LOW-COMPLEXITY SYNCHRONIZATION IN A NAVIGATION RECEIVER UNDER MULTIPATH INTERFERENCE

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ABSTRACT

The time-of-arrival estimation error produced by multipath interference in a navigation receiver causes a strong degradation of the positioning accuracy. We present a synchronisation technique, operative in a navigation receiver under multipath interference, that takes into account its special conditions: low signal-to-noise ratio, DS-CDMA signals with long spreading codes, and very low data rates. The paper begins introducing the signal model of a deterministic antenna array. In this context, the system-specific features translate into the assumtion that all times of arrival must be inside a delay interval in which a truncated series approximation of the reference signal is valid. The introduction of this series in the signal model leads to an implementation based on a bank of correlators. Finally, the associated minimization problem is solved using three minimization algorithms: ESPRIT, IQML and a modification of Newton's Method. The latter calculates the maximum Likelihood estimator with a low computational burden.

1. INTRODUCTION

In a GPS or GNSS navigation receiver, the multipath interference produces a bias in the time-of-arrival estimation delivered by the DLL (Delay-Lock Loop), that results in a degradation of the positioning accuracy. The simple mofications of the receiver, like reducing the early-late spacing or changing the antenna pattern, do not eliminate this effect, (see [1]). Under these circumstances, the DLL precision is good enough to detect the data due to the long spreading codes, but is too coarse for positioning. For example, in the GPS C/A signal, the DLL precision is approximately of 1 chip, and one data symbol is composed of 1023 chips, while 1 chip corresponds to 300 meters in pseudorange (distance) accuracy. Several features of the navigation receiver allow to simplify the synchronisation problem. First, we can assume that the data modulation has been removed using the estimation provided by the DLL. With this, the synchronization consists in estimating the time of arrival of a known signal with the interference of several delayed replicas. Second, any signal replica with a delay greater than approximately 1.5 chips is eliminated, due to the cross-correlation properties of the spreading code. Thus, we can fix a delay interval around the DLL timing estimation in which all times of arrival are contained. Third, if we use a sampling frequency close to or above the Nyquist rate, (like 2 samples/chip), the navigation signal varies slowly in the delay interval. All this implies that the reference signal is quite regular in the delay interval, and can be approximated using a truncated series. The selection of the functions in the series depends on the complexity of the resulting minimization problem. If we use sincs or undamped exponentials, the technique would be similar to a technique applied at the output of a matched filter. In this paper, we develop the signal model of a deterministic antenna array, which approximates the navigation signal using a series of integral powers instead, because they provide a good approximation when the reference signal varies slowly in the delay interval, and the resulting optimization problem has a smaller size.

2. SIGNAL MODEL

Consider an array of sensors with arbitrary geometry and equal directional patterns. A direct wave and several delayed replicas $s(t-\tau_1), s(t-\tau_2), \ldots, s(t-\tau_n)$ impinge the antenna array. The signal at the i-th sensor, ($i = 1, \ldots, m$), is

$$y_i(t) = \sum_{k=1}^{n} a_{ik} s(t - \tau_k) + n_i(t), \tag{1}$$

where the known reference signal $s(t)$ contains the spreading code and no data modulation. The remaining elements are:

- a_{ik} Coefficient that depends on the i-th sensor pattern, and on the complex amplitude and direction of arrival of the k-th impinging signal.
- τ_k Delay of the k-th signal replica.
- $n_i(t)$ Complex White Gaussian noise process with variance σ^2 and uncorrelated among antennas.

The receiver takes N samples, relative to its own time reference, at epochs $t = t_1, \dots, t_N$. In what follows, we denote with $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ the transpose and Hermitian operations respectively.

The samples can be arranged in a matrix $\mathbf{Y}$, in which time varies column-wise and the sensor row-wise, (i.e. $(\mathbf{Y})_{li}$ is the sample at time t_l from the i-th sensor). The k-th wave adds to this observation matrix $\mathbf{s}(\tau_k)\mathbf{a}_k^{\mathrm{T}}$, where

$\mathbf{a}_k \quad \equiv [a_{1k}, a_{2k}, \dots, a_{mk}]^{\mathrm{T}}$,
$\mathbf{s}(\tau) \quad \equiv [s(t_1 - \tau), s(t_2 - \tau), \dots, s(t_N - \tau)]^{\mathrm{T}}$

are the time and spatial signatures respectively. So, $\mathbf{Y}$ can be written as

$$\mathbf{Y} = \sum_{k=1}^{n} \mathbf{s}(\tau_k)\mathbf{a}_k^{\mathrm{T}} + \mathbf{N} = \mathbf{S}(\boldsymbol{\tau})\mathbf{A}^{\mathrm{T}} + \mathbf{N}, \qquad (2)$$

with

$\boldsymbol{\tau} \quad \equiv [\tau_1, \tau_2, \dots, \tau_n]^T$. Vector of delays.
$\mathbf{S}(\boldsymbol{\tau}) \equiv [\mathbf{s}(\tau_1), \mathbf{s}(\tau_2), \dots, \mathbf{s}(\tau_n)]$. Time signatures.
$\mathbf{A} \quad \equiv [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_m]$. Spatial signatures.
$\mathbf{N} \quad \equiv \{n_i(t_l)\}_{li}$. Noise matrix.

At this stage, we apply the following assumption: The delays belong to an interval $[\tau_a, \tau_b]$ in which the $s(t_l - \tau)$ function can be approximated using the following truncated series with negligible error:

$$s(t_l - \tau) \approx \sum_{p=0}^{N_s-1} c_p(t_l)\phi_p(\tau), \;\; l = 1, \dots, N. \qquad (3)$$

This equation can be rewritten in matrix form by collecting the coefficients $c_p(t_l)$ and the functions $\phi_p(\tau)$ in separate matrices. Define:

$$\begin{aligned} \mathbf{c}_p &\equiv [c_p(t_1), \dots, c_p(t_N)]^{\mathrm{T}}, \\ \mathbf{C} &\equiv [\mathbf{c}_0, \dots, \mathbf{c}_{N_s-1}], \\ \boldsymbol{\phi}(\tau) &\equiv [\phi_0(\tau), \dots, \phi_{N_s-1}(\tau)]^{\mathrm{T}}, \\ \boldsymbol{\Phi}(\tau) &\equiv [\boldsymbol{\phi}(\tau_1), \dots, \boldsymbol{\phi}(\tau_n)]. \end{aligned} \qquad (4)$$

Then, from (3):

$$\mathbf{S}(\boldsymbol{\tau}) = [\mathbf{C}\boldsymbol{\phi}(\tau_1), \dots, \mathbf{C}\boldsymbol{\phi}(\tau_n)] = \mathbf{C}\Phi(\boldsymbol{\tau}). \qquad (5)$$

Equation (5) can be substituted into (2) to obtain

$$\mathbf{Y} = \mathbf{C}\boldsymbol{\Phi}(\boldsymbol{\tau})\mathbf{A}^{\mathrm{T}} + \mathbf{N}. \qquad (6)$$

This model equation shows that the signal replicas are contained in the span of $\mathbf{C}$, or equivalently, that the projection onto the span of $\mathbf{C}$ is a sufficient statistic. We can make this fact explicit by using the qr decomposition, $\mathbf{C} = \mathbf{QR}$, where $\mathbf{Q}$ and $\mathbf{C}$ have the same size, $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}$, and $\mathbf{R}$ is full-rank, square and upper triangular. Now, we multiply (6) by $\mathbf{Q}^{\mathrm{H}}$ to condense the information in a smaller matrix $\mathbf{Y}_{\mathrm{q}}$:

$$\mathbf{Y}_{\mathrm{q}} \equiv \mathbf{Q}^{\mathrm{H}}\mathbf{Y} = \mathbf{R}\boldsymbol{\Phi}(\boldsymbol{\tau})\mathbf{A}^{\mathrm{T}} + \mathbf{Q}^{\mathrm{H}}\mathbf{N}. \qquad (7)$$

The columns of $\mathbf{Q}$ provide the correlators to be used in a practical implementation.

3. MAXIMUM LIKELIHOOD ESTIMATOR

The Maximum Likelihood Estimator can be obtained operating with the delays only, if we eliminate the $\mathbf{A}$ matrix using the Conditional Maximum Likelihood equation, (see [2]):

$$\hat{\boldsymbol{\tau}} = \arg\max_{\boldsymbol{\tau}} \mathrm{tr}\left\{\boldsymbol{\Phi}\left[\boldsymbol{\Phi}^{\mathrm{H}}\mathbf{R}^{\mathrm{H}}\mathbf{R}\boldsymbol{\Phi}\right]^{-1}\boldsymbol{\Phi}^{\mathrm{H}}\mathbf{R}^{\mathrm{H}}\mathbf{Y}_{\mathrm{q}}\mathbf{Y}_{\mathrm{q}}^{\mathrm{H}}\mathbf{R}\right\}, \qquad (8)$$

where "tr" is the trace operator, and we have omitted the $\boldsymbol{\tau}$ dependency for clarity. This equation can be restated in terms of a matrix $\boldsymbol{\Phi}_{\perp}$ that spans the orthogonal complement to $\boldsymbol{\Phi}$, if such matrix is available:

$$\hat{\boldsymbol{\tau}} = \arg\min_{\boldsymbol{\tau}} \mathrm{tr}\left\{\boldsymbol{\Phi}_{\perp}\left[\boldsymbol{\Phi}_{\perp}^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{R}^{-1})^{\mathrm{H}}\boldsymbol{\Phi}_{\perp}\right]^{-1}\boldsymbol{\Phi}_{\perp}^{\mathrm{H}} \cdot \mathbf{R}^{-1}\mathbf{Y}_{\mathrm{q}}\mathbf{Y}_{\mathrm{q}}^{\mathrm{H}}(\mathbf{R}^{-1})^{\mathrm{H}}\right\}. \qquad (9)$$

4. MINIMIZATION ALGORITHMS

In this section, we adapt the ESPRIT, IQML and Newton's algorithms in order to calculate $\hat{\boldsymbol{\tau}}$ in either (8) or (9) when $\phi_p(\tau) = \tau^p$. The application of ESPRIT and IQML to a sum of undamped exponentials can be found in [3] and [4], respectively.

4.1. The ESPRIT algorithm

This algorithm exploits the shift invariance property of the matrix $\boldsymbol{\Phi}$. We present an adaptation of the algorithm in [3]. Let us denote with two sub-indexes $(\cdot)_{rs}$ the sub-matrix that contains from the r-th to the s-th column, $r \leq s$. Since $\boldsymbol{\Phi}$ is a Vandermonde matrix, the rank of

$$\boldsymbol{\Phi}_{r+1,s+1} - \lambda\boldsymbol{\Phi}_{r,s} \qquad (10)$$

is reduced by one if $\lambda = \tau_k$ for any $k = 1, \dots, n$. Recalling (7), we can repeat the same procedure with the matrix $R^{-1}\mathbf{Y}_{\mathrm{q}}$, and look for an approximated rank reduction of

$$(R^{-1}\mathbf{Y}_{\mathrm{q}})_{r+1,s+1} - \lambda(R^{-1}\mathbf{Y}_{\mathrm{q}})_{r,s}. \qquad (11)$$

These λ values are the generalized eigenvalues of the pencil,

$$\left[(R^{-1}\mathbf{Y}_{\mathrm{q}})_{r+1,s+1}(R^{-1}\mathbf{Y}_{\mathrm{q}})_{r,s}^{\mathrm{H}}, (R^{-1}\mathbf{Y}_{\mathrm{q}})_{r,s}(R^{-1}\mathbf{Y}_{\mathrm{q}})_{r,s}^{\mathrm{H}}\right], \qquad (12)$$

and provide the estimation $\hat{\boldsymbol{\tau}}$. Reference [5] contains further details on the definition and properties of the pencil of two matrices.

4.2. The IQML algorithm

The Vandermonde structure of $\mathbf{\Phi}$ can be used to form a real polynomial $b_0 + b_1\tau + \ldots + b_n\tau^n$ with roots $\tau_1, \ldots, \tau_n$. Then, the vector $\mathbf{b}^{\mathrm{H}} \equiv [b_0, \ldots, b_n]$ follows $\mathbf{b}^{\mathrm{H}}\mathbf{\Phi}_{1,n+1} = 0$. This allows us to obtain a matrix that spans the orthogonal complement to $\mathbf{\Phi}$, by placing shifted replicas of $\mathbf{b}$ in consecutive columns, i.e.,

$$\mathbf{\Phi}_{\perp} \equiv \begin{bmatrix} \mathbf{b} & 0 & \ldots & 0 \\ 0 & \mathbf{b} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \mathbf{b} \end{bmatrix}. \tag{13}$$

Given the special structure of $\mathbf{\Phi}_{\perp}$, we can see that the product $\mathbf{\Phi}_{\perp}^{\mathrm{H}}\mathbf{v}$ depends linearly on $\mathbf{b}$ for any vector $\mathbf{v}$. So, we can reorder the elements of $\mathbf{v}$ in a matrix $\mathcal{M}\{\mathbf{v}\}$ that follows,

$$\mathbf{\Phi}_{\perp}^{\mathrm{H}}\mathbf{v} = \mathcal{M}\{\mathbf{v}\}\mathbf{b}. \tag{14}$$

Now, we can operate on (9) using this equation and the properties of the trace to obtain the minimization problem in terms of $\mathbf{b}$:

$$\hat{\boldsymbol{\tau}} = \arg \min_{\mathbf{b} \text{ real}} \mathbf{b}^{\mathrm{H}}\mathbf{K}\mathbf{b}, \tag{15}$$

where

$$\mathbf{K} \equiv \sum_{i=1}^{m} \mathcal{M}\{\mathbf{y}_{\mathrm{q},i}\}^{\mathrm{H}} \left[\mathbf{\Phi}_{\perp}^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{R}^{-1})^{\mathrm{H}}\mathbf{\Phi}_{\perp}\right]^{-1} \mathcal{M}\{\mathbf{y}_{\mathrm{q},i}\}, \tag{16}$$

and $\mathbf{y}_{\mathrm{q},i}$ is the i-th column of $\mathbf{Y}_{\mathrm{q}}$. Note that $\mathbf{K}$ depends on $\mathbf{b}$ through $\mathbf{\Phi}_{\perp}$.

The IQML (Iterative Quadratical Maximum Likelihood) algorithm iterates on $\mathbf{b}$. Given the result of the q-th iteration $\mathbf{b}_{(q)}$, it calculates $\mathbf{K}$ first using $\mathbf{b}_{(q)}$, and then $\mathbf{b}_{(q+1)}$ by minimizing (15), which is a quadratical problem when $\mathbf{K}$ is fixed. The application of this algorithm with undamped exponentials can be found in [4].

4.3. The Modified Newton's Method

The classical Newton's Method, updates the approximation $\boldsymbol{\tau}_q$ with the formula $\boldsymbol{\tau}_{q+1} = \boldsymbol{\tau}_q + \mathbf{H}_q^{-1}\mathbf{g}_q$, where $\mathbf{H}_q$ is the Hessian and $\mathbf{g}_q$ the gradient of the cost function in (8). This method fails if $-\mathbf{H}_q$ is not positive definite. In order to avoid this problem, we load the diagonal using the iteration $\boldsymbol{\tau}_{q+1} = \boldsymbol{\tau}_q + (\mathbf{H}_q^{-1} + \lambda_q\mathbf{I})\mathbf{g}_q$, where λ_q is chosen to make $-(\mathbf{H}_q^{-1} + \lambda_q\mathbf{I})$ positive definite.

5. SIMULATION RESULTS

The three algorithms described in the previous section have been simulated in a multipath scenario in which a direct signal and one multipath replica impinge the antenna array. We summarize the technical details for each parameter in the signal model:

- $s(t)$. DS-CDMA signal composed of a Gold code with length 1023. The pulse shape is a root-raise cosine with roll-off factor $\beta = 0.2$. The sampling rate is 2 samples/chip.
- a_{ik}. Spatial signatures corresponding to the angles of arrival relative to the broadside of $\theta_1 = 30^o$ for the direct signal, and $\theta_2 = 80^o$ for the multipath replica. The antenna array is linear with sensors separated $\lambda/2$. The direct signal at the output of the sensors is $10\mathrm{dB}$ stronger. The carrier phases of both signals are chose randomly in each trial of the simulation.
- n_{it}. The signal-to-noise ratio in the samples after averaging the correlation during N code periods is approximately $\mathrm{S/N(dB)} = -23 + 10\log_{10}(\mathrm{N})$. This is a typical value of the GPS C/A signal, in which averaging $\mathrm{N} = 200$ code periods produces $\mathrm{S/N} = 0\mathrm{dB}$. (See [1]).
- τ_1, τ_2, $[\tau_a, \tau_b]$. Signal delays equal to 0.1 and 0.4 chips respectively. The delay interval is $[-2, 2]$ chips.
- n, m, N_s. The number of signal replicas is known ($n = 2$), the number of sensors is $m = 10$, and the approximation degree is $N_s = 14$.
- t_l, N. The sample epochs are taken with a rate of 2 samples/chip during an integer number of code words.

The truncated series approximation in (3) has been obtained from an initial high order Taylor series of $s(t_k - \tau)$ for all $k = 1, \ldots, N$. Then, the Chebyshev polynomials have been used to generate a lower order approximation with an error that is uniformly distributed in $[\tau_a, \tau_b]$, which is very close to the optimal Remez approximation. In this procedure, we have followed the chapter dedicated to the evaluation of functions in [6].

5.1. Accuracy performance

Figure 5.1 shows the Root Mean Square delay error for different signal-to-noise ratios. We can see that the IQML and the Modified Newton's Method perform much better that the ESPRIT algorithm. The Modified Newton's Method is always slightly better than the IQML method, because the latter uses the constraint set $\{\mathbf{b} : b_i \text{ real}\}$, while the exact constraint set is $\{\mathbf{b} : b_0 + b_1\tau + \ldots + b_n\tau^n \text{ with real roots}\}$. In the simulation, the ESPRIT estimation was used to initialize the IQML algorithm and the Modified Newton's Method.

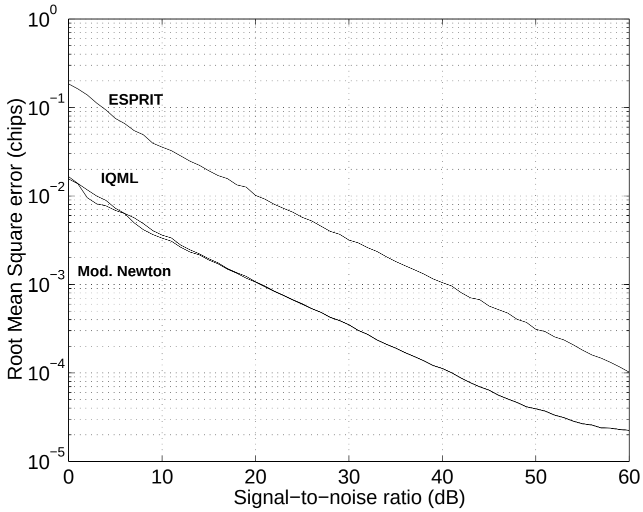

Fig. 1. Root Mean Square error of the direct signal delay estimator, $\sqrt{\mathrm{E}\{(\tau_1 - \hat{\tau}_1)^2\}}$.

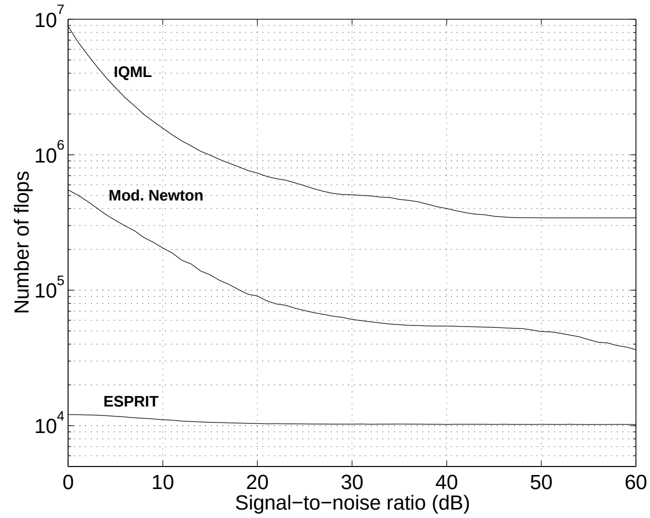

Fig. 2. Computational burden in number of floating point operations.

5.2. Computational Burden performance

Figure 5.2 shows the computational burden of the three algorithms. We can see that ESPRIT has almost a constant burden, while the Modified Newton's Method is about 8 times faster that the IQML. This is because the IQML algorithm must recalculate in each iteration the $\mathbf{K}$ matrix in (16). In the Modified Newton's Method, the calculation of the value, the gradient and the Hessian of the cost function using the analytical expressions is very efficient; in the current simulation, it takes only about three times the number of flops required to calculate the value of the cost function alone. All algorithms require a higher number of iterations to converge for lower signal-to-noise ratios, and consequently a higher computational burden.

6. CONCLUSION

We have introduced a signal model that takes into account the special features of a satellite navigation system. Its specific feature is the introduction of a truncated series expansion that approximates the reference signal. The model leads to an implementation based on a bank of correlators. The simulations have compared the performances of the ESPRIT, IQML and Modified Newton's algorithms when applied to solving the associated minimization problem. The results show that the Modified Newton's Method actually calculates the Maximum Likelihood estimator, and the IQML algorithm achieves almost the same root mean square error. In terms of complexity, ESPRIT has an almost constant computational burden, while the Modified Newton's Method shows a much smaller (8 times) burden that the IQML algorithm, due to the efficient calculation of the gradient and the Hessian of the cost function.

7. REFERENCES

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