Then define $\mathbf{\Pi}_i = \mathbf{A}_i(\mathbf{A}_i^H\mathbf{A}_i)^{-1}\mathbf{A}_i^H$ to be the projection matrix onto the interference subspace and $\mathbf{\Pi}_i^{\perp} = \mathbf{I} - \mathbf{\Pi}_i$ to be the projection matrix onto the orthogonal subspace correspondingly.

With these conventions we are ready to present our first result.

Proposition 1 *Let $\mathbf{R}_1$ and $\mathbf{R}_2$ be given by (3) and assume*

(AS4) *(Identifiability)* $\frac{\sigma_{k1}^2}{\sigma_{k2}^2} \neq \frac{\sigma_{l1}^2}{\sigma_{l2}^2}$, $\forall k, \ l = 1, \dots, K$ *and* $k \neq l$.

Then $\lambda_i = \frac{\sigma_{i1}^2}{\sigma_{i2}^2}$, $\mathbf{g}_i = \frac{\mathbf{\Pi}_i^{\perp}\mathbf{a}_i}{\mathbf{a}_i^H\mathbf{\Pi}_i^{\perp}\mathbf{a}_i}$, $i = 1, \dots, K$, *satisfy (4).* □

Before proceeding to the proof, we want to indicate that $\mathbf{g}_i$ falls into the orthogonal complement of the interference subspace to the ith source. It can be easily verified that[1]. $\mathbf{g}_i^H\mathbf{a}_k = \delta(i-k)$. Therefore, $\mathbf{g}_i$ represents a zero-forcing beamformer for the ith source, $\mathbf{g}_i^H\mathbf{x}(n) = \mathbf{g}_i^H\sum_k \mathbf{a}_k s_k(n) = s_i(n)$.

Proof: Rewrite (4) as $\sum_{i=1}^K(\sigma_{i1}^2 - \lambda\sigma_{i2}^2)\mathbf{a}_i\mathbf{a}_i^H\mathbf{g} = 0$. Using the fact that $\mathbf{a}_k^H\mathbf{g}_i = 0, \ \forall k \neq i$, then we obtain $(\sigma_{i1}^2 - \lambda_i\sigma_{i2}^2)\mathbf{a}_i\mathbf{a}_i^H\mathbf{g}_i = 0$ which is satisfied with $\lambda_i = \sigma_{i1}^2/\sigma_{i2}^2$. □

A remark on the second-order source separation method based on Proposition 1 is appropriate on the topic of distinguishing those zero-forcing eigenvectores (one for each source) from the noise eigenvectors. It is impossible to resort to the eigenvalues as is generally used because the eigenvalues corresponding to the noise eigenvectors can assume any value. This can be achieved by noting that zero-forcing eigenvectors are in the signal subspace of the received signal while the noise eigenvectors are in the noise subspace. Therefore, the distances of the generalized eigenvectors to the two orthogonal complementary subspaces provides information to distinguish them.

[1] $\delta(\cdot)$ is the Kronecker delta function.

3.2. More on Identifiability

Proposition 1 exploits the sources' power change in order to separate them. It also states the identifiability condition that no two sources change their power by the same proportion (see (AS 4)). It is natural therefore to investigate what happens when two or more sources violate (AS 4). The answer is given by the following result.

Proposition 2 *Let without loss of generality the first K' sources violate (AS 4), i.e., $\sigma_{l1}^2/\sigma_{l2}^2 = \lambda$, $l = 1, \dots, K'$, $K' \leq K$. Then λ is a generalized eigenvalue of $(\mathbf{R}_1,\mathbf{R}_2)$ with multiplicity K' and the corresponding eigenvectors lie in the space spanned by $\{\mathbf{\Pi}_{K'+1:K}^{\perp}\mathbf{a}_1, \dots, \mathbf{\Pi}_{K'+1:K}^{\perp}\mathbf{a}_L\}$, where $\mathbf{\Pi}_{K'+1:K}^{\perp} = \mathbf{I} - \mathbf{\Pi}_{K'+1:K}$ and $\mathbf{\Pi}_{K'+1:K} = \mathbf{A}(K'+1:K)[\mathbf{A}^H(K'+1:K)\mathbf{A}(K'+1:K)]^{-1}\mathbf{A}^H(K'+1:K)$ is the projection matrix onto the subspace $\{\mathbf{a}_{K'+1}, \dots, \mathbf{a}_K\}$.* □

The proof is along the same lines as that of Propostion 1.

