A MAXIMUM-LIKELIHOOD PARAMETRIC APPROACH TO SOURCE LOCALIZATIONS

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ABSTRACT

Source localization using passive sensor arrays has been an active research problem for many years. Most near-field source localization algorithms involve two separate estimations, namely, relative time-delay estimations and source location estimation. In this paper, a one-step maximum-likelihood parametric source localization algorithm is proposed based on the maximum correlation between time shifted sensor data at the true source location. The performance of the algorithm is evaluated and shown to approach the Cramér-Rao bound asymtotically in simulations.

1. INTRODUCTION

Source localization and tracking using a passive array of sensors support many applications in underwater acoustics, navigation, geophysics, electronic warfare, and surveillance. Most near-field source localization algorithms [1]-[4] include two steps, namely, estimating relative time-delays between sensor data and then source location based on the time-delay estimates. Usually, the cross-correlation between sensor data has been used to extract the time-delay information, but have not been exploited to directly estimate the source location.

In this paper, we propose a maximum-likelihood parametric algorithm which directly estimates the source location without time-delay estimation by exploiting the cross-correlation of the sensor data. An optimization metric is derived from the cross-correlations of all sensors, which is also shown to be optimal in the maximum-likelihood sense, and the resulting form involves only the FFT of the sensor data and appropriate weighting for the given location. This approach not only avoids estimating time-delays but also avoids computing the cross-correlations directly. Since the maximum value of the metric ideally resides at the source location, the estimation becomes a peak finding problem. The metric may be computed on a coarse grid of points, and many iterative schemes can be used to find the true peak. For source tracking, the estimate of the previous frame may serve as the initial estimate of the next frame.

The paper is organized as follows. In section 2, the proposed parametric source localization algorithm is introduced. Then, the derivation of the Cramér-Rao bound is given in section 3. In section 4, simulated examples are given to show the effectiveness of the proposed algorithm. Finally, we draw our conclusions.

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2. MAXIMUM-LIKELIHOOD PARAMETRIC SOURCE LOCALIZATION ALGORITHM

The proposed maximum-likelihood parametric algorithm exploits the cross-correlations between the wideband sensor data evaluated in the frequency-domain. It is inspired by fact that source location information is contained in the linear phase shift of the sensor data spectrum. Let R be the number of sensors, L be the number of data samples in a block, N be the zero-padded DFT length, where $N \geq L + \tau$, $\tau = \max(t_{pq})$, $p, q = 1, \ldots, R$, $t_p = |\mathbf{r}_s - \mathbf{r}_p| / v$ is the time-delay from the source located at $\mathbf{r}_s$ to the pth sensor located at $\mathbf{r}_p$, $t_{pq} = t_p - t_q = (|\mathbf{r}_s - \mathbf{r}_p| - |\mathbf{r}_s - \mathbf{r}_q|) / v$ is the relative time-delay between the pth and the qth sensors, and v is the speed of propagation in length unit per sample. The received sensor array data can be modeled by $\mathbf{x}(n) = [x_1(n), \ldots, x_R(n)]^T = [s_1(n), \ldots, s_R(n)]^T + \overline{\eta}(n)$, where $s_p(n) = a_p s_0(n - t_p)$, $s_0(n)$ is the source signal, a_p is the signal gain level at the pth sensor, and $\overline{\eta}(n)$ is the zero mean white Gaussian distributed system noise with variance σ^2. The frequency representation of the received sensor array data can be given by

$$\mathbf{X}(k) = \mathbf{S}(k) + \eta(k), \text{ for } k = 0, \ldots, N-1, \tag{1}$$

where $\mathbf{X}(k) = [X_1(k), \ldots, X_R(k)]^T$, $\mathbf{S}(k) = [S_1(k), \ldots, S_R(k)]^T$, $S_p(k) = \sum_{n=0}^{L-1} s_p(n) e^{-j2\pi nk/N} = S_0(k) a_p e^{-j2\pi k t_p/N}$, and $\eta(k)$ is zero mean complex white Gaussian distributed with variance $L\sigma^2$. Note $\eta(k)$ approaches Gaussian distribution even if $\overline{\eta}(n)$ is any arbitrary i.i.d. sequence by the Central Limit Theorem. The circular time shift of the DFT is equivalent to the linear time shift given that the zero-padding length is larger than the maximum time-delay, i.e., $N - L \geq \tau$.

The weighted cross-correlation function of the pth and the qth sensors is defined by

$$\begin{aligned} c_{pq}(\tau) &= \sum_{n=0}^{L-1} w_{pq} x_p(n) x_q(n-\tau) \\ &= \frac{1}{N} \sum_{k=0}^{N-1} C_{pq}(k) e^{j2\pi\tau k/N} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} w_{pq} X_p(k) X_q^*(k) e^{j2\pi\tau k/N}, \end{aligned} \tag{2}$$

where the weight $w_{pq} = \overline{a}_p \overline{a}_q$, $\overline{a}_p = a_p \Big/ \sqrt{\sum_{p=1}^{R} a_p^2}$ is the normalized gain at the pth sensor, $C_{pq}(k) = w_{pq} X_p(k) X_q^*(k)$ is the weighted cross-power spectral densities, and superscript $*$ denotes the complex conjugate operation. The cross-power spectral density representation avoids directly computing the cross-correlation functions, which may have significant computational savings.

A natural way of defining an optimization metric is the sum of the weighted cross-correlations with the relative time-delays for parameter Θ, which can be given by

$$\begin{aligned} J(\Theta) &= \sum_{p=1}^{R}\sum_{q=1}^{R} c_{pq}(t_{pq}(\Theta)) \\ &= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{p=1}^{R}\overline{a}_p X_p(k) e^{j2\pi k t_p/N}\sum_{q=1}^{R}\overline{a}_q X_q^*(k) e^{-j2\pi k t_q/N} \\ &= \frac{1}{N}\sum_{k=0}^{N-1}\left|B(k,\Theta)\right|^2, \end{aligned} \quad (3)$$

and $B(k,\Theta) = \mathbf{w}(k)^H \mathbf{X}(k) = \sum_{p=1}^{R}\overline{a}_p X_p(k) e^{j2\pi k t_p/N}$ is the beam-steered beamformer output in the frequency-domain, where the array weight $w_p(k) = \overline{a}_p e^{-j2\pi k t_p/N}$ and superscript H denotes complex conjugate transpose. The source location can be estimated based on where (3) is maximized for the given set of locations. By only considering the positive frequency bins or a subset of the bins with significant spectral densities, we obtain the normalized metric that is given by

$$J_{\mathrm{N}}(\Theta) \equiv \frac{\sum_{k=0}^{N/2}\left|B(k,\Theta)\right|^2}{J_{\max}} \leq 1, \quad (4)$$

where $J_{\max} = \sum_{k=0}^{N/2}\left[\sum_{p=1}^{R}\overline{a}_p |X_p(k)|\right]^2$, which is useful to verify estimated peak values. To avoid further computational complexities, recursive gradient or other more advanced techniques [5] may be applied to find the true peak. The initial location estimate can be given by the peak of a coarse grid or from *a priori* information. For source tracking, the initial location estimate can also be given by the estimate of the previous frame. The gradient of the normalized $J_{\mathrm{N}}(\Theta)$ is given by

$$\nabla_{\mathbf{r}_s} J_{\mathrm{N}}(\Theta) = \frac{-4\pi}{J_{\max} N v}\sum_{k=0}^{N/2} k\,\mathrm{Im}\left[B^*(k,\Theta)\mathbf{C}(k,\Theta)\right], \quad (5)$$

where $\mathbf{C}(k,\Theta) = \sum_{p=1}^{R}\overline{a}_p X_p(k) e^{j2\pi k t_p/N}\mathbf{u}_p$, and $\mathbf{u}_p = (\mathbf{r}_s - \mathbf{r}_p)/|\mathbf{r}_s - \mathbf{r}_p|$ is a unit vector indicating the direction of the source from the pth sensor. One simple iterative gradient algorithm can be formulated by

$$\mathbf{r}_s^{(i+1)} = \mathbf{r}_s^{(i)} + \mu[\nabla_{\mathbf{r}_s} J_{\mathrm{N}}(\mathbf{r}_s^{(i)})], \quad (6)$$

where μ is the step size which depends on the shape of $J_{\mathrm{N}}(\mathbf{r}_s)$ and initial estimate $\mathbf{r}_s^{(0)}$. Other advanced methods may be used without specifying a step size. When the speed of propagation is unknown, we may expand the unknown parameter space to include it, i.e., $\Theta = [\mathbf{r}_s^T, v]^T$.

The proposed parametric algorithm can be shown to be optimal in the maximum-likelihood sense. From the signal model in (1), the log-likelihood function for the kth frequency bin can be given by $\mathcal{L}_k(\Theta) = -\left(1/L\sigma^2\right)\left\|\mathbf{X}(k) - \mathbf{S}(k)\right\|^2$. Then, the maximum-likelihood source localization solution with unknown source signal can be given by

$$\max_{\mathbf{r}_s, \mathbf{S}_0}\mathcal{L}(\Theta) = \min_{\mathbf{r}_s, \mathbf{S}_0}\sum_{k=0}^{N-1}\left\|\mathbf{X}(k) - \mathbf{S}(k)\right\|^2, \quad (7)$$

which is equivalent to finding $\min_{\mathbf{r}_s, S_0(k)}\left\|\mathbf{X}(k) - \mathbf{S}(k)\right\|^2$ for all k bins. Define $f(k) = \left\|\mathbf{X}(k) - \mathbf{S}(k)\right\|^2$. For any source location $\mathbf{r}_s$, the minima of $f(k)$ with respect to the source signal must satisfy

$$\frac{\partial f(k)}{\partial S_0(k)} = -\sum_{p=1}^{R} a_p X_p^*(k) e^{-j2\pi k t_p/N} + \sum_{p=1}^{R} a_p^2 S_0^*(k) = 0, \quad (8)$$

which yields the maximum-likelihood solution for the source signal at true $\mathbf{r}_s$ given by

$$\widehat{S}_0^{\mathrm{ML}}(k) = \frac{1}{\sqrt{\sum_{p=1}^{R} a_p^2}} B(k,\Theta). \quad (9)$$

By substituting $\widehat{S}_0^{\mathrm{ML}}(k)$ back into $f(k)$, we obtain

$$\begin{aligned} \min_{\mathbf{r}_s} f(k) &= \min_{\mathbf{r}_s}\left\{-2\,\mathrm{Re}\left[|B(k,\Theta)|^2\right] + |B(k,\Theta)|^2\right\} \\ &= \max_{\mathbf{r}_s}|B(k,\Theta)|^2, \end{aligned} \quad (10)$$

which is equivalent to the proposed parametric algorithm. Therefore, the proposed parametric algorithm is optimal in the maximum-likelihood sense. It can also be shown to be optimal for additional unknown parameters such as the speed of propagation.

3. CRAMÉR-RAO BOUND FOR SOURCE LOCALIZATION

The Cramér-Rao bound is most often used as a theoretical lower bound for any unbiased estimator. Most of the derivations of the Cramér-Rao bound for source localization found in the literature are in terms of relative time-delay estimation error. In this section, we derive a more general Cramér-Rao bound directly from the signal model. By developing a theoretical lower bound in terms of signal characteristics and noise level, we not only bypass the involvement of the intermediate time-delay estimator, but also offer useful insights to the physics of the problem.

By stacking up the N frequency bins of the signal model in (1) into a single column, we can re-write the sensor data into a $NR \times 1$ space-temporal frequency vector as $\mathbf{X} = \mathbf{G}(\Theta) + \xi$, where $\mathbf{G}(\Theta) = \left[\mathbf{S}(0)^T, \ldots, \mathbf{S}(N-1)^T\right]^T$, $\xi = [\eta(0), \ldots, \eta(N-1)]^T$, and $\mathbf{R}_\xi = E[\xi\xi^H] = L\sigma^2\mathbf{I}_{NR}$. The log-likelihood function of the complex Gaussian noise ξ is given by $\mathcal{L}(\Theta) = -\left(1/L\sigma^2\right)\left\|\mathbf{X} - \mathbf{G}(\Theta)\right\|^2$. The Fisher information matrix can be given by

$$\mathbf{F} = 2\,\mathrm{Re}\left[\mathbf{H}^H\mathbf{R}_\xi^{-1}\mathbf{H}\right] = \left(2/L\sigma^2\right)\mathrm{Re}\left[\mathbf{H}^H\mathbf{H}\right], \quad (11)$$

where $\mathbf{H} = \left[\frac{\partial S_1(0)}{\partial \mathbf{r}_s}, \frac{\partial S_1(1)}{\partial \mathbf{r}_s}, \ldots, \frac{\partial S_R(N-2)}{\partial \mathbf{r}_s}, \frac{\partial S_R(N-1)}{\partial \mathbf{r}_s}\right]^T$, assuming $\mathbf{r}_s$ is the only unknown. In this case, $\mathbf{F}_{\mathbf{r}_s} = \zeta\mathbf{A}$, and $\zeta = \left(2/L\sigma^2 v^2\right)\sum_{k=0}^{N-1}\left(2\pi k\left|S_0(k)\right|/N\right)^2$ is the scale factor that is proportional to the total power in the derivative of the source signal,

$$\mathbf{A} = \sum_{p=1}^{R} a_p^2 \mathbf{u}_p \mathbf{u}_p^T \tag{12}$$

is the *array matrix*. The $\mathbf{A}$ matrix provides a measure of geometric relations between the source and the sensor array. Poor array geometry may lead to degeneration in the rank of matrix $\mathbf{A}$. It is clear from the scale factor ζ that the performance does not solely depend on the SNR, but also the signal bandwidth and spectral density. Thus, source localization performance is better for signals with more energy in the high frequencies.

When the source signal is also unknown, i.e., $\Theta = [\mathbf{r}_s^T, \mathbf{S}_0^T]^T$, the new $\mathbf{H}$ matrix is given by

$$\mathbf{H} = \begin{bmatrix} \frac{\partial S_1(0)}{\partial \mathbf{r}_s^T} & \frac{\partial S_1(0)}{\partial \mathbf{S}_0^T} \\ \vdots & \vdots \\ \frac{\partial S_R(N-1)}{\partial \mathbf{r}_s^T} & \frac{\partial S_R(N-1)}{\partial \mathbf{S}_0^T} \end{bmatrix}, \tag{13}$$

and $\mathbf{S}_0 = [S_0(0), \ldots, S_0(N-1)]^T$. The Fisher information matrix can then be explicitly given by

$$\mathbf{F}_{\mathbf{r}_s,\mathbf{S}_0} = \begin{bmatrix} \zeta\mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{D} \end{bmatrix}, \tag{14}$$

where $\mathbf{B}$ and $\mathbf{D}$ are not explicitly given for brevity. By applying the block matrix inversion lemma, the leading 2×2 submatrix of the inverse Fisher information block matrix can be given by

$$\left[\mathbf{F}^{-1}_{\mathbf{r}_s,\mathbf{S}_0}\right]_{11:22} = \frac{1}{\zeta}(\mathbf{A}-\mathbf{Z})^{-1}, \tag{15}$$

where the *penalty matrix* due to the unknown source signal is defined by

$$\mathbf{Z} = \frac{1}{\sum_{p=1}^{R} a_p^2} \left(\sum_{p=1}^{R} a_p^2 \mathbf{u}_p\right) \left(\sum_{p=1}^{R} a_p^2 \mathbf{u}_p\right)^T. \tag{16}$$

The Cramér-Rao bound with unknown source signal is always larger than that with known source signal. It can be easily shown since the penalty matrix $\mathbf{Z}$ is always a non-negative definite matrix. The $\mathbf{Z}$ matrix acts as a penalty term since it is the average of the square of weighted $\mathbf{u}_p$ vectors. The estimation variance is larger when the source is far away since the $\mathbf{u}_p$ vectors are similar in directions to generate a larger penalty matrix. When the source is inside the convex hull of the sensor array, the estimation variance is smaller since $\mathbf{Z}$ approaches zero. The Cramér-Rao bound for the distance error bound from the true source location can be given by

$$\sigma_d^2 = \sigma_{x_s}^2 + \sigma_{y_s}^2 \geq \left[\mathbf{F}^{-1}_{\mathbf{r}_s,\mathbf{S}_0}\right]_{11} + \left[\mathbf{F}^{-1}_{\mathbf{r}_s,\mathbf{S}_0}\right]_{22}, \tag{17}$$

where $d^2 = (\widehat{x}_s - x_s)^2 + (\widehat{y}_s - y_s)^2$. By further expanding the parameter space, the Cramér-Rao bound for source localization given unknown source location, unknown source signal, and unknown speed of propagation can also be derived in the similar manner.

4. SIMULATION EXAMPLES

For all simulation examples, we use simulated array data generated by an acoustic waveform of a tank signature. The speed of propagation is known to be 345 (m/sec). For a randomly distributed array of five acoustic sensors depicted in Figure 1, the Nelder-Mead direct search algorithm (Matlab's *fmins.m*) is applied to estimate the source location from an initial estimate that is within the vicinity of the true source location. The $J_{\mathrm{N}}(\mathbf{r}_s)$ curve for a source inside the convex hull of this array is depicted in Figure 2 under 20dB SNR, and a high peak shows up at the source location. The shape of $J_{\mathrm{N}}(\mathbf{r}_s)$ depends on the geometry of the source and sensors. When the source moves away from the sensors, the peak broadens and results in more range estimation errors since it is more sensitive to noise. As depicted in Figure 3, the range estimation error is likely to occur in the source direction. In Figure 4, the source tracking scenario is depicted. The performance of the ML algorithm is compared to the conventional Least-Squares algorithm and the Cramér-Rao bound. Circular arrays of 5 and 7 sensors are considered in the comparison. At high SNR region and short range we observe the asymptotic approaching behavior for both algorithms, as depicted in Figure 5, but the ML out performs the LS algorithm. The Cramér-Rao bound for the known source signal and speed of propagation, unknown speed of propagation, and unknown source signal cases using 7 sensors are plotted in Figure 6. The unknown source signal has been shown to be a much more significant parameter factor than the unknown speed of propagation.

5. CONCLUSIONS

In this paper, a one-step maximum-likelihood parametric source localization algorithm is proposed. It maximizes the cross-correlations of the array signals and also the beam-steered beamformer output. It has been shown to be efficient with respect to the Cramér-Rao bound via simulations. It can also be extended to estimate more unknown parameters such as the speed of propagation and unknown sensor locations. For multiple sources case, the maximum-likelihood algorithm needs to be expanded to higher dimensions, which results in greater complexities.

6. REFERENCES

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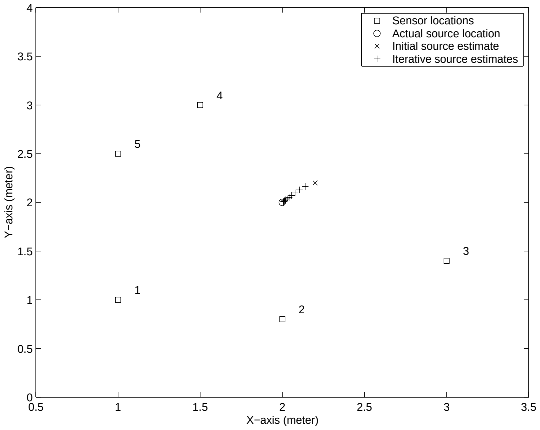

Fig. 1. Converging ML source localization example

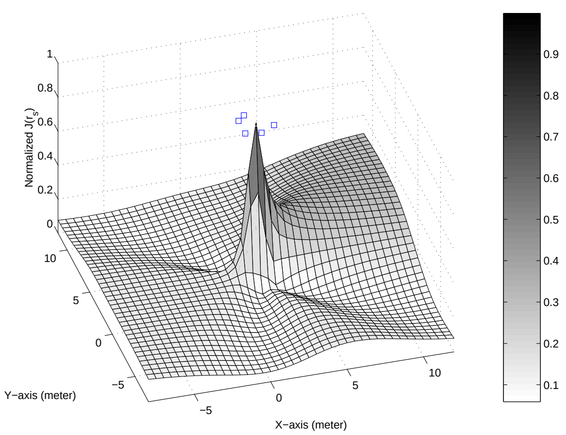

Fig. 2. The 3-D plot of $J_{\mathrm{N}}(\mathbf{r}_s)$ for a near source

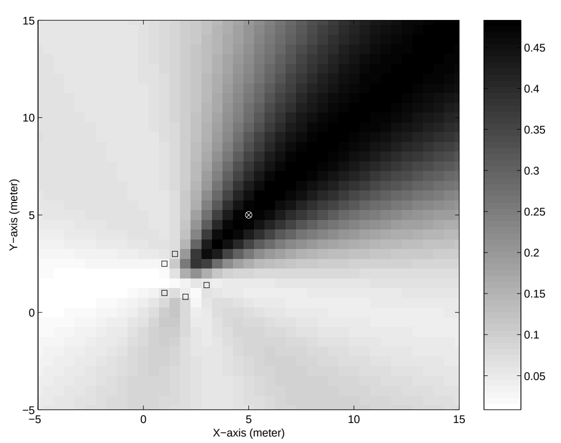

Fig. 3. The image plot of $J_{\mathrm{N}}(\mathbf{r}_s)$ for a distant source

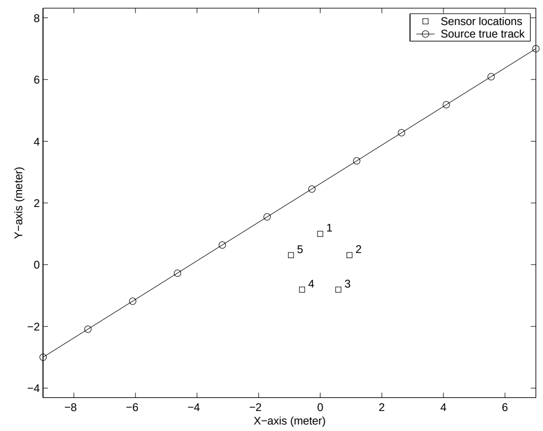

Fig. 4. Source tracking scenario

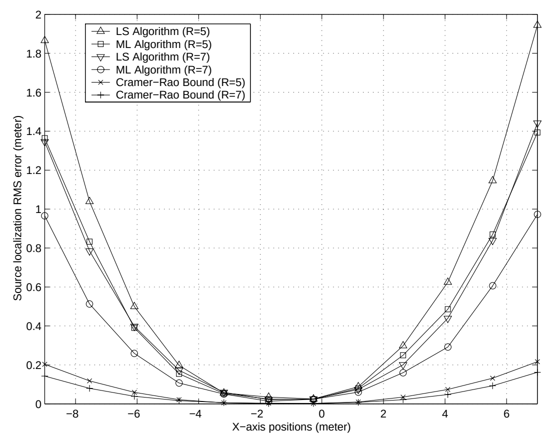

Fig. 5. Source tracking performance comparison

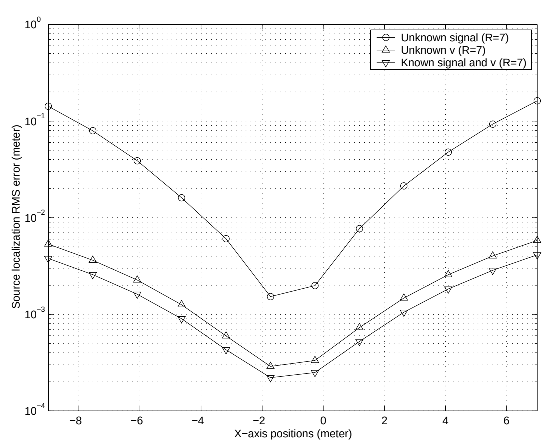

Fig. 6. Cramér-Rao bound comparison for source tracking