AVERAGING BLIND SIGN ALGORITHMS FOR ADAPTIVE MULTIUSER DETECTION

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ABSTRACT

This paper illustrates the use of "averaging" to improve the convergence rate of adaptive sign regressor and sign error multiuser detectors. The ingenious concept of averaging was invented by Polyak in 1990 – this paper analyses the performance of averaging in the sign error and sign regressor adaptive blind multiuser detection algorithms in DS/CDMA Systems.

1. INTRODUCTION

Demodulating a given user in a DS/CDMA network requires processing the received signal to minimize two types of interference, namely, narrow-band interference (NBI) and wide-band multiple access interference (MAI) caused by other spread-spectrum users in the channel–as well as ambient channel noise. Recently, *blind multiuser detection* techniques [4], [8] have been developed that allow one to use a linear multiuser detector for a given user, with no knowledge beyond that required for implementation of the conventional detector for that user. Blind multiuser detection is useful in mobile wireless channels when the desired user can experience a deep fade or if a strong interferer suddenly appears. In [4] a blind least mean square (LMS) algorithm is given for linear minimum mean square error (MMSE) detection. In [8] a code-aided blind recursive least squares algorithm for jointly suppressing MAI and NBI is given together with convergence analysis.

The main idea in this paper is to derive and analyse accelerated convergent low complexity adaptive sign algorithms for joint MAI and NBI suppression. Our contributions are twofold:

1. We derive sign-error and sign-regressor based multiuser detection algorithms. We show how the convergence of the sign-error and sign-regressor blind multiuser detectors can be accelerated using the ingenious procedure of averaging invented by Polyak in 1990, see also [6].

The ingenious idea behind this approach is to introduce a second round of averaging on the sign algorithm. For the LMS algorithm based blind multiuser detector it has recently been shown that averaging results in asymptotic convergence rate and performance identical to recursive least squares (RLS) based multiuser detectors [5]. In this paper we show that averaging also yields substantial improvements for the sign-error and sign-regressor algorithms in blind multiuser detection.

2. Expressions are derived for the asymptotic excess mean square error and the signal to interference ratio of these averaged signed algorithms. Numerical studies are presented which illustrate the performance of the averaged algorithms.

Sign error and sign regressor algorithms have been studied extensively [2], [1]. Their main advantage is the low complexity in implementation compared to the LMS algorithm. To our knowledge their use in adaptive multiuser detection and accelerating their convergence via averaging have not been studied.

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2. DS/CDMA SIGNAL MODEL

Consider a synchronous K-user binary DS/CDMA communication system. After the received continuous-time signal is preprocessing and sampled at the CDMA receiver the resulting discrete-time received signal at time n, denoted by r_n, is given by

$$r_n = \sum_{k=1}^{K} \sqrt{P_k} b_k(n) s_k + \varsigma_n + \sigma \varpi_n. \tag{2.1}$$

Here r_n is an N-dimensional vector where N denotes the spreading gain; s_k is the normalized signature sequence of the kth user ($s_k' s_k = 1$); the iid ± 1 sequence $b_k(n)$ denotes the transmitted data bit of the user k at time n; P_k is the received power of the kth user; ς_n is the NBI signal N-vector, which is assumed to be a wide-sense stationary autoregressive process with mean zero and covariance matrix R_ς; σ is the standard deviation of the channel noise; and ϖ_n is a white Gaussian vector with mean zero and covariance matrix I, where I denotes the $N \times N$ identity matrix.

Assume that user 1 is the user of interest. A linear blind multiuser detector demodulates the bits of user 1 according to $\widehat{b}_1(n) = \text{sgn}(c_*' r_n)$ where $\widehat{b}_1(n)$ denotes the estimate of the transmitted bit $b_1(n)$ at time n, and the "weight vector" c_* is chosen to minimize the Mean output error (MOE) cost function

$$\zeta_n \triangleq \mathbf{E}\{(c' r_n)^2\} \qquad \text{subject to the constraint} \quad c' s_1 = 1. \tag{2.2}$$

The blind MOE detector yields the following estimate $\widehat{b}_1(n)$ of the transmitted signal (see [8] for details)

$$\widehat{b}_1(n) = \text{sgn}(c_*' r_n) \quad \text{where} \quad c_* = \frac{R^{-1} s_1}{s_1' R^{-1} s_1} \tag{2.3}$$

Here $R = \mathbf{E}\{r r'\}$. In the above equation c_* is the optimal linear MOE "weight vector". The output signal-to-interference ratio (SIR) for an arbitrary weight vector c is defined as

$$\text{SIR} \triangleq \frac{P_1}{\sigma^2 c' c + \sum_{k=2}^{K} P_k (c' s_k)^2} \tag{2.4}$$

3. ADAPTIVE SIGN ALGORITHMS FOR BLIND MULTIUSER DETECTION

In *adaptive* blind multiuser detection problems, we are interested in recursively adapting the weight vector c_n to minimize the MOE ζ_n (2.2). It is necessary to use a constant step size tracking algorithm due to the time-varying nature of c^* caused by the birth and death of users (MAI interferers).

It is convenient to work with an unconstrained optimization problem rather than (2.2). Let $c_{n,i},\ i \in [1, ..., N]$ denote the elements of c_n. The constrained optimization problem (2.2) may be transformed into an unconstrained optimization problem by solving for one of the elements $c_{n,i},\ i \in [1, \dots, N]$ using the constraint (2.2). With no loss of generality, we solve for the first element $c_{n,1}$ and obtain

$$c_{n,1} = \frac{1}{s_{1,1}}\left(1 - \sum_{i=2}^{N} s_{1,i}c_{n,i}\right) \tag{3.5}$$

By defining the $(N-1)$-dimensional vector $\theta_n = (c_{n,2}, \dots, c_{n,N})'$, we obtain the equivalent unconstrained optimization problem:

$$\text{Compute } \min_{\theta} J_n \quad \text{where } J_n = \mathbf{E}\big(y_n - \theta'\varphi_n\big)^2 . \tag{3.6}$$

Here $y_n = -r_{n,1}/s_{1,1}$ and φ_n denotes the vector

$$\varphi_n = (r_{n,2} - r_{n,1}s_{1,2}/s_{1,1}, \dots, r_{n,N} - r_{n,1}s_{1,N}/s_{1,1})' \tag{3.7}$$

Let θ_* denote the MMSE solution $\theta_* = \{\mathbf{E}\{\varphi_n\varphi_n'\}^{-1}\mathbf{E}\{\varphi_n y_n\}$. It is straightforward but tedious to show that the components of θ_* are indeed the last $(N-1)$ elements of optimal weight vector c_* defined in (2.3).

We now present constant step-size versions of the sign-regressor and sign-error algorithms for blind adaptive multiuser detection.

Averaged Sign-regressor Blind Multiuser Detector

$$\begin{aligned} \theta^{\varepsilon}_{n+1} &= \theta^{\varepsilon}_n + \varepsilon \mathrm{Sgn}(\varphi_n)(y_n - \theta^{\varepsilon,\prime}_n\varphi_n) \\ \overline{\theta}_{n+1} &= (1-\rho_n)\overline{\theta}_n + \rho_n\theta^{\varepsilon}_n, \end{aligned} \tag{3.8}$$

where $0 < \rho_n < 1$ denotes a forgetting factor applied to the averaging procedure.

Averaged Sign-Error Blind Multiuser Detector

$$\begin{aligned} \theta^{\varepsilon}_{n+1} &= \theta^{\varepsilon}_n + \varepsilon\varphi_n \mathrm{sgn}(y_n - \theta^{\varepsilon,\prime}_n\varphi_n) \\ \overline{\theta}_{n+1} &= (1-\rho_n)\overline{\theta}_n + \rho_n\theta^{\varepsilon}_n, \end{aligned} \tag{3.9}$$

Note that because ς_n is a finite variance stationary Gaussian AR process and $\{b_k(n)\}$, ϖ_n are iid processes, it straightforwardly follows that $\{\varphi_n, y_n\}$ is a stationary mixing process. As a result it can be proved that the above algorithms converge in mean to the optimal weight vector c_* [2].

Canonical Coordinates. In [4] the constraint (2.2) is taken care of by introducing canonical coordinates. The essential idea is to replace the unconstrained gradient of the MOE in (2.2) by its component orthogonal to s_1 namely, $r_n'\,c_n\,(r_n - r_n's_1s_1)$. The blind averaged LMS and blind averaged sign-error algorithm can be expressed in canonical coordinates straightforwardly. However, it is not possible to derive a sign-regressor algorithm in canonical coordinates that satisfies the constraint (2.2). For the convergence analysis however, it is more convenient to work with the equivalent algorithm derived for the unconstrained cost function.

4. PERFORMANCE ANALYSIS OF AVERAGED ALGORITHMS

In this section expressions for the asymptotic excess mean square error of the averaged and un-averaged signed LMS algorithms are derived for the DS/CDMA signal model. In the sequel the following covariance matrices

$$D_{\varphi\varphi} = \mathbf{E}\{\varphi_n\varphi_n'\}, \qquad D_{\mathrm{SR}} = \mathbf{E}\{\mathrm{Sgn}(\varphi_n)\varphi_n'\} \tag{4.10}$$

will be used. As is common in the analysis of blind multiuser detection algorithms, we make the following simplifying assumptions, both of which hold for the DS/CDMA model.

(i) For slow adaptation, small ε, $\mathrm{Sgn}(\varphi_n)\varphi_n'$ behaves like its time or ensemble average D_{SR}. Similarly $\varphi_n\varphi_n' \sim D_{\varphi\varphi}$. This ergodicity is at heart of stochastic averaging results; see [7].

(ii) The input data r_n and the previous weight vector c_{n-1} are assumed to be statistically independent [3, Chapter 9]. This assumption is satisfied if the interference consist only of MAI and white noise.

(iii) Some fourth order statistics are approximated in terms of second order statistics [3, Chapter 9].

We first quote the following result in Ljung [7].

Result 4.1 (Lemma 3.1 and Eq.25, [7]) *Consider the following averaged constant step size algorithm with $0 < \rho, \mu < 1$,*

$$\theta_{n+1} = (1 - \varepsilon D)\theta_n + \varepsilon w_n \tag{4.11}$$

$$\overline{\theta}_{n+1} = (1-\rho)\overline{\theta}_n + \rho\theta_n. \tag{4.12}$$

Here either $D = CD_{\varphi\varphi}$ and $w_n = C\varphi_n y_n$ ($C > 0$ denotes a constant) or $D = D_{SR}$ and $w_n = \mathrm{Sgn}(\varphi_n)y_n$. Under the above assumptions, it follows that for large n,

$$\overline{\theta}_{n+1} = (1-\rho)\overline{\theta}_n + \rho D^{-1}\varphi_n y_n. \tag{4.13}$$

The main point to note is that (4.13) is independent of the step size μ. It only depends on ρ, the forgetting factor used in the averaging and on D. The proof for the case $D = C\mathbf{E}\{\varphi_n\varphi_n'\}$ and $w_n = C\varphi_n y_n$ is given in [7]. For the case $D = \mathbf{E}\{\mathrm{Sgn}(\varphi_n)\varphi_n'\}$ and $w_n = \mathrm{Sgn}(\varphi_n)y_n$, the proof is very similar and hence omitted.

Also we will need to the following principle of orthogonality for the MMSE solution θ_*: The estimation error e_n^* of the MMSE (Wiener) solution θ_* , i.e,

$$e_n^* = y_n - \varphi_n'\theta_* \tag{4.14}$$

is a zero mean iid process. Let $\sigma_e^2 \triangleq \mathbf{E}\{e_n^{*\,2}\}$.

Averaged LMS algorithm: The LMS algorithm can be written as

$$\theta_{n+1} = (I - \varepsilon\varphi_n\varphi_n')\theta_n + \varepsilon\varphi_n y_n$$

By assumption (i) above, this behaves for large n as

$$\theta_{n+1} = (I - \varepsilon D_{\varphi\varphi})\theta_n + \varepsilon\varphi_n y_n.$$

Using Result 4.1, the averaged estimate behaves for large n as

$$\overline{\theta}_{n+1} = (1-\rho)\overline{\theta}_n + \rho D_{\varphi\varphi}^{-1}\varphi_n y_n.$$

Finally using the principle of orthogonality (4.14) yields

$$\tilde{\theta}_{n+1} = (1-\rho)\tilde{\theta}_n + \rho D_{\varphi\varphi}^{-1}\varphi_n\varepsilon_n^*. \tag{4.15}$$

Averaged Sign-regressor Algorithm. Using the Result 4.1 and the principle of orthogonality (4.14) yields

$$\tilde{\theta}_{n+1} = (1-\rho)\tilde{\theta}_n + \rho D_{\text{SR}}^{-1}\text{Sgn}(\varphi_n)e_n^*. \qquad (4.16)$$

Averaged Sign-error Algorithm. Here we use the following additional assumptions that are widely used in the analysis of sign error algorithms [1]. These assumptions are that $\{\varphi_n, y_n\}$ is jointly Gaussian, and that $\mathbf{E}\{{e_n^*}^2|\theta_n\} = \mathbf{E}\{{e_n^*}^2\} = \sigma_e^2$. The Gaussian assumption on (φ_n, y_n) is valid if the number of users K is large. Indeed for large K the Central limit theorem implies that φ_n defined in (3.7) is a zero mean Gaussian random vector.

Under these approximations, [1, Eq.39] shows that θ_n behaves according to (4.11) with

$$C = \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_e(n)}, \qquad D_{\text{SE}} = CD_{\varphi\varphi}, \qquad w_n = \varphi_n y_n.$$

Using (4.13) this yields

$$\theta_{n+1} = (1-\rho)\theta_n + \rho D_{\varphi\varphi}^{-1}\theta_n y_n,$$

which is exactly the same as that for the averaged LMS algorithm. Using the orthogonality result (4.14) yields (4.15).

Remark 4.2. The above result which shows that the averaged LMS and averaged sign error algorithm behave similarly is somewhat surprising. However, one should keep in mind that several approximations were used to get [1, Eq.39]. Still, the above equation justifies numerical examples which show that the averaged LMS and averaged sign error algorithm perform similarly.

Weight Error Correlation Matrix.

Let $K_n \triangleq \mathbf{E}\{\tilde{\theta}_n\tilde{\theta}_n'\}$ denote the weight error correlation matrix. Note that K_n is an $(N-1)\times(N-1)$ positive definite matrix. To derive a recursion for K_n, we start with (4.15). As in [3, Chapter 9], we expand $\mathbf{E}\{\tilde{\theta}_n\tilde{\theta}_n'\}$. First consider the averaged LMS case (which is identical to the averaged sign error case as shown above). This involves the following four terms:
(i) $(1-\rho)^2\, E\{\tilde{\theta}_{n-1}\tilde{\theta}_{n-1}'\} = (1-\rho)^2 K_{n-1}$;
(ii) $\rho(1-\rho)\mathbf{E}\{\overline{\theta}_{n-1}'\Sigma^{-1}\overline{r}_n e_n\} = \rho(1-\rho)\mathbf{E}\{\tilde{\theta}_{n-1}'\}D^{-1}\mathbf{E}\{\varphi_n e_n^*\} = 0$ since e_n^* is a zero mean iid process;
(iii) $\rho(1-\rho)\mathbf{E}\{e_n\varphi_n{}'D^{-1}\tilde{\theta}_{n-1}\} = 0$, similarly to (ii);
(iv) $\rho^2 D^{-1}\mathbf{E}\{\varphi_n e_n^* e_n^* \varphi_n{}'\} = \rho^2\mathbf{E}\{{e_n^*}^2\}D^{-1}$. Recalling that $\overline{\zeta}$ is the MOE it follows that $\rho^2\Sigma^{-1}\mathbf{E}\{\varphi_n e_n^* e_n^*\varphi_n{}'\} = \rho^2\overline{\zeta}\Sigma^{-1}$.
Thus for large n, K_n satisfies the following recursion:

$$K_n = (1-\rho)^2 K_{n-1} + \rho^2\overline{\zeta}D_{\varphi\varphi}^{-1}. \qquad (4.17)$$

Since $(1-\rho)^2 < 1$, K_n converges to $K_\infty = \frac{\rho}{2-\rho}\overline{\zeta}D_{\varphi\varphi}^{-1}$.

Next consider the averaged sign regressor algorithm. Expanding out K_n into four terms as above, the fourth term now is $\rho^2\overline{\zeta}D_{\text{SR}}^{-1}\mathbf{E}\{\text{Sgn}(\varphi_n)\text{Sgn}(\varphi_n)'\}D_{\text{SR}}^{-1}{}'$. But $\mathbf{E}\{\text{Sgn}(\varphi_n)\text{Sgn}(\varphi_n)'\} = I_{(N-1)\times(N-1)}$. Thus for the averaged sign regressor algorithm we have

$$K_\infty = \frac{\rho}{2-\rho}\overline{\zeta}D_{\text{SR}}^{-1}D_{\text{SR}}^{-1}{}'. \qquad (4.18)$$

Asymptotic Excess Mean Square Error.

The MOE ζ_n defined in (2.2) can be re-expressed as

$$\begin{aligned}\zeta_n &= \mathbf{E}\left(y_n - \varphi_n'(\theta_* - \tilde{\theta}_n)\right)^2 \\ &= \overline{\zeta} + \text{tr}[D_{\varphi\varphi}K_n] - 2\mathbf{E}\left\{(y_n - \varphi_n'\theta_*)\varphi_n'\right\}\tilde{\theta}_n.\end{aligned}$$

Since $\mathbf{E}\{\tilde{\theta}_n\} \to 0$ as $n \to \infty$, the last term is transient. Hence for large n, $\zeta_n = \overline{\zeta} + \epsilon_{ex}(n)$ where the excess mean square error is defined as

$$\epsilon_{\text{ex}}(n) = \text{tr}[D_{\varphi\varphi}K_n]. \qquad (4.19)$$

Below we compute expressions for the asymptotic excess mean square error $\epsilon_{\text{ex}}(\infty)$.

Consider first the averaged blind LMS and sign error algorithm. It follows from (4.17) that

$$\begin{aligned}\text{tr}\,[D_{\varphi\varphi}K_n] &= (1-\rho)^2\text{tr}\,[D_{\varphi\varphi}K_{n-1}] + \rho^2\,\overline{\zeta}\text{tr}\left[I_{(N-1)\times(N-1)}\right] \\ &= (1-\rho)^2\text{tr}\,[D_{\varphi\varphi}K_{n-1}] + \rho^2\,\overline{\zeta}(N-1).\end{aligned} \qquad (4.20)$$

Since $(1-\rho)^2 < 1$, $\text{tr}\,[D_{\varphi\varphi}K_n]$ converges. The steady state excess mean square error is then given by

$$\epsilon_{\text{ex}}(\infty) = \frac{\rho}{2-\rho}\overline{\zeta}(N-1) \qquad (4.21)$$

Note that the above equation is identical to that of the Blind RLS, see [8, Eq.40]. As with blind RLS, the convergence of the MSE and the steady state misadjustment of BAG are independent of the eigenvalue distribution of the data auto-correlation matrix.

Next consider the averaged sign regressor algorithm. To evaluate $\text{tr}[D_{\text{SR}}]$ we first assume that φ_n is an iid zero mean Gaussian vector random variable with covariance $D_{\varphi\varphi}$. This Gaussian assumption follows from the central limit theorem if the number of users K is large. Then Price's formula yields

$$D_{\text{SR}} = \sqrt{\frac{2}{\pi}}\mathcal{D}D_{\varphi\varphi}$$

$$\mathcal{D} \triangleq \text{diag}\,\left(1/\sqrt{\mathbf{E}\{\varphi_n(1,1)^2\}}, \ldots, 1/\sqrt{\mathbf{E}\{\varphi_n^2(N-1,N-1)\}}\right).$$

Using (4.19) yields

$$\epsilon_{\text{ex}}(\infty) = \text{tr}[D_{\varphi\varphi}K_\infty] = \frac{\rho}{2-\rho}\overline{\zeta}\frac{\pi}{2}\text{tr}\,[\mathcal{D}D_{\varphi\varphi}\mathcal{D}] \qquad (4.22)$$

We can easily compute a lower bound for $\epsilon_{\text{ex}}(\infty)$ by bounding $\text{tr}\,[\mathcal{D}D_{\varphi\varphi}\mathcal{D}]$ in terms of $\text{tr}\,[\mathcal{D}D_{\varphi\varphi}\mathcal{D}]^{-1}$ as follows: Let λ_i, $i = 1,\ldots,N-1$ denote the eigenvalues of the positive definite symmetric matrix $[\mathcal{D}D_{\varphi\varphi}\mathcal{D}]^{-1}$. Since all the diagonal elements of this matrix are 1, $\text{tr}\,[\mathcal{D}D_{\varphi\varphi}\mathcal{D}]^{-1} = \sum_{i=1}^{N-1}\lambda_i = (N-1)$. Using the well-known inequality that the harmonic mean is less than the arithmetic mean we obtain

$$\frac{N-1}{\text{tr}\,[\mathcal{D}D_{\varphi\varphi}\mathcal{D}]} = \frac{N-1}{\sum_{i=1}^{N}1/\lambda_i} \le \frac{1}{N-1}\sum_{i=1}^{N}\lambda_i = 1,$$

which implies that

$$\text{tr}\,[\mathcal{D}D_{\varphi\varphi}\mathcal{D}] \ge N-1, \text{ and } \epsilon_{\text{ex}}(\infty) \ge \frac{\rho}{2-\rho}\overline{\zeta}\frac{\pi}{2}(N-1).$$

It is illustrative to compare the asymptotic excess mean square error of the averaged sign algorithms with their standard (un-averaged) counterparts. Expressions for the asymptotic excess mean square error of the standard sign-error algorithms have been derived in [1] and for the sign-regressor algorithm in [2]. The following table summarizes the results.

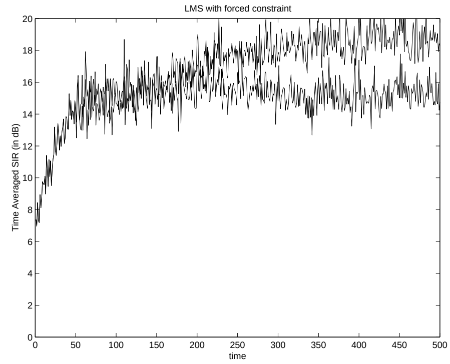
Algorithm	Standard	Averaged
Blind LMS	$\frac{\mu}{2}\overline{\zeta}\mathrm{tr}[D_{\varphi\varphi}]$	$\frac{\rho}{2-\rho}\overline{\zeta}(N-1)$
Sign Regressor	$\frac{\mu}{2}\overline{\zeta}\sqrt{\frac{\pi}{2}}(N-1)\max_i\sqrt{D_{\varphi\varphi}(i,i)}$	$\frac{\rho}{2-\rho}\overline{\zeta}\frac{\pi}{2}(N-1)$
Sign Error	$\frac{\mu}{2}\sqrt{\frac{\pi}{2}}\mathrm{tr}[D_{\varphi\varphi}]$	$\frac{\rho}{2-\rho}\overline{\zeta}(N-1)$

Table 1. Asymptotic Excess Mean Square Errors $\epsilon_{\text{ex}}(\infty)$

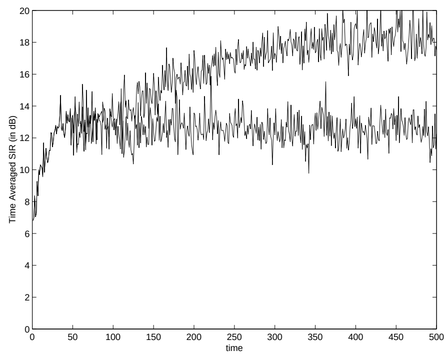
Remarks: (i) All the expressions for the standard algorithms above assume that $\mu \ll 1$. In particular, terms involving μ^2 are negligible. More precise expressions are available in [1] and [2]. The expressions for the sign regressor algorithm given are lower bounds. (ii) $\epsilon_{\text{ex}}(\infty)$ for the averaged algorithms do not depend on the eigen-distribution of $D_{\varphi\varphi}$. This is particularly useful in dynamic mobile environments where the eigen-structure of $D_{\varphi\varphi}$ can change rapidly. In [8] a similar property is shown for the blind Recursive Least squares (RLS) algorithm.

5. NUMERICAL EXAMPLES

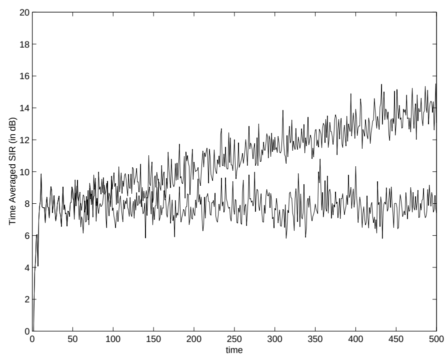
All the signal and noise powers are given in dB relative to the channel noise variance σ_n^2, see (2.1). The simulations assume a synchronous DS/CDMA system with processing gain N=31. The desired user's signature is generated as an m-sequence. The signature sequences of the other MAIs are generated randomly. The user of interest has SNR of 20 dB. There are 7 multiple access interferers: 5 users each of SNR 20 dB, and two users of SNR 40 dB. Fig. 1 shows the SIR versus time for the following six algorithms, averaged over 100 independent simulations. (a) Blind LMS; (b) Blind Averaged LMS; (c) Blind Sign Regressor; (d) Blind Averaged Sign Regressor; (e) Blind Sign Error; (f) Blind Averaged Sign Error. Fig. 1 shows that the averaged algorithms exhibit faster convergence than the un-averaged algorithms.
Remarks: For details of convergence proofs of the sign algorithms and optimal asymptotic convergence via averaging see [9].



(a) Blind LMS and Averaged LMS algorithms with $\varepsilon = 10^{-4}$, $\rho = 5e-3$



(b) Blind Sign regressor and Averaged Sign Regressor algorithms with $\varepsilon = 10^{-4}$, $\rho = 5e-3$



(c) Blind Sign regressor and Averaged Sign Regressor algorithms with $\varepsilon = 10^{-4}$, $\rho = 5e-3$

Fig. 1. Average SIR vs. time for MAI-suppression. The parameters are specified in Sec.5

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