Propositon 2 indicates that a group of sources with proportionally equal power change can be separated from the rest, but cannot be separated from each other. On the other hand the existence of such a group does not inhibit the separation of other sources, the associated eigenvalues of which have multiplicity one.

A special case of the above remark is the situation where a group of L sources remains stationary (i.e., $\sigma_{l1}^2/\sigma_{l2}^2 = 1$) while the rest change their power. The method can separate the non-stationary sources but not the stationary ones. Finally, the case where all but one source remain stationary is the setup addressed in [10, 7, 9, 11].

4. PERFORMANCE ANALYSIS

In this section, we are going to analyze the performance of the proposed method when the two correlation matrics can only be estimated from average of finite samples. We will use the output siganl-to-interference-ratio (SIR) as the figure of merit.

Let us consider the sample average estimates $\hat{R}_1 = \frac{1}{N_1}\sum_{n=M_1}^{N_1-M_1+1}\mathbf{x}_1(n)\mathbf{x}_1^H(n)$, $\hat{R}_2 = \frac{1}{N_2}\sum_{n=M_2}^{N_2-M_2+1}\mathbf{x}_2(n)\mathbf{x}_2^H(n)$, and the associated generalized eigenvectors $\hat{\mathbf{g}}_i$ and eigenvalues $\hat{\lambda}_i$, $i = 1, \dots, L$. According the Proposition 1, the ith source is extracted by filtering the received signal with a vector matched to $\hat{\mathbf{g}}_i$, i.e., $\hat{\mathbf{s}}_i(n) = \hat{\mathbf{g}}_i^H\mathbf{x}(n)$. Then the output SIR of the ith source in the first interval is given by

$$\mathrm{SIR}_{i1} = \frac{\sigma_{i1}^2\mathbf{a}_i^H\mathrm{E}[\hat{\mathbf{g}}_i\hat{\mathbf{g}}_i^H]\mathbf{a}_i}{\sum_{j\neq i}\sigma_{j1}^2\mathbf{a}_j^H\mathrm{E}[\hat{\mathbf{g}}_i\hat{\mathbf{g}}_i^H]\mathbf{a}_j} \ . \quad (5)$$

Similarly, we can define SIR_{i2} for the second interval. SIR_{i1} and SIR_{i2} may be different due to the power change of the sources. However, the asymptotic performance solely depends on the correlation matrix $\mathrm{E}[\hat{\mathbf{g}}_i\hat{\mathbf{g}}_i^H]$, hence the procedure and results of the analysis will apply to both SIR_{i1} and SIR_{i2}.

Suppose the estimate $\hat{\mathbf{g}}_i$ deviates from the real $\mathbf{g}_i$ by a small zero-mean error $\Delta\mathbf{g}_i = \hat{\mathbf{g}}_i - \mathbf{g}_i$, due to imperfect estimation. Then substituting $\hat{\mathbf{g}}_i = \mathbf{g}_i + \Delta\mathbf{g}_i$ into (5), we get

$$\mathrm{SIR}_{i1} = \frac{\sigma_{i1}^2\{1 + \mathbf{a}_i^H\mathrm{E}[\Delta\mathbf{g}_i\Delta\mathbf{g}_i^H]\mathbf{a}_i\}}{\sum_{j\neq i}\sigma_{j1}^2\mathbf{a}_j^H\mathrm{E}[\Delta\mathbf{g}_i\Delta\mathbf{g}_i^H]\mathbf{a}_j} \ , \quad (6)$$

where we have used the fact that $\Delta\mathbf{g}_i$ is zero-mean and $\mathbf{g}_i^H\mathbf{a}_j = \delta(i-j)$. In order to evaluate (6) the expression for the correlation matrix $\mathrm{E}[\Delta\mathbf{g}_i\Delta\mathbf{g}_i^H]$ is necessary. Next, we employ perturbation analysis tools to develop it.

