Nonlinear Cumulant Based Adaptive Filter for Simultaneous Removal of Gaussian and Impulsive Noises in Images

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Abstract:-This correspondence presents an edge-preserving higher order statistics (HOS) based filter called Nonlinear Cumulant Based Adaptive Filter (NCBAF) for noise suppression in images. The NCBAF algorithm combines the linear (Averaging) characteristics of the Two-Dimensional Cumulant Based Adaptive Enhancer (2DCBAE) and the nonlinear characteristics of the median filter .The proposed algorithm allows the simultaneous removal of impulsive and Gaussian noise types in images, processed in a single filtering pass. The performance of the proposed method is compared to the commonly used median filter and the 2DCBAE.

I INTRODUCTION

The process of image smoothing seeks to remove unwanted noise from images while at the same time preserving all of the essential details in the original image data. In images, edges very often contain essential information; furthermore, our visual perception is heavily based on edge information [1, 2]. Therefore, any filtering should preserve the edges.

Many different approaches have been proposed for removing Gaussian noise. The two-dimensional least mean square (2DLMS) [3, 4] and the adaptive correlation enhancer [5] have drawn attention for their simplicity. The two-dimensional Cumulant Based Adaptive Enhancer (2DCBAE) [6, 7] proved excellent performance in suppressing white/colored Gaussian noise. However, like most linear FIR filtering techniques the 2DCBAE tends to smear edges and its performance deteriorates in the presence of impulsive noise.. .

In the presence of impulsive noise, only nonlinear filters can achieve reasonable noise suppression. One of the most popular nonlinear filters for noise removal is the median filter [8-11]. The median filter is well known for being able to remove impulsive noise and preserve image edges. Furthermore its simple nonlinear action gives it some very interesting characteristic. However, median filter does not in general allow the user a sufficient degree of control over its characteristic [8]. It has the disadvantage of its damaging of thin lines and sharp corners [12]. Furthermore, median filters do not have the averaging operation that is particularly appropriate in reducing additive Gaussian noise components in noisy data; thus, they may perform poorly in Gaussian noise.

When the signal has both edges and details and the noise has both Gaussian and impulsive components the restoration process becomes more complex. For a better overall performance in this case it is, in general, highly desirable to implement a filtering scheme with an algorithm that has both linear (averaging) and nonlinear characteristics. The order statistics or L-filters, the M-filters, and the MTM filters [8], the k-nearest neighbor (KNN) filters [12] are typical examples of such techniques.

In this correspondence a new higher order statistics (HOS) based filter, called Nonlinear Cumulant Based Adaptive Filter (NCBAF) is proposed. The proposed algorithm combines the linear (averaging) characteristics of the 2DCBAE and the nonlinear characteristics of the median filter. The new algorithm utilizes an impulse detector to exclude the impulsive points in the input from updating the filter coefficients. Therefore the deterioration occurs due to the presence of impulsive noise could be avoided. A comparison with the conventional median filter and the 2DCBAE shows the favorable performance of the proposed filter.

The organization of this paper is as follows. In section II, a review for the 2DCBAE algorithm is given. Section III describes the suggested NCBAF algorithm. Simulated results are given in section IV where comparisons between our suggested method and the results of the median filter and the 2DCBAE are presented. The presented simulation utilizes a synthetic image and the real Lena image for comparing the different methods. Conclusions are given in section V.

II. REVIEW OF THE 2DCBAE ALGORITHM

The 2D Cumulant Based Adaptive Enhancer (2DCBAE) algorithm [4,13] consists of a 2D FIR filter whose input is the 2D noisy signal x(m,n) with mean removed. The signal model is given by:

$$x(m,n) = d(m,n) + v(m,n) \qquad 0 \le m,n \le M-1 \qquad (1)$$

Where *d (m, n) is* the noise free image data and *v (m, n)* is Gaussian noise. The 2DCBAE output is the enhanced signal *y (m, n)*. A conceptual scheme of this enhancer is shown in Fig.1. The adaptive filter coefficient matrix at each adaptation index k, w_k , is recursively estimated using the following relation:

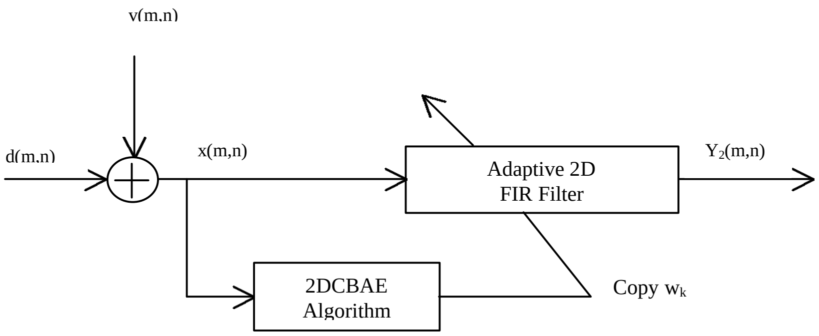

Fig. 1 A conceptual scheme of the 2DCBA enhancer

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$$w_k = \lambda w_{k-1} + (1-\lambda) X_k x^2(m,n) \qquad (2)$$

Where $k = mM + n$ denotes the iteration number and imply that the 2D signal is scanned row-by-row, left-to-right, downward. w_k is the (2L + 1) x (2L + 1) filter coefficients matrix given by:

$$w_k = \begin{bmatrix} w_k(-L,-L) & w_k(-L,-L+1) & \cdots & w_k(-L,L) \\ w_k(-L+,-L) & w_k(-L+1,-L+1) & \cdots & w_k(-L+1,L) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ w_k(L,-L) & w_k(L,-L+1) & \cdots & w_k(L,L) \end{bmatrix} \qquad (3)$$

X_k is the (2L+ 1) x (2L + 1) observation matrix given by:

$$X_k = \begin{bmatrix} x(m-L,n-L) & x(m-L,n-L+1) & \cdots & x(m-L,n+L) \\ x(m-L+1,n-L) & x(m-L+1,n-L+1) & \cdots & x(m-L+1,n+L) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ x(m+L,n-L) & x(m+L,n-L+1) & \cdots & x(m+L,n+L) \end{bmatrix} \qquad (4)$$

L is the maximum lag and λ is a forgetting factor which controls the rate of adaptation of the coefficient matrix w_k. It lies in the range:

$$0 << \lambda < 1 \qquad (5)$$

The final value of the coefficient matrix w_k computed from (1) is equal to the (i, j ; 0, 0) -slice of the third-order mixed cumulant of x(m,n) which is equal to the third-order mixed cumulant of d(m,n), where d(m, n) is the noise free signal. The output of the adaptive filter is computed using the well-known convolution summation, that is:

$$y(m,n) = \sum_{i=-L}^{L} \sum_{j=-L}^{L} w_k(i,j) x(m+i, n+j) \qquad (6)$$

III. THE NONLINEAR CUMULANT BASED ADAPTIVE FILTER (NCBAF) ALGORITHM

The schematic diagram for the proposed NCBAF algorithm is shown in Fig.2. It consists of two main stages. The first stage is an adaptive linear multi-output estimator whose coefficients are produced and updated using the 2DCBAE algorithm. The second (output) stage is a nonlinear selector represented by a median filter. The median filter selects the median value of the estimates, produced by the adaptive linear multi-output estimator, as the final output of the overall algorithm.

The input signal to the adaptive filter *x (m, n)* can be modeled as

$$x(m,n) = d(m,n) + v(m,n) + p(m,n), \; 0 \le m,n \le M-1 \quad (7)$$

Where *d (m, n)* is the signal of interest, *v (m, n)* is additive Gaussian noise of zero mean and variance σ_n^2, and *p (m, n)* is an impulsive noise. The amplitude and probability distributions of the impulsive noise are as given in [13].

Let the (2L + 1) x (2L + 1) coefficient matrix w_k, given in (3), be the coefficient matrix of the adaptive linear multi-output estimator. Where *k* and L are defined in section II. The recursive estimating formula, given in (2), for updating the coefficient matrix w_k is proved to be optimal under the condition of white/color Gaussian noise corruption [6, 7]. The process of updating the coefficients is greatly disturbed in the presence of impulsive noise. So, why the performance of the 2DCBAE is poor in the case of impulsive noise interference. To alleviate the problem of estimating the filter coefficients when both impulsive and Gaussian noise corrupt the image, a modified formula for updating $w_k(i, j)$ at each iteration (time index) *k*, is proposed

$$w_k = \begin{cases} w_{k-1}, & if \quad G_i(m,n) = 0 \\ \lambda w_{k-1} + (1-\lambda) X_k x^2(m,n), & if \quad G_i(m,n) = 1 \end{cases} \qquad (8)$$

Where λ is given in (5), and G_I is a coefficient that indicates weather an impulse occurs at the point *(m, n)* or not. The impulse detector is used to produce $G_i(m, n)$ at each iteration index *k* as shown in Fig.2. To detect whether a pixel *x (m, n)* is affected by an impulse or not, the following criterion is applied.

$$G_i(m,n) = \begin{cases} 0, & if \quad \left| x(m,n) - \dfrac{x1 + x2}{2} \right| \ge T \\ 1, & if \quad \left| x(m,n) - \dfrac{x1 + x2}{2} \right| < T \end{cases} \qquad (9)$$

Where *x1* and *x2* are the pixel values which are most similar to the pixel *x (m, n)* in the 3 x 3 window centered at (m, n), and *T* is appropriately chosen threshold. According to equation (9) $G_i(m, n)$ will be equal 0 if *x(m, n)* is affected by an impulse, otherwise $G_I(m, n)$ will be equal 1.

The $\underline{lth}$ output estimate from the adaptive linear multi-output estimator is given by

$$y_l(m,n) = \sum_{i=i_{\min}}^{i_{\max}} \sum_{j=j_{\min}}^{j_{\max}} w_k(i,j) G_i(m+i,n+j) x_k(m+i,n+j) \quad , \quad 1 \le l \le q \qquad (10)$$

Where $w_k(i,j), \; -L \le i,j \le L$ are the coefficient matrix given in (3), x_k is the (2L + 1) x (2L +1) observation matrix given in (4), $G_i(m + i, n + j)$ is the impulsive coefficient given in (9), *q* is the number of estimates produced by the adaptive linear multi-output estimator and it must be an odd number, and i_{min}, i_{max}, j_{min}, and j_{max} are summation limits that lie in the range

$$-L \le i_{\min} \le i_{\max} \le L, \qquad -L \le j_{\min} \le j_{\max} \le L \qquad (11)$$

Assume that w_{lk} is a subwindow whose limits i_{min}, i_{max}, j_{min}, and j_{max} where $1 \le l \le q$. Equation (11) implies that

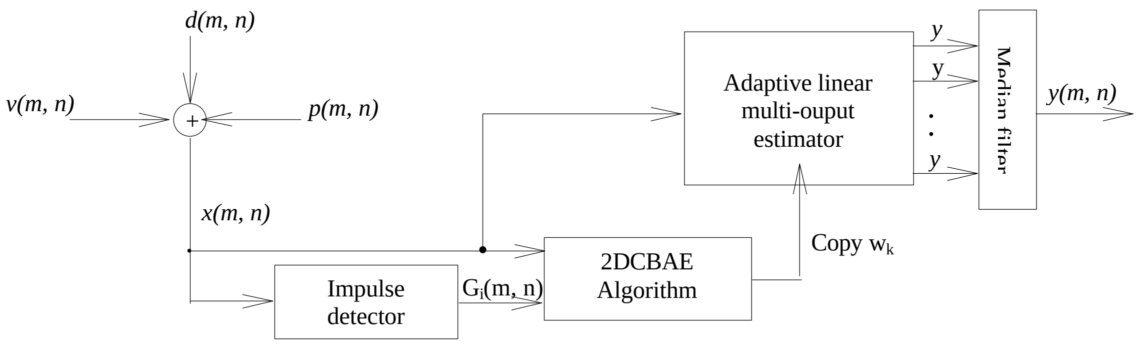


Fig.2 Image enhancement using the NCBAF algorithm.

$w_{lk} \in w_k, \quad 1 \le l \le q$. Now, for the best use of the proposed algorithm, the number of subwindows *q* and the limits of each subwindow should be chosen under the constraint that

$$w_k = w_{1k} \cup w_{2k} \cup \ldots \cup w_{lk} \cup \ldots \cup w_q \qquad (12)$$

Where $\cup$ denotes the union of two sets. The number of subwindows can range from $q = 1$ to $q = (2L + 1) \times (2L + 1)$. For the special case of which $q = 1$, the overall NCBAF algorithm reduces to the 2DCBAE algorithm. Also, for the special case of which $q = (2L + 1) \times (2L + 1)$, the overall NCBAE algorithm reduces to the simple median filter. If the number of subwindows (estimates) *q* is chosen to have an odd value between but not include the last two special cases. Then the NCBAF algorithm will have the advantages of the 2DCBAE in removing Gaussian noise and the advantages of the median filter in preserving edges in addition to the elimination of positive and/or negative impulsive noise.

Now, as a special case the number of subwindows is chosen to be $q = 7$ and the limits of each subwindow are chosen such that the seven output estimates from the adaptive linear multi-

$$y_1 = \sum_{i=-L}^{-1} \sum_{j=-L}^{-1} w_k(i, j)\, x(m-i, n-j)$$

$$y_2 = \sum_{i=-L}^{-1} \sum_{j=1}^{L} w_k(i, j)\, x(m-i, n-j)$$

$$y_3 = \sum_{i=1}^{L} \sum_{j=-L}^{-1} w_k(i, j)\, x(m-i, n-j)$$

$$y_4 = \sum_{i=1}^{L} \sum_{j=1}^{L} w_k(i, j)\, x(m-i, n-j) \qquad (13)$$

$$y_5 = x(m, n)$$

$$y_6 = \sum_{i=-L}^{L} w_k(i, 0)\, x(m-i, n)$$

$$y_7 = \sum_{j=-L}^{L} w_k(0, j)\, x(m, n-j)$$

output estimator at each iteration (time index) *k*, are given by:

Therefore, the overall filter output at iteration index *k*, is given by:

$$y(m,n) = median(y1, y2, \ldots\ldots, y7) \qquad (14)$$

The definitions for the number of subwindows and their limits, the output estimates, and the overall filter output given in (13) and (16) will be used through the rest of this paper.

IV. SIMULATED RESULTS

To show the effectiveness of the NCBAF algorithm compared with the 2DCBAE and the median filters we consider two test images. The first image is a synthetic one, which contains sharp edges. The other image is the real Lena image. Two measuring factors will be used to assess the filter performance in noise reduction and edge preservation, respectively. These are the signal to noise ratio improvement (SNRimprov) and the transition rate (TR) defined respectively as follows:

$$SNR_{improv} = 10 Log_{10}\left((x(m,n) - d(m,n))^2 / (y(m,n) - d(m,n))^2 \right) \qquad (15)$$

Where $d(i, j)$ is the noise free image, $x(i, j)$ is the noisy image and $y(i, j)$ is the processed image.

And *TR* is given by:

$$TR = | E[Y(k)] - E[Y(k-1)] | / 100 \qquad (16)$$

Where *Y* is the filtered output of a noisy edge function *f*. Where *f* is defined as follows; $f(i) = 100$ for $i < k$; $f(i) = 200$ for $i \ge k$. The transition rate *TR is* defined as the percentage of edge reconstruction from its disturbed version. Where E [•] is estimated by averaging 128 different trial outputs. Therefore it can be used as a performance measure of edge preservation, 1 0 represents full damaging of the edge.

White Gaussian noise of variable variance and impulsive noise of variable percentage corrupt the synthetic image, which contains an edge function *f*. Where *f* is described in the last paragraph. The noisy synthetic image is applied to each of the NCBAF, the median, and the 2DCBAE filters respectively. The window size used for both the NCBAF and the 2DCBAE is of size (5x5) and the parameter λ is 0.98. The used median filter is of size 3 x 3. Fig 3-a shows the SNR improvement versus the variance of Gaussian noise at constant percentage of impulsive noise equal to 10%. Fig.3-b shows the SNR improvement versus the percentage of impulsive noise at constant Gaussian noise variance equal to 1000. As shown in Fig.3-a, the NCBAF has better SNR improvement than median filter except when the variance of the Gaussian noise is very low (i.e. the degradation is mainly caused by impulsive noise). Also it is clear from Fig.3-b that, for constant variance of Gaussian noise, the NCBAF is superior to the median filter whatever the percentage of the impulsive noise. As shown in Fig. 3 (a-b) the NCBAF has better performance than the 2DCBAE under all conditions.

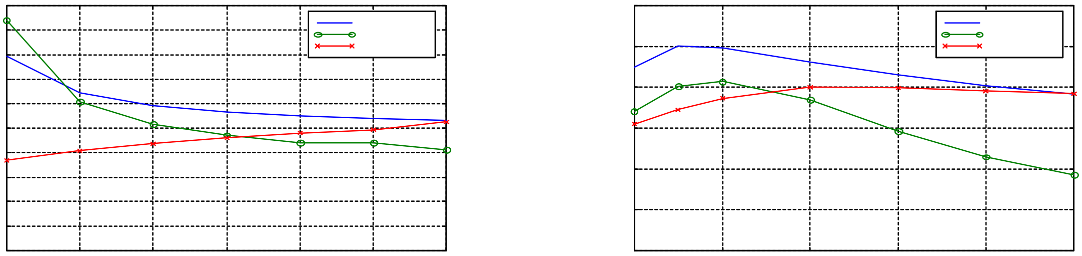
Fig 4-a shows the transition rate versus the variance of Gaussian noise at constant percentage of impulsive noise equal to 10%. Fig.4-b shows the transition rate versus the percentage of impulsive noise at constant Gaussian noise variance equal to 1000. As shown in Fig.4-a, the NCBAF has better transition rate than the median filter except when the variance of the Gaussian noise is very low (i.e. the degradation is mainly caused by impulsive noise). Also it is clear from Fig.4-b that, when the percentage of the impulsive noise is low (i.e. Gaussian noise is dominant), the NCBAF has little better transition rate than the median filter. As the percentage of impulsive noise increases the median filter becomes little better than the NCBAF. However, both of them have approximately the same transition rate performance. As shown in Fig. 4(a-b) the 2DCBAE has very poor performance in preserving edges.

Fig 5(a) shows the original Lena image and Fig. 5(b) shows a noisy version of Fig. 5(a) after adding white Gaussian noise of variance 700 in addition to 10% impulsive noise. Figures 5(c) - (e) show output of the NCBAF, median, and the 2DCBAE filters respectively. It is clear from Fig. 5 that the NCBAF is superior to both the 2DCBAE and the median filters in removing Gaussian and impulsive noise and preserving edges in real images.

A comparison between the NCBAF and the commonly used median filter and the 2DCBAE is summarized in table1.

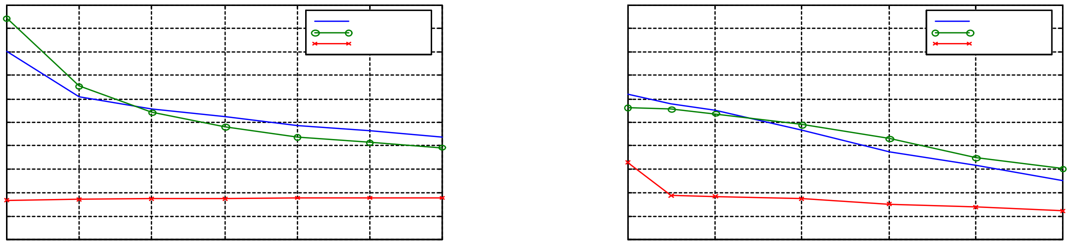
Table 1.

Image	Noise		SNR improvement dB		
	Gaussian	impulses	2DCBAE	median	NCBAF
Synth.	10e-8	-	-28.6	-18.64	-12.3
Synth.	1000	10%	7.49	10.31	11.83
Lena	-	10%	9.02	16.58	14.77
Lena	700	20%	10.42	11.57	12.43



(a) (b)

Fig 3. (a) SNR improvement Vs. variance of Gaussian noise (10% impulsive noise)
(b) SNR improvement Vs. percentage of impulsive noise (variance of Gaussian noise = 1000)



(a) (b)

Fig.4 (a) Transition rate Vs. variance of Gaussian noise (10% impulsive noise)
(b) Transition rate Vs. percentage of impulsive noise (variance of Gaussian noise = 1000)

(a)

(c)

(b)

(d)

Fig.5 (a) Original
(b) Noisy image with Gaussian noise of σ_n^2 =700 and 10% impulsive noise
(c) Output of NCBAF
(d) Output of median filter
(e) Output of 2DCBAE

(e)

V. CONCLUSION

The NCBAF for edge-preserving and simultaneous removal of Gaussian and impulsive noise types in images has been suggested. The proposed method combines the linear characteristics of the 2DCBAE and the nonlinear characteristics of the median filter. The NCBAF is superior to the median filter in attenuating Gaussian noise and preserving edges even if impulsive noise is found. The NCBAF is also superior to the 2DCBAE in preserving edges and removing impulsive noise. Simulated results were included to show the effectiveness of the NCBAF.

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