ROBUST BLIND MULTIUSER DETECTION AGAINST CDMA SIGNATURE MISMATCH

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ABSTRACT

A common problem with the existing blind multi-user CDMA detectors is that their performance is very sensitive to the Signature Waveform Mismatch (SWM) caused by channel distortion. In this paper we consider the problem of designing a blind multi-user CDMA detector which is robust to the SWM. We present a convex formulation for this problem by using the Second Order Cone (SOC) programming. We also propose the use of recently developed interior point methods to efficiently solve the resulting SOC problem. Computer simulations indicate that the performance of our new robust blind multi-user detector is superior.

1. INTRODUCTION

Multiuser detection is mainly being used for the demodulation of digitally modulated signals in the presence of multi-access interference, and it has now become one of the basic techniques in Code-Division Multiple-Access (CDMA) receiver design [1, 4, 5]. However, a common problem with the existing multi-user CDMA detectors is that their performance is very sensitive to the Signature Waveform Mismatch (SWM) caused by channel distortion. Since channel distortion exists in most environments where CDMA is used (e.g., cellular mobile telephony), mitigation of SWM is essential (or may even be required) when we design a practical multi-user CDMA detector [2, 6].

One approach to deal with SWM is by using training sequences. An alternative strategy of mitigating SWM is to design a blind multiuser detector which has strong robustness to SWM. In [2] (see also [3]), a formulation is presented for the design of robust blind multi-user CDMA detectors which calls for the minimization of the detector's output energy. Moreover, two gradient descent algorithms (the Stochastic Gradient (SG) algorithm and the Least Squares (LS) algorithm) were proposed in [2] for achieving the Minimum Output Energy (MOE) under the constraint that the so called "surplus energy" created by SWM is bounded. However, constraining the "surplus energy" is an indirect way of achieving robustness. A more natural (and perhaps also more desirable) formulation is to directly maximize the worst case system performance given a specific range of SWM. This is the approach taken by the present paper. Another drawback of the two iterative algorithms proposed in [2] is that they require some channel/data-dependent parameters which are not easy to choose, and a poor choice would lead to poor performance.

In this paper we present a new formulation for the design of robust blind multi-user CDMA detectors. Our formulation is direct in the sense that it allows explicit control of the amount of required robustness in the detector. Moreover, the new robust blind multi-user detector can be obtained using the highly efficient interior point methods recently developed in the optimization community.

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2. PROBLEM FORMULATION

Consider an antipodal K-user direct sequence CDMA channel corrupted by some additive and white Gaussian noise $n(t)$. Suppose the standard deviation of $n(t)$ is $\sigma > 0$. Let $s_k(t)$ denote the signature waveform for the k-th user which is assumed to have unit energy ($\|s_k(t)\| = 1$), and let $\{b_k\}$ denote the transmitted data bits which are independent BPSK signals. Suppose the data bits are transmitted at the rate of $1/T$, with $T > 0$ being the bit duration. Then the synchronous received signal can be written as

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \qquad t \in [0, T] \tag{2.1}$$

where A_k is the received signal amplitude for the k-th user.

Although asynchronous CDMA is the reality in practice, it is often beneficial to consider synchronous CDMA systems first since they provide a useful simplified framework to develop our algorithms and carry out the analysis. Furthermore, algorithms designed for the synchronous CDMA can still be used in the asynchronous case provided the timing offsets $\{\tau_k\}$ are small.

Sampling the received signal $y(t)$ at chip rate $1/\Delta$, where $\Delta > 0$ is the chip interval, we obtain the signal vector (2.1):

$$\mathbf{y} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n}, \tag{2.2}$$

where

$$\mathbf{y} = \begin{bmatrix} y(0) \\ y(\Delta) \\ \vdots \\ y(N\Delta) \end{bmatrix}, \quad \mathbf{s}_k = \begin{bmatrix} s(0) \\ s(\Delta) \\ \vdots \\ s(N\Delta) \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} n(0) \\ n(\Delta) \\ \vdots \\ n(N\Delta) \end{bmatrix}$$

with N being the code spreading factor. Note that $T = N\Delta$.

Without loss of generality, suppose that the user 1 is our desired user whose signature waveform is $\mathbf{s}_1$. Our goal is to select a vector $\mathbf{c}_1$ which, upon correlating with the received vector $\mathbf{y}$ and passing through a hard limiter, will recover the data bits $\{b_1[i]\}$ sent by user 1. The Minimum Output Energy (MOE) based multi-user detector introduced in [2] can be described as follows.

$$\begin{array}{ll} \text{minimize} & E\,|\langle \mathbf{y}, \mathbf{c}_1 \rangle|^2 = \mathbf{c}_1^T \mathbf{R} \mathbf{c}_1 \\ \text{subject to} & \mathbf{c}_1^T \mathbf{s}_1 = 1 \end{array} \tag{2.3}$$

where $\mathbf{c_1}$ is the vector to be determined, and $\mathbf{R} = E(\mathbf{y}\mathbf{y}^T) \in \Re^{N\times N}$. In practice we have only finite number of snapshots of the received data. Thus, we need to replace $\mathbf{R}$ in (2.3) with the sample covariance matrix $\hat{\mathbf{R}} = \frac{1}{N_s}\sum_{n=1}^{N_s} \mathbf{y}[n](\mathbf{y}[n])^T$, where N_s is the number of snapshots and $\mathbf{y}[n]$ is the nth received data vector. This leads to the following implementable version of (2.3):

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T \hat{\mathbf{R}} \mathbf{c}_1 \\ \text{subject to} & \mathbf{c}_1^T \mathbf{s}_1 = 1. \end{array} \tag{2.4}$$

When SWM is present, the actual received signature waveforms becomes

$$\hat{\mathbf{s}}_k = \mathbf{s}_k + \mathbf{e}_k \tag{2.5}$$

where $\mathbf{e}_k$ is the mismatch error vector. Clearly, $\|\mathbf{e}_k\|$ is a measure of the magnitude of signal waveform mismatch. Note that $\mathbf{e}_k$ may be different among different users. It is well known that the MOE solution to (2.4) is highly sensitive to SWM and often lead to poor BER performance. To overcome this sensitivity to SWM, Honig *et. al.* [2] introduced an energy-constrained MOE detector. The robustness against SWM is achieved indirectly by constraining the surplus energy. In what follows, we describe a more direct (and perhaps more natural) way to construct a robust solution for the MOE formulation (2.4) whose global optimal solution can be found efficiently.

Suppose we have estimated that the norm of signal waveform distortion $\mathbf{e}_1$ is bounded by some constant $\delta > 0$, that is $\|\mathbf{e}_1\| \le \delta$. Then the actual received signal waveform $\hat{\mathbf{s}}_1$ can be described as a vector in the set

$$S_1(\delta) = \{\hat{\mathbf{s}}_1 \mid \hat{\mathbf{s}}_1 = \mathbf{s}_1 + \mathbf{e}_1,\ \|\mathbf{e}_1\| \le \delta\}.$$

Since $\hat{\mathbf{s}}_1$ can be any vector in $S_1(\delta)$, we must ensure that the detector gain for all signals in $S_1(\delta)$ should be greater than 1, that is, $\mathbf{c}_1^T\hat{\mathbf{s}}_1 \ge 1$ for all vectors $\hat{\mathbf{s}}_1 \in S_1(\delta)$. Such a constraint ensures that we can extract the data bits from user 1 regardless how its signature waveform is distorted, as long as the distortion is bounded by δ. Now suppose this gain constraint is enforced, then our goal remains to pick a vector $\mathbf{c}_1$ that minimizes $\mathbf{c}_1^T\hat{\mathbf{R}}\mathbf{c}_1$. Thus, a robust version of (2.4) can be described as follows:

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T \hat{\mathbf{R}} \mathbf{c}_1 \\ \text{subject to} & \mathbf{c}_1^T \hat{\mathbf{s}}_1 \ge 1 \ \text{ for all } \hat{\mathbf{s}}_1 \in S_1(\delta), \end{array} \tag{2.6}$$

where δ is an upper bound on the norm of the signal mismatch error vector.

For each choice of $\hat{\mathbf{s}}_1 \in S_1(\delta)$, the condition $\mathbf{c}_1^T\hat{\mathbf{s}}_1 \ge 1$ represents a linear constraint on $\mathbf{c}_1$. Since there are infinite number of $\hat{\mathbf{s}}_1$ in $S_1(\delta)$, the constraints in (2.6) are semi-infinite and linear. To facilitate the computation of optimal $\mathbf{c}_1$, we will convert these semi-infinite linear constraints into a so called second-order cone constraint. This is achieved by considering the worst case performance as follows. Note that the optimal solution of the minimization problem

$$\min_{\hat{\mathbf{s}}_s \in S_1(\delta)} \mathbf{c}_1^T \hat{\mathbf{s}}_1 \qquad \text{or equivalently} \qquad \min_{\|\mathbf{e}_1\| \le \delta} \mathbf{c}_1^T(\mathbf{s}_1 + \mathbf{e}_1)$$

is given by $-\delta\mathbf{c}_1/\|\mathbf{c}_1\|$. This can be easily verified by Cauchy-Schwartz inequality. Therefore, the constraint

$$\mathbf{c}_1^T\hat{\mathbf{s}}_1 \ge 1 \ \text{ for all } \hat{\mathbf{s}}_1 \in S_1(\delta)$$

can be equivalently described by

$$\mathbf{c}_1^T\left(\mathbf{s}_1 - \delta\frac{\mathbf{c}_1}{\|\mathbf{c}_1\|}\right) \ge 1$$

or

$$\mathbf{c}_1^T\mathbf{s}_1 - \delta\|\mathbf{c}_1\| \ge 1. \tag{2.7}$$

Substituting (2.7) into (2.6), we obtain

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T \hat{\mathbf{R}} \mathbf{c}_1 \\ \text{subject to} & \mathbf{c}_1^T \mathbf{s}_1 - \delta\|\mathbf{c}_1\| \ge 1 \end{array} \tag{2.8}$$

Notice that the constraint in (2.8) of the form

$$\|\mathbf{P}\mathbf{c}_1\| \le \mathbf{p}^T\mathbf{c}_1 + q, \quad \text{for some given } \mathbf{P} \in \Re^{N\times N},\ \mathbf{p} \in \Re^N$$

and $q \in \Re$, is called a second-order cone constraint.

Next we convert the quadratic objective function of (2.8) into a linear one. To do so, we first notice that $\mathbf{c}_1^T\mathbf{R}\mathbf{c}_1 = \|\mathbf{L}\mathbf{c}_1\|^2$, where $\mathbf{L}^T\mathbf{L} = \mathbf{R}$ is the Cholesky factorization. Obviously, minimizing the quadratic norm $\|\mathbf{L}\mathbf{c}_1\|^2$ is equivalent to minimizing $\|\mathbf{L}\mathbf{c}_1\|$. Introducing a new variable t and a new constraint $\|\mathbf{L}\mathbf{c}_1\| \leq t$, we can convert (2.8) into the following:

$$\begin{aligned} &\text{minimize} \quad t \\ &\text{subject to} \quad \|\mathbf{L}\mathbf{c}_1\| \leq t, \\ &\qquad\qquad\quad \|\delta^{1/2}\mathbf{c}_1\| \leq \mathbf{s}_1^T\mathbf{c}_1 - 1 \end{aligned} \tag{2.9}$$

The above formulation (2.9) is now in the standard form of a second-order cone programming problem. This is because the objective function is linear and the two constraints are both second-order cone constraints (which are convex).

Recently there have been some highly efficient interior point methods developed to solve the above second-order cone problem (2.8). Below we briefly describe a primal-dual potential reduction method [7] for solving the SOC problem. To do so, we first introduce the *dual* problem of (2.9) given by

$$\begin{aligned} &\text{maximize} \quad w_2 \\ &\text{subject to} \quad \mathbf{A}_1^T\mathbf{z}_1 + \mathbf{A}_2^T\mathbf{z}_2 + w_1\mathbf{f} + w_2\mathbf{b} = \mathbf{f} \\ &\qquad\qquad\quad \|\mathbf{z}_1\| \leq w_1 \\ &\qquad\qquad\quad \|\mathbf{z}_2\| \leq w_2 \end{aligned} \tag{2.10}$$

where $\mathbf{f} = [0, 0, \ldots, 1]^T \in \Re^{N+1}$, $\mathbf{A}_1 = [\mathbf{L}, \mathbf{0}] \in \Re^{N\times(N+1)}$, $\mathbf{b} = [\mathbf{s}_1^T, 0]^T \in \Re^{N+1}$, and $\mathbf{A}_2 = [\delta\mathbf{I}, \mathbf{0}] \in \Re^{N\times(N+1)}$. The dual optimization variables are the vectors $\mathbf{z} = [\mathbf{z}_1, \mathbf{z}_2] \in \Re^{N\times 2}$, and $\mathbf{w} = [w_1, w_2] \in \Re^2$. The difference between the primal and dual objectives is called the *duality gap* associated with t, $\mathbf{c}_1$, $\mathbf{z}$ and $\mathbf{w}$, and will be denoted by $\eta(t, \mathbf{c}_1, \mathbf{z}, \mathbf{w})$, or simply η:

$$\eta(t, \mathbf{c}_1, \mathbf{z}, \mathbf{w}) = t - w_2 \tag{2.11}$$

It is known that the duality gap is nonnegative for each feasible t, $\mathbf{c}_1$, $\mathbf{z}$ and $\mathbf{w}$ (i.e., satisfy the constraints of (2.9) and (2.10)), and is minimized at the optimal point $t^\star$, $\mathbf{c}_1^\star$, $\mathbf{z}^\star$ and $\mathbf{w}^\star$.

A useful tool in solving the SOC problem is the *logarithmic barrier function* which can be optimized by the primal-dual potential reduction method [7]. The detail of this approach is omitted here.

3. SIMULATIONS

We consider a synchronous CDMA system using Gold codes of length $N = 31$ with the number of users $K = 7$. The SNR is 10dB while the interference to signal ratio (ISR) is 20dB, representing a severe near-far effect. In our simulations, the length of the data sequence is set to be $i = 100$, so that the total data sample size is 3100. Such a choice of i is to ensure adequate iterative convergence of both LS and SG methods which we shall compare with our SOC method, and to ensure the sample covariance matrix $\hat{\mathbf{R}}$ is a close approximation. We perform a total of $L = 200$ Monte Carlo runs. Random data, random distortion and random noise are chosen for each run. To measure the effect of interference cancellation, we calculate the *bit error rate* (BER) of the detector output.

We now compare the averaged BER of our new SOC method with those of the existing methods which include the classical Matched Filtering method (MF), the standard (non-robust) MOE detector, the Least Square (LS) method and the Stochastic Gradient (SG) method (both from [2]). In our simulations, we assume the value of δ is known to the detector and we use this value in the SOC formulation (2.9). To solve (2.9) we have used a Matlab-based tool called SeDuMi [8] which is an efficient implementation of a primal-dual interior point method for solving SOC problems. For the LS and the SG methods, we have experimented with a large selection of different values of χ (the "surplus energy" parameter) and have chosen the one which gives the best SINR. For $\delta = 0.4$, the BER comparison is given in Fig. 1.

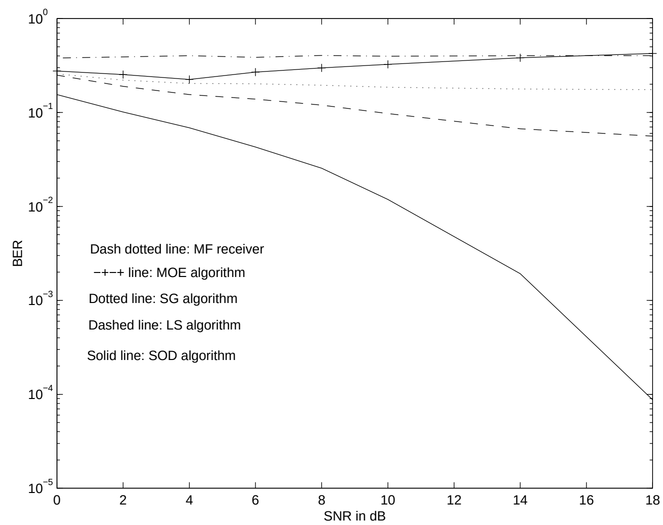

Fig. 1: BER averaged over 100 random runs

It can be seen from Fig. 1 that the SOC method has the best BER performance, followed by LS, SG, MOE and MF. Notice that the BER for the MOE detector worsens when the SNR increases. This is because for a non-robust detector like the MOE method, a part of the signal power will be contributed towards the interference when SWM exists, leading to larger interference power and worse BER performance as the signal power increases.

So far we have seen that the SOC detector has superior performance when δ is assumed to be known. It is important to see how sensitive the SOC detector is to the value of δ. We consider two cases when the SWM bound δ is over-estimated and under-estimated, respectively. The results are shown in Fig. 2. and Fig. 3, respectively.

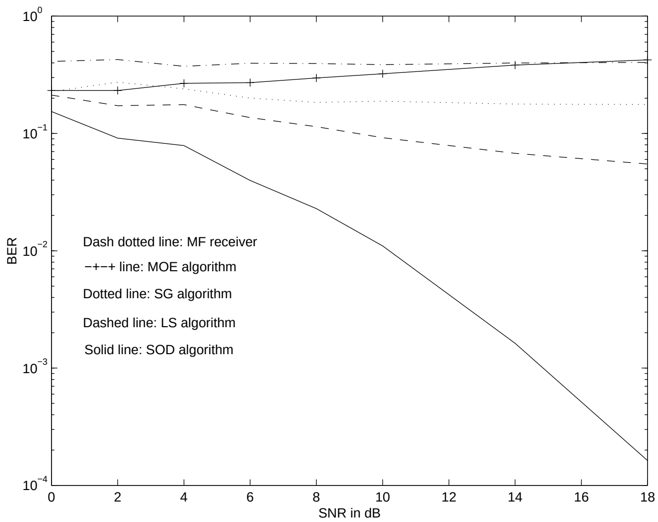

Fig. 2: BER versus SNR, averaged over 100 random runs with $\delta = 0.4$ and $\hat{\delta} = 0.6$

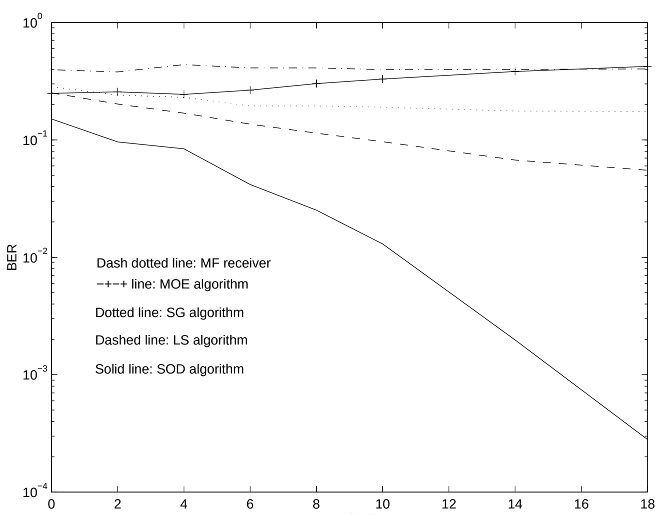

Fig. 3: BER versus SNR, averaged over 100 random runs with $\delta = 0.4$ and $\hat{\delta} = 0.2$

It can be seen from Figs. 2 and 3 that SOC detector is robust even when the bound of SWM is unknown. In contrast, we have found the performance of the LS and SG methods to be rather sensitive to the choice of surplus energy χ. Finally, we remark that, in our simulations, solving each SOC problem (2.9) with the Matlab tool SeDuMi [8] takes less than a second on a 600 MHz Pentium III PC.

4. CONCLUDING REMARKS

In this paper we have proposed a new robust blind multiuser detector for synchronous CDMA in the presence of signature waveform mismatch (SWM). Our method is based on a robust formulation of the Minimum Output Energy (MOE) detector using the Second-Order-Cone (SOC) programming technique. The SOC formulation (2.9) is convex and can be efficiently solved by the recently developed interior point methods. The new SOC detector can be considered blind since it only requires the knowledge of the signature waveform of the desired signal.

5. REFERENCES

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