Besides $\Delta\mathbf{g}_i$, let us further define the errors $\Delta\mathbf{R}_1 = \hat{\mathbf{R}}_1 - \mathbf{R}_1$, $\Delta\mathbf{R}_2 = \hat{\mathbf{R}}_2 - \mathbf{R}_2$ and $\Delta\lambda_i = \hat{\lambda}_i - \lambda_i$. Applying standard perturbation techniques on (4), we obtain the following first order approximations:

$$\Delta\lambda_i = \frac{\mathbf{g}_i^H(\Delta\mathbf{R}_1 - \lambda_i\Delta\mathbf{R}_2)\mathbf{g}_i}{\mathbf{g}_i^H\mathbf{R}_2\mathbf{g}_i} \quad (7)$$

and

$$(\mathbf{R}_1 - \lambda_i\mathbf{R}_2)\Delta\mathbf{g}_i = (\Delta\lambda_i\mathbf{R}_2 - \Delta\mathbf{R}_1 + \lambda_i\Delta\mathbf{R}_2)\mathbf{g}_i \ . \quad (8)$$

Since no additive noise was assumed, $\hat{\mathbf{R}}_i$ and $\mathbf{R}_i$, $i = 1, 2$, share identical noise subspaces (if the sources are sufficient rich). Therefore the noise eigenvectors can be perfectly estimated from finite data and (8) only applies to $\mathbf{g}_i$ in the signal subspace. Hence $\Delta\mathbf{g}_i$ is also in the signal subspace and (8) can be solved using $(\mathbf{R}_1 - \lambda\mathbf{R}_2)^\dagger$ where $\dagger$ denotes the pseudo-inverse.

A more useful version of (7) and (8) can be obtained if we apply the expressions of λ_i and $\mathbf{g}_i$ from Proposition 1, as well as the definitions of $\Delta\mathbf{R}_1$ and $\Delta\mathbf{R}_2$ and the model equations (2).

Lemma 1 *(7) and (8) can be expressed as*

$$\Delta\lambda_i = \frac{1}{\sigma_{i2}^2}(\hat{\sigma}_{i1}^2 - \lambda_i\hat{\sigma}_{i2}^2) \ , \quad (9)$$

and

$$\Delta\mathbf{g}_i = (\mathbf{R}_1 - \lambda_i\mathbf{R}_2)^\dagger[(\hat{\sigma}_{i1}^2 - \lambda_i\hat{\sigma}_{i2}^2)\mathbf{a}_i - \mathbf{A}(\hat{\mathbf{r}}_{i1} - \lambda_i\hat{\mathbf{r}}_{i2})] \ , \quad (10)$$

where $\hat{\sigma}_{i1} = \frac{1}{N_1}\sum_{n=M_1}^{N_1+M_1-1}|\mathbf{s}_{i1}(n)|^2$, $\hat{\mathbf{r}}_{i1} = \frac{1}{N_1}\sum_{n=M_1}^{N_1+M_1-1}s_{i1}^*(n)\mathbf{s}_1(n)$ *denote the sample source variance and cross-correlation vector respectively;* $\hat{\sigma}_{i2}$, $\hat{\mathbf{r}}_{i2}$ *are defined similarly.* □

The proof is straightforward though tedious.

According to (10), $\mathrm{E}[\Delta\mathbf{g}_i\Delta\mathbf{g}_i^H]$ has several auto- and cross-correlation terms to be evaluated. To avoid unnecessary complication and make the results conceptually clear, we assume that there is no overlap between the two observation intervals of $\mathbf{s}(n)$ so that $\mathbf{s}_1(n)$ and $\mathbf{s}_2(n)$ are independent. We further assume that $\mathbf{s}_1(n)$ and $\mathbf{s}_2(n)$ are Gaussian. Then the following results can be obtained.

Lemma 2 *If the sources are independent* **(AS2)** *and Gaussian, and the two segments of each source do not overlap, then* $\mathrm{E}[\hat{\sigma}_{i1}^4] = (1 + \frac{2}{N_1})\sigma_{i1}^4$, $\mathrm{E}[\hat{\sigma}_{i2}^4] = (1 + \frac{2}{N_2})\sigma_{i2}^4$, $\mathrm{E}[\hat{\sigma}_{i1}^2\hat{\sigma}_{i2}^2] = \sigma_{i1}^2\sigma_{i2}^2$, $\mathrm{E}[\hat{\mathbf{r}}_{i1}\hat{\sigma}_{i1}^2] = (1 + \frac{2}{N_1})\sigma_{i1}^4\mathbf{e}_i$, $\mathrm{E}[\hat{\mathbf{r}}_{i1}\hat{\sigma}_{i2}^2] = \sigma_{i1}^2\sigma_{i2}^2\mathbf{e}_i$, $\mathrm{E}[\hat{\mathbf{r}}_{i2}\hat{\sigma}_{i1}^2] = \sigma_{i1}^2\sigma_{i2}^2\mathbf{e}_i$, $\mathrm{E}[\hat{\mathbf{r}}_{i2}\hat{\sigma}_{i2}^2] = (1 + \frac{2}{N_2})\sigma_{i2}^4\mathbf{e}_i$, $\mathrm{E}[\hat{\mathbf{r}}_{i1}\hat{\mathbf{r}}_{i1}^H] = \frac{\sigma_{i1}^2}{N_1}\mathbf{D}_1 + (1 + \frac{2}{N_1})\sigma_{i1}^4\mathbf{e}_i\mathbf{e}_i^T$, $\mathrm{E}[\hat{\mathbf{r}}_{i2}\hat{\mathbf{r}}_{i2}^H] = \frac{\sigma_{i1}^2}{N_2}\mathbf{D}_2 + (1 + \frac{2}{N_2})\sigma_{i2}^4\mathbf{e}_i\mathbf{e}_i^T$, $\mathrm{E}[\hat{\mathbf{r}}_{i1}\hat{\mathbf{r}}_{i2}^H] = \sigma_{i1}^2\sigma_{i2}^2\mathbf{e}_i\mathbf{e}_i^T$, *where* $\mathbf{e}_i = [0, \ldots, 0, 1, 0, \ldots, 0]^T$ *has 1 in the* i^{th} *position and* $\mathbf{D}_1 = diag\{\sigma_{11}^2, \ldots, \sigma_{K1}^2\}$, $\mathbf{D}_2 = diag\{\sigma_{12}^2, \ldots, \sigma_{K2}^2\}$. □

The proof of Lemma 2 is also a straightforward procedure if we apply the Gaussian and independence assumption. Similar but more complicated expressions can be derived for the non-Gaussian case.

Finally, with the two lemmas, we have

$$\begin{aligned}\mathrm{E}[\Delta\mathbf{g}_i\Delta\mathbf{g}_i^H] \ &= \ \frac{\sigma_{i1}^2}{N}(\mathbf{R}_1 - \lambda_i\mathbf{R}_2)^\dagger(\mathbf{R}_1 + \lambda_i\mathbf{R}_2 - 2\sigma_{i1}^2\mathbf{a}_i\mathbf{a}_i^H) \\ &\quad (\mathbf{R}_1 - \lambda_i\mathbf{R}_2)^\dagger \ . \end{aligned} \quad (11)$$

In (11), we set $N_1 = N_2 = N$ for convenience. Substituting (11) into (6), we can see that the SIR increases linearly with respect to the data length. This effect will be verified by the simulations as explained next.

5. MODEL EXTENSION AND PARAFAC ANALYSIS

Before going to simulation results, we want to indicate an interesting extension of the signal model of (2). To exploit the nonstationarity of the source signals, (2) uses the data of two observation intervals. It is natural then to think of using more. Suppose that P intervals are considered,

$$\mathbf{x}_p(n) = \mathbf{A}\mathbf{s}_p(n), \ n = M_p, \ldots, N_p + M_p - 1, \ p = 1, \ldots P, \quad (12)$$

where $\mathbf{x}_p(n)$ and $\mathbf{s}_p(n)$, $p = 1, \ldots, P$ are the segments of $\mathbf{x}(n)$ and $\mathbf{s}(n)$ in the pth interval. Again, $\mathbf{s}_p(n)$'s are assumed spatially independent, temporally i.i.d., zero-mean stationary processes with different second-order statistics (due to power variation of each source signals). Then the second-order statistical information is provided by the P correlation matrices of the output signal,

$$\mathbf{R}_p = \mathrm{E}[\mathbf{x}_p(n)\mathbf{x}_p^H(n)] = \mathbf{A}\mathbf{D}_p\mathbf{A}^H \ , \ \ p = 1, \ldots, P \ , \quad (13)$$

where $\mathbf{D}_p = diag\{\sigma_{1p}^2, \ldots, \sigma_{Kp}^2\}$ and σ_{ip}^2 is the power of the ith source in the pth interval. σ_{ip}^2 should be different for different i and p, according to the nonstationarity assumption. The question now is how to obtain $\mathbf{s}_p(n)$'s based on (13).

Let us write (12) down to the scalar form

$$r_{ijp} = \sum_{k=1}^{K}\alpha_{ik}\sigma_{kp}^2\alpha_{kj}^* \ , \ \ i, j = 1, \ldots, K, \ p = 1, \ldots, P, \quad (14)$$

where r_{ijp} is the (i, j)th element of $\mathbf{R}_p$, α_{ik} is the (i, k)th element of $\mathbf{A}$, and σ_{kp}^2 is the kth diagonal element of $\mathbf{D}_p$. (14) is a K-component *trilinear decomposition* of the *three-way* $M \times M \times P$ array with typical element x_{ijp}, being special in that there are two dimensions sharing the same entry set. Trilinear decomposition is usually referred as PARAllel FACtor (PARAFAC) analysis, a tool recently introduced into the communications and signal processing fields [6]. Under some mild conditions, the PARAFAC model is unique, i.e., $\mathbf{A}$ and $\mathbf{D}_p$ can be uniquely identified (modulo inherent permutation and scalar of the columns) from $\mathbf{R}_p$. The identifiability is actually a corollary of uniqueness of low-rank decomposition of three-way arrays.

To solve the PARAFAC model, the alternating least-square (ALS) fitting algorithm is usually used. The ALS algorithm updates one of the estimate $\hat{\mathbf{A}}$ or $\hat{\mathcal{D}}$ ($\mathcal{D} = [\sigma_{kp}^2]_{K\times P}$), each at a time given the initial estimate of the other, in the sense of least-square fitting to the data $[r_{ijp}]_{M\times M\times P}$. It resorts to the symmetry of the PARAFAC model of (14) to implement the round-robin updating.

Once the estimate of $\mathbf{A}$ is available, the estimation of $\mathbf{s}_p(n)$ from (12) becomes a simple LS-inverse problem.

6. SIMULATIONS

The simulation is performed in the scenario of using 2 antennas to separate 2 BPSK signals transmitted through Rayleigh fading channels with maximum normalized Doppler frequency of $\frac{\text{bit rate}}{400}$. The steering (mixing) matrix of the antenna array is fixed, $A = [0.30.87; 0.660.15]$. The noise is AWGN.

A block of length N of data are used in each experiment. They are broken into two equal length non-overlapping segments to be used in the algorithm. Fig. 1 and Fig. 2 show the output SINR versus the data length N for the two pieces of the two signals. Fig. 1 is the noiseless case and Fig. 2 the noisy. Comparing the output SINR of each source in each piece with their respective input SINR (given under the plots), we can see the excellent performance of the proposed technique in interference cancellation. Fig. 1 also shows the consistency of the experiment and theoretic results; while in Fig. 2 the noisy results is compared with the noiseless ones (copied from Fig. 1) to show the impact of the presence of noise on the output SINR. This result is more clearly shown in Fig. 3 which gives the output SINR versus input SNR for each sources in each interval. The output SINR there is obtained when 400 data points are used so that the asymptotic (with respect to N) value is achieved. We can see that the output SINR linearly increases with the input SNR.

7. CONCLUSIONS

In this work, a method of blind source separation using only second-order statistics is proposed. This method assumes power variations of the sources to provide identifiability. Also, the method provides zero-forcing beamforming vectors directly in a single step. A limitation of the proposed approach is its inability to handle additive noise. Adaptations of this framework to the noisy case is an interesting future research topic.

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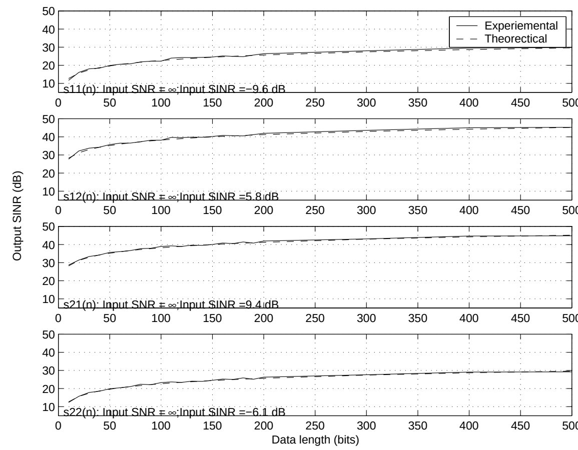

Figure 1. Ouput SINR vs Data length in noiseless cases

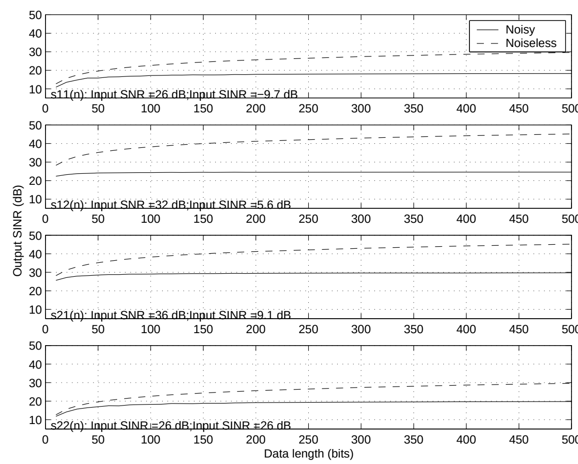

Figure 2. Ouput SINR vs Data length in noisy cases

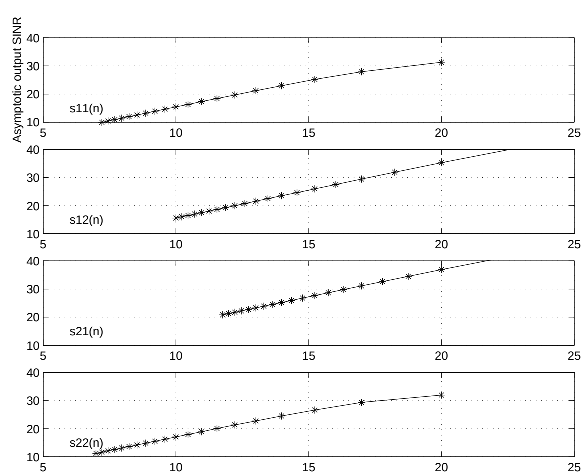

Figure 3. Effects of additive noise on performance input SNR