SEMIBLIND DECISION-FEEDBACK MULTIUSER INTERFERENCE CANCELLATION BASED ON THE MAXIMUM LIKELIHOOD PRINCIPLE

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ABSTRACT

This paper addresses the problem of interference suppression in Direct Sequence Code Division Multiple Access (DS CDMA) systems. We propose a novel semiblind Decision Feedback (DF) receiver based on the Maximum Likelihood (ML) principle that simultaneously exploits the transmission of training sequences and the statistical information concerning the unknown transmitted symbols. The Space Alternating Generalized Expectation Maximization (SAGE) algorithm allows an efficient iterative implementation of the receiver. Computer simulations show that the resulting multiuser detector attains practically the same performance as the theoretical DF Minimum Mean Square Error (MMSE) receiver.

1. INTRODUCTION

Third generation wideband mobile communication systems rely on the Direct Sequence Code Division Multiple Access (DS CDMA) scheme because it provides an efficient and flexible use of the spectrum [1]. In practice, however, the capacity of CDMA systems is severely limited by the Multiple Access Interference (MAI) and the Inter-Symbol Interference (ISI).

It is well known [2] that optimum multiuser detection results from the Maximum Likelihood (ML) estimation of the symbols transmitted by the desired user. Implementation of the ML detectors proposed in the literature [2], however, is limited by the need of knowing the channel characteristics and their prohibitive computational complexity. Therefore, low-complexity alternative approaches based on linear filtering have been investigated. Conventional linear Minimum Mean Square Error (MMSE) receivers [2] implicitly estimate the channel parameters using a training sequence. Therefore, the longer this training sequence is the better the receiver performance is. However, in burst transmission systems each block of received data consists of a training part and a sequence of unknown symbols. In this context, it is desirable to minimize the length of the training part in order to use the channel efficiently.

In this work we introduce a novel nonlinear approach to MAI and ISI cancellation in multiuser communication systems that is based on the application of the ML principle. A Decision Feedback (DF) receiver structure is considered and the Space Alternating Generalized Expectation-Maximization (SAGE) [3] algorithm is used to compute the optimum (according to the ML criterion) values of the receiver parameters. The resulting detector is a *semiblind* one because it exploits both the transmission of training sequences and the known statistical features of the transmitted symbols and the noise in the channel. As a consequence, the proposed scheme allows to achieve a very advantageous trade-off between training sequence length and receiver performance when compared to conventional approaches.

It is important to remark that the proposed semiblind DF-SAGE approach substantially differs from existing DF ML Sequence Detectors (DF-MLSD) [4] because the latter are based on the Viterbi algorithm whereas the former simply consists of two linear filters (one forward filter and one backward filter) with an intercalated threshold detector. Thus, our approach is similar in complexity to conventional linear multiuser receivers and considerably less computationally demanding than DF-MLSD. Furthermore, computer simulations reveal that the DF-SAGE receiver widely improves the theoretical performance limit of linear multiuser receivers due to the nonlinearity introduced by the threshold detector.

The remaining of the paper is organized as follows. Section 2 describes the signal model of an asynchronous time-dispersive DS CDMA system. Section 3 introduces the DF semiblind multiuser receiver based on the ML criterion. In section 4 we develop the SAGE algorithm that iteratively solves the ML optimization problem. Section 5 presents simulation results and section 6 is devoted to the conclusions.

2. SIGNAL MODEL

Figure 1 shows the asynchronous baseband discrete-time equivalent model of a Direct Sequence (DS) CDMA system with time dispersive channels. When the i-th user transmits an isolated symbol, b_i, it is multiplied by a unique binary-valued spreading sequence with L chips per symbol, $c_i(k),\ \ k = 0, ..., L-1$. The resulting signal passes through a linear time-dispersive channel, $h_i(k),\ \ k = 0, ..., P-1$, that accounts not only for the channel response but also for the relative time delays of the different users and the transmit and receiver front-end filters. The received sequence is the superposition of the transmitted signals from the N users plus the Additive White Gaussian Noise (AWGN) sequence $g(k)$, i.e.,

$$x(k) = \sum_{i=1}^{N} d_i(k) b_i + g(k),\ \ k = 0, ..., L+P-2. \quad (1)$$

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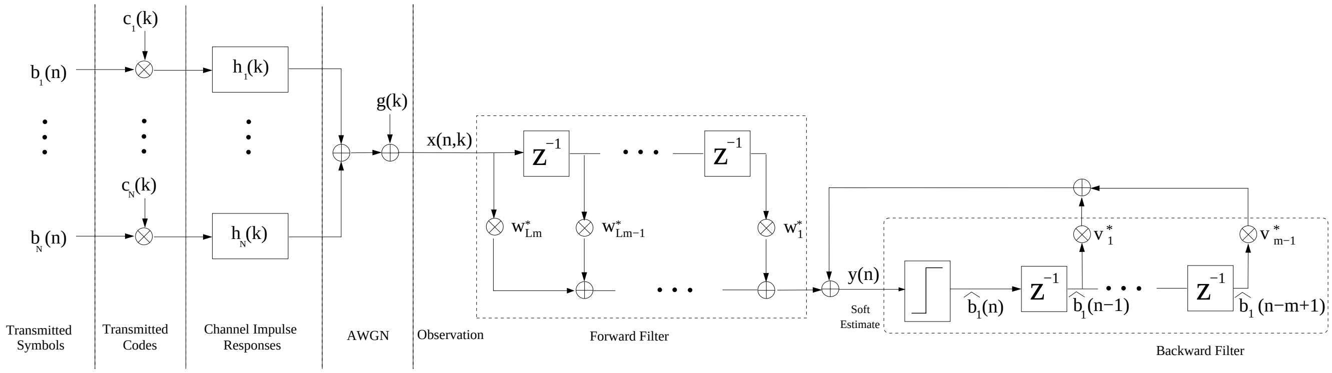

Fig. 1. Baseband discrete-time equivalent model of a DS CDMA system with time dispersive channels.

where $d_i(k) = c_i(k) * h_i(k) = \sum_{p=0}^{P-1} h_i(p)c_i(k-p)$, has length $L+P-1$ and will be termed *received code.* Due to the channel time dispersive effect, when a symbol, $b_i(n)$, is transmitted, it interferes with $b_i(n-1)$, $b_i(n-2), ..., b_i(n-m+1)$, where $m = \lceil \frac{L+P-1}{L} \rceil$ is the channel memory size. The received sequence for the n-th symbol period is

$$x(n,k) = x(nL+k) = \sum_{i=1}^{N} \sum_{r=0}^{m-1} d_i(r,k) b_i(n-r) + g(n,k), \quad k = 0, ..., L-1 \tag{2}$$

where $d_i(r,k) = d_i(rL+k)$. Bringing together all the observations that involve the symbol of interest, $b_i(n)$, we obtain an expresion for the overall received sequence using vector notation as

$$\mathbf{x}(n) = \mathbf{D}\mathbf{b}(n) + \mathbf{g}(n) \tag{3}$$

where $\mathbf{x}(n) = [x(n,0), \cdots, x(n, L-1), \cdots, x(n+m-1, 0), \cdots, x(n+m-1, L-1)]^T$ is the $Lm \times 1$ observation vector,

$$\mathbf{D} = \begin{bmatrix} \underline{D}_{m-1} & \underline{D}_{m-2} & \cdots & \underline{D}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \underline{D}_{m-1} & \underline{D}_{m-2} & \cdots & \underline{D}_0 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \cdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \underline{D}_{m-1} & \underline{D}_{m-2} & \cdots & \underline{D}_0 \end{bmatrix}$$

is the $Lm \times N(2m-1)$ received code matrix, composed of the submatrices $\underline{D}_r = [\mathbf{d}_1(r), \cdots, \mathbf{d}_N(r)]^T$, and $\mathbf{d}_i(r) = [d_i(r,0), \cdots, d_i(r, L-1)]^T$. The transmitted symbol vector is $\mathbf{b}(n) = [b_1(n-m+1), \cdots, b_N(n-m+1), \cdots, b_1(n), \cdots, b_N(n), \cdots, b_1(n+m-1), \cdots, b_N(n+m-1)]^T$ and $\mathbf{g}(n) = [g(n,0), \cdots, g(n+m-1, L-1)]^T$ is a vector of independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance σ_g^2.

The DF multiuser receiver consists of a linear FIR forward filter, $\mathbf{w} = [w_1, \cdots, w_{Lm}]^T$, and a backward filter, $\mathbf{v} = [v_1, \cdots, v_{m-1}]^T$. Therefore, the n-th symbol soft estimate is

$$y(n) = \mathbf{w}^H \mathbf{x}(n) + \mathbf{v}^H \hat{\mathbf{b}}_1(n) \tag{4}$$

where the superindex ^{H} denotes Hermitian transposition and $\hat{\mathbf{b}}_1(n) = [\hat{b}_1(n-1), \cdots, \hat{b}_1(n-m+1)]$ is an $(m-1) \times 1$ vector that corresponds to an estimate of the *causal* ISI. The symbols in $\hat{\mathbf{b}}_1(n)$ are easily obtained from the available soft estimates, $y(n-1), \cdots, y(n-m+1)$ using a threshold detector.

3. SELECTION OF THE FILTER COEFFICIENTS

Let us assume that both the MAI and the ISI are totally suppressed by the receiver. Then, the soft symbol estimate, $y(n)$, consists of just two components: the desired user symbol and an additive Gaussian noise term. Denoting $\mathbf{w}_*$ and $\mathbf{v}_*$ as the filters that completely eliminate both MAI and ISI, we can write

$$y(n) = \mathbf{w}_*^H \mathbf{x}(n) + \mathbf{v}_*^H \hat{\mathbf{b}}_1(n) = b_1(n) + g_f(n) \tag{5}$$

where $b_1(n)$ is the desired user symbol and $g_f(n)$ is a complex Gaussian random variable with zero mean and variance σ_f^2. In this section, the filtered noise variance, σ_f^2, will be considered a constant for the sake of simplicity. In section 4, however, we will also propose an easy-to-implement updating rule for σ_f^2.

Let K be the number of observation vectors available at the receiver. Since in digital communications the transmitted symbols are usually modelled as discrete, independent and identically distributed (i.i.d.) random variables with known probability density function (p.d.f.) and finite alphabet, the optimum symbol estimates $y(0), ..., y(K-1)$ computed as in (5) are also i.i.d. and the joint p.d.f. of $\mathbf{y} = [y(0), ..., y(K-1)]^T$ is [5]

$$f_{\mathbf{y};\Theta}(\mathbf{y}) = \left(\frac{1}{\pi \sigma_f^2} \right)^K \prod_{n=0}^{K-1} E_b \left[e^{-\frac{|y(n)-b|^2}{\sigma_f^2}} \right] \tag{6}$$

where $\Theta = [\mathbf{w}_*, \mathbf{v}_*]$ and $E_b[\cdot]$ denotes statistical expectation with respect to (w.r.t.) the desired user symbol. Notice that $E_b[\cdot]$ can be analytically calculated because it reduces to a simple summation. Using (6), the ML estimate of Θ turns out to be

$$\hat{\Theta} = \arg \max_{\Theta} \{\mathcal{L}(\Theta)\} \tag{7}$$

where $\hat{\Theta} = [\hat{\mathbf{w}}, \hat{\mathbf{v}}]$, and

$$\mathcal{L}(\Theta) = \sum_{n=0}^{K-1} \log E_b \left[e^{-\frac{|y(n)-b|^2}{\sigma_f^2}} \right] \tag{8}$$

is the log-likelihood of Θ w.r.t. the soft estimates $\mathbf{y}$.

The log-likelihood $\mathcal{L}(\Theta)$ is a non quadratic function with several maxima. In particular, the solutions to problem (7) guarantee that the soft estimates, $y(n)$, have a p.d.f. close to $f_{b+g_f}(\cdot)$. However, this is not enough to ensure that the desired user is extracted. Since in CDMA all users transmit symbols with the same modulation format, the p.d.f. of the i-th interference at the receiver is also $f_{b+g_f}(\cdot)$, which does not differ from the desired user p.d.f.. Therefore, solving the optimization problem (7) may lead to the *capture* of an interference.

The capture problem can be considerably alleviated if we exploit the transmission of a short training sequence of $M < K$ symbols, as it is done in currently standardized mobile communication systems. Indeed, let us assume that the first M symbols (i.e., $\mathbf{b}_t = [b_1(0), ..., b_1(M-1)]^T$) are known *a priori* by the receiver. Conditioning the expectations in (7) w.r.t. the known symbols, $\mathbf{b}_t$, we arrive at a *semiblind* receiver where the filter coefficients are computed as the solution to

$$\begin{aligned} \hat{\Theta} &= \arg \max_{\Theta} \{ \mathcal{L}(\Theta) \mid_{\mathbf{b}_t} \} \\ &= \arg \min_{\Theta} \left\{ \sum_{n=0}^{M-1} \mid y(n) - b_1(n) \mid^2 \right. \\ &\quad \left. - \sum_{n=M}^{K-1} \log E_b \left[e^{-\frac{|y(n)-b|^2}{\sigma_f^2}} \right] \right\}. \end{aligned} \tag{9}$$

The computer simulations in section 5 show that short training sequences ($M \simeq 30$ symbols) are enough to avoid the capture problem. This is because the first term in (9) is a purely quadratic form with an only minimum that corresponds to the rough extraction of the desired user. Note that Θ is still computed according to the ML principle. All the available statistical information is employed to obtain the filter coefficients and, hence, the proposed semiblind DF receiver outperforms conventional DF-MMSE multiuser detectors [2] that only exploit the training sequence $\mathbf{b}_t$.

4. ITERATIVE IMPLEMENTATION

Since it is not possible to find a closed form solution to problem (9), we propose to compute the parameter estimates $\hat{\Theta}$ using the Expectation Maximization (EM) algorithm [6]. The EM approach postulates the existence of some missing (unobserved) data that, if known, would aid in the estimation problem.

Let the soft estimates, $y(n), \quad n = 0, ..., K-1$, be the incomplete data set and let be the *extended* vectors $\mathbf{y}_e(n) = [y(n) \;\; b_1(n)]^T, \quad n = 0, ..., K-1$, the complete data set. The complete data sufficient statistics is provided by the function [5]

$$U\left(\Theta, \hat{\Theta}_{i,i}\right) = \sum_{n=0}^{K-1} E_{b_1(n)|y(n),\mathbf{b}_t;\hat{\Theta}_{i,i}} \left[\log\left(f_{\mathbf{y}_e;\Theta}(\mathbf{y}_e(n))\right)\right] \tag{10}$$

where $\hat{\Theta}_{i,j} = [\hat{\mathbf{w}}(i), \hat{\mathbf{v}}(j)]$, $\mathbf{y}_e(n) \mid y(n), \mathbf{b}_t$ denotes conditioning of the complete data to the observed data and the training sequence, and

$$f_{\mathbf{y}_e;\Theta}\left(\mathbf{y}_e(n)\right) = \frac{1}{\pi\sigma_f^2} \; f_b\left(b_1(n)\right) \; e^{-\frac{|y(n)-b_1(n)|^2}{\sigma_f^2}} \tag{11}$$

is the likelihood of Θ w.r.t. a single complete data vector [5]. Substituting (11) into (10), the E and M steps can be combined into the equivalent single iteration

$$\begin{aligned} \hat{\Theta}_{i+1,i+1} &= \arg \max_{\Theta} \left\{ U\left(\Theta, \hat{\Theta}_{i,i}\right) \right\} = \\ &= \arg \min_{\Theta} \left\{ \sum_{n=0}^{M-1} \mid y(n) - b_1(n) \mid^2 \right. \\ &\quad \left. + \sum_{n=M}^{K-1} E_{b_1(n)|y(n);\hat{\Theta}_{i,i}} \left[\mid y(n) - b_1(n) \mid^2 \right] \right\}. \end{aligned} \tag{12}$$

Thus, we have cast problem (9), which does not have a closed form solution, into a sequence of quadratic problems that can be analytically solved.

Nevertheless, solving (12) w.r.t. the joint parameter vector Θ is rather involved. The Space Alternating Generalized EM (SAGE) algorithm [3] is a suitable modification of the conventional EM approach that consists of succesively maximizing function $U(\cdot,\cdot)$ w.r.t. different parameter subsets [3]. In our case, it is straightforward to find separate updating rules for $\hat{\mathbf{w}}(i+1)$ and $\hat{\mathbf{v}}(i+1)$ (see (13) and (14)), respectively) where $\hat{\mathbf{R}}_x = \sum_{n=0}^{K-1} \mathbf{x}(n)\mathbf{x}^H(n)$, $\hat{\mathbf{R}}_{\hat{b}_1} = \sum_{n=0}^{K-1} \hat{\mathbf{b}}_1(n)\hat{\mathbf{b}}_1^H(n)$ and $\varepsilon_{i,j} = E_{b_1(n)|y(n);\hat{\Theta}_{i,j}}[b_1^*(n)]$, which is calculated using the Bayes theorem [5]. The hard symbol estimates $\hat{b}_1(n-q), q = 1, \cdots, m-1$, which are used to build vector $\hat{\mathbf{b}}_1(n)$, are computed from the corresponding soft estimates $y(n-q)$ obtained using the past iteration filter coefficients $\hat{\mathbf{w}}(i)$ and $\hat{\mathbf{v}}(i)$.

Finally, note that the filtered noise variance parameter, σ_f^2, is required in order to compute $\varepsilon_{i,j}$ in equations (13) and (14). Since $\sigma_f^2 = \mathbf{w}_*^H \mathbf{w}_* \sigma_g^2$ is not known *a priori*, it must be estimated. A very simple estimation method consists of iteratively updating σ_f^2 using the estimates of $\mathbf{w}_*$ obtained from (13), i.e.,

$$\hat{\sigma}_f^2(i+1) = \hat{\mathbf{w}}^H(i)\hat{\mathbf{w}}(i)\hat{\sigma}_f^2(i). \tag{15}$$

Thus, the updated value of $\hat{\sigma}_f^2(i+1)$ can be used to compute $\hat{\mathbf{w}}(i+1)$ and $\hat{\mathbf{v}}(i+1)$. According to our computer simulations, an adequate initialization of (15) is $\hat{\sigma}_f^2 = \hat{\sigma}_g^2$, where $\hat{\sigma}_g^2$ is a rough estimate of the AWGN variance.

5. COMPUTER SIMULATIONS

We have carried out computer simulations to illustrate the performance of the proposed semiblind receiver in an asynchronous time dispersive DS CDMA system with $N = 4$ users transmitting QPSK symbols, length $L = 6$ binary codes and length $P = 10$ complex unknown user channels. Both the spreading codes and the channel coefficients of all users have been chosen randomly. Figure 2 plots Symbol Error Rate (SER) curves for several values of the Signal to Noise Ratio (SNR) defined as, SNR= $10 \log_{10} \frac{E[|b_1|^2]\mathbf{d}_1^H \mathbf{d}_1}{\sigma_g^2}$, when the number of observation vectors available to estimate the receiver coefficients is $K = 300$ and the length of the training sequence is $M = 30$ symbols. In this figure we compare the theoretical linear and DF MMSE receivers (labeled LMMSE and DF-MMSE, respectively) that assume perfect knowledge of the channel parameters of all users with the conventional DF-MMSE and semiblind DF-SAGE detectors. The former uses only the training sequence to select the receiver coefficients whereas the latter is given by equations (13) and (14). It is apparent that the proposed DF-SAGE receiver practically matches the performance limit of the DF-MMSE receiver and clearly outperforms both the LMMSE detector and the conventional DF-MMSE receiver. The poor

$$\hat{\mathbf{w}}(i+1) \;=\; \hat{\mathbf{R}}_x^{-1}\left(\sum_{n=0}^{M-1}\mathbf{x}(n)b_1^*(n) + \sum_{n=M}^{K-1}\mathbf{x}(n)\varepsilon_{i,i} - \sum_{n=0}^{K-1}\mathbf{x}(n)\hat{\mathbf{b}}_1^H(n)\hat{\mathbf{v}}(i)\right) \tag{13}$$

$$\hat{\mathbf{v}}(i+1) \;=\; \hat{\mathbf{R}}_{\hat{b}_1}^{-1}\left(\sum_{n=0}^{M-1}\hat{\mathbf{b}}_1(n)b_1^*(n) + \sum_{n=M}^{K-1}\hat{\mathbf{b}}_1(n)\varepsilon_{i+1,i} - \sum_{n=0}^{K-1}\hat{\mathbf{b}}_1(n)\mathbf{x}^H(n)\hat{\mathbf{w}}(i+1)\right) \tag{14}$$

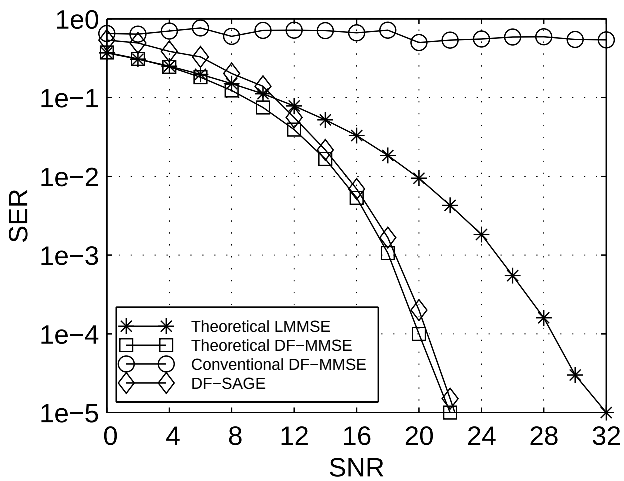

Fig. 2. SER for several values of SNR.

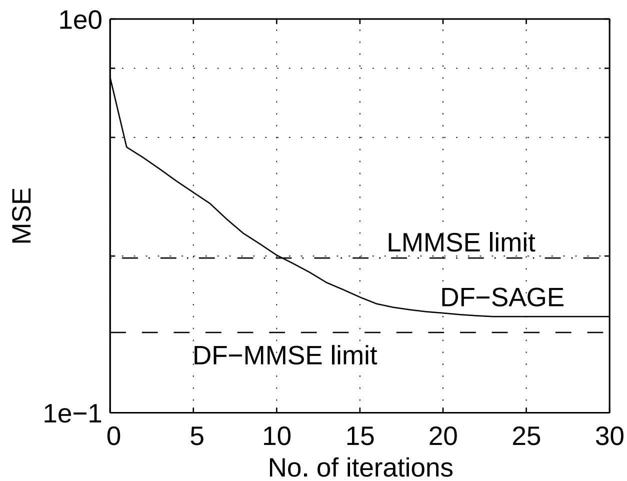

Fig. 3. MSE vs. the number of iterations with SNR=12 dB.

performance of the latter detector is due to the insufficient length of the training sequence.

The convergence speed of the DF-SAGE iterative algorithm is illustrated in figure 3, in terms of the Mean Square Error (MSE), for a SNR of 12 dB. It is observed that a few iterations ($\simeq 20$) are enough to practically attain the minimum MSE.

6. CONCLUSIONS

We have introduced a new semiblind approach to MAI and ISI rejection in DS CDMA. The proposed method uses the ML principle to estimate the coefficients of a DF receiver. It is termed semiblind because it uses short training sequences but also exploits the statistical information of the unknown transmitted symbols and AWGN in the channel. Computer simulations show that the proposed semiblind DF receiver attains the same performance as the theoretical DF MMSE multiuser receiver using short training sequences. Thus, it clearly outperforms linear multiuser receivers at the expense of a very modest increase in computational complexity.

7. REFERENCES